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Energetic and Dynamic Performance Enhancements for Compliant Robot Actuation

by

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Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation contains fewer than 65,000 words including appendices, bibliography, footnotes, tables and equations and has fewer than 150 figures.

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Abstract

The vast repertoire of human skills enable nimble and graceful execution of several habitual tasks. On the contrary, despite many advances, robots are still far from matching human capabilities. Robots can be envisioned to take up the oft-touted dull, dirty and dangerous humans jobs only with breakthroughs in their energetic and dynamic characteristics. A typical bio-inspired approach that can potentially help robots achieve the aforesaid performance aspects is joint compliance. Compliant actuation technology has vastly diversified in the last decade. A survey of the various actuator methodologies hints at possible advances for series-clutched and series-parallel multi-articulated actuation. To address them, the thesis builds upon these existing actuation concepts and focuses on specifically improving their energetic and dynamic performance characteristics.

Energetic benefits of series elastic actuation is often marred by gearbox friction. An instance is when friction dissipates the link energy impeding gravity-driven link motions. At such instances, clutches can help stem undesired power-flows by decoupling the link and motor. In addition, when natural link motions are to be damped while not driving the actuator, clutches can be used to actively exploit slippage to dissipate the excess mechanical energy. Such a continuous clutching action has significant implications for energy economy. Therefore, first contributions of this thesis is towards deriving an energy-based model and an optimal controller for a series clutched actuator. In addition, an optimization-based approach is sought to obtain design parameters for a physical implementation of the actuator.

Parallel actuators can often augment series elastic actuators as secondary torque sources. Owing to their energy storage capabilities, they have been used to greatly enhance robot energetics. However, the joint torque resolution problem when employing dissimilar (series and parallel) actuators is difficult, more so when the parallel actuators are biarticulated. Therefore, a second contribution of the thesis is towards deriving an energetic criteria-driven, torque resolution controller. An energetic analysis of the various criteria predicts that allocation of maximum possible torque demand to the higher efficiency actuators may not necessarily be the best strategy at all times. The analysis when extended to a wider range of motion frequencies predicts progressively lesser utilization of parallel actuators can contribute to higher energy-economy.

Through energy-recycling, mono and biarticulated parallel compliance can amplify jumping performance of series elastic actuated robots. While the principle augments robot performance from the mechanical domain, joint velocities powered by series actuators yet suffer from limited voltage. Field weakening is applied at the electrical level to alleviate voltage constraints. In order to maximize energetic economy during such highly dynamic motions, trajectory optimization is further employed with knowledge of actuation capabilities and novel power constraints. Therefore, the confluence of these methods is proposed and experimentally demonstrated to significantly enhance jumping performance. Finally, the efficacy of these concepts towards enhancing explosive motions are quantified through a

centroidal dynamic manipulability analysis. These results lead to a third broad contribution from this thesis.

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Nomenclature

Abbreviations

ACA	Asymmetric compliant actuation	MTPA	Maximum torque per ampere
CoM	Center of mass	NLP	Nonlinear Programming problem
CoP	Center of pressure	OCP	Optimal control problem
DCM	Dynamic manipulability of CoM	PB	Power branch
DME	Dynamic manipulability ellipse	PEA	Parallel elastic actuator
DoF	Degrees of freedom	QP	Quadratic programming
DSP	Digital signal processing	SCA	Series clutched actuator
ESB	Energy storage branch	SCD	Stance CoM displacement
FCAP	Feasible CoM acceleration polygons	SEA	Series elastic actuator
FW	Field weakening	SPEA	Series-parallel elastic actuator
GMS	Generalized Maxwell-slip	SPMSM	Surface permanent magnet synchronous motor
iLQR	iterative Linear Quadratic Regulator	TO	Trajectory Optimization
LCP	Linear complementarity problem		
MELS	Maximize ESB load share		
MPC	Model-predictive control		
MR	Magneto-rheological		
MSLC	Minimize sum of squared load currents		
MSPL	Minimize sum of load power losses		
MSTP	Minimize sum of total powers		

Symbols - Clutched actuator

α	Cone clutch angle	d_i	Cone clutch inner diameter
$\dot{\phi}$	Slip velocity	d_l	Link damping coefficient
η	Cone clutch efficiency	$e_{(i)}$	effort at port i
μ_C	LuGre Coulomb friction coefficient	F_a	Clutching Force
μ_s	LuGre Static friction coefficient	$f_{(i)}$	flow at port i
μ_{st}	Sticking friction coefficient	g	LuGre friction conductance
σ_0	Bristle stiffness	H	Hamiltonian
σ_1	Bristle damping	I_1	Driving motor inertia
σ_2	Viscous friction coefficient	I_2	Clutching motor inertia
τ_c	Clutch torque	I_l	Link inertia
τ_F	Friction torque	J_c	Cost function
τ_g	Link gravity torque	k_τ	Torque constant
τ_m	Motor electromotive torque	k_m	Motor constant
τ_{ext}	External torque	K_r	Clutch geometry coefficient
$\tau_{m,1}$	Driving motor torque	K_s	Spring stiffness
$\tau_{m,2}$	Clutching motor torque	N_c	Ballscrew reduction ratio
$\mathbf{G}$	Input mapping matrix	P_L	Resistive power loss
$\mathbf{J}$	Interconnection matrix	q_1	Driving motor coordinate
$\mathbf{R}$	Dissipation matrix	q_2	Clutching motor coordinate
$\mathbf{u}$	System input	q_l	Link coordinate
$\mathbf{x}$	Generalized coordinate	q_s	Ballscrew coordinate
$\mathbf{y}$	System output	R_L	Motor winding resistance
C	Driving motor Coulomb coefficient	v_s	Stribeck velocity
d_1	Driving motor damping	z	Bristle deflection

Symbols - *eLeg*

		$\mathbf{J}_c$	Center of mass Jacobian
ΔT	Optimization phase durations	$\mathbf{J}_q$	Joint torque - joint acceleration mapping
Δt	Time step duration	$\mathbf{M}$	Joint space inertia matrix
Δ	Tendon deflection	$\mathbf{n}$	Joint space bias forces
Λ	Set of friction cone feasible forces	$\mathbf{P}_M$	Null-space projection of contact Jacobian
$\mathcal{W}$	DME peripheral coordinate	$\mathbf{S}$	Selection matrix for SEA
μ	Ground static friction coefficient	$\mathbf{T}$	Parallel articulation topology
ψ	Motor flux linkage	$\mathbf{W}_\tau$	Weighing matrix
τ^*	Joint reference torque	i_d	Direct current
τ_d	Power branch friction torque	I_l	Link inertia
τ_e	External torque	i_p	Current from Energy Storage Branch
τ_g	Gravity torque	i_q	Quadrature current
τ_p	Torque from Energy Storage Branch	i_{max}	Maximum available current
τ_m	Power branch motor torque	i_{PB}	Current from Power Branch
τ_{PB}	Torque from Power Branch	J	Cost function
θ	Motor coordinate	k_G	Robot centroidal angular momentum
$\tilde{\Delta}$	Tendon deflection approximation	K_{PB}	Power branch motor stiffness
λ	Foot contact forces	K_p	Tendon stiffness
$\mathbf{B}$	Selection matrix	L	Winding inductance
$\mathbf{c}$	Center of mass coordinate vector	N	Gear ratio
$\mathbf{F}_p$	Tendon force	n_p	Number of poles
$\mathbf{J}$	Contact Jacobian	p	Pretension coordinate
$\mathbf{J}^+$	Joint-space inertia weighted pseudoinverse of contact Jacobian	P_e	Joint electrical power
$\mathbf{J}_\tau$	Joint torque - CoM acceleration mapping	P_m	Joint mechanical power
		q	Joint/Link coordinate
		q_a	Actuated joint coordinate

R_p	Energy storage branch motor resistance	t_{ij}	Topology matrix element
R_{PB}	Power branch motor resistance	v_d	Direct voltage
t_0	Initial time	v_q	Quadrature voltage
t_f	Final time	v_{max}	Inverter voltage
$t_{i,f}$	Terminal time of phase i		

Notes on notation

In this thesis, bold symbols denote matrices. Lower-case bold symbols denote uni-column matrices or vectors while upper-case bold symbols denote multi-column matrices. Non-bold symbols denote scalar quantities.

Chapter 1

Introduction

THE last decade has witnessed a drastic shift in the application domains of robots - from heavy-duty industrial sectors involving repetitive tasks to service-based sectors involving an array of versatile tasks. Common contemporary use-cases for service robots include locomotion, rehabilitation, object manipulation, human-interaction and assistive tasks. In particular, robot locomotion features multi-contact scenarios that involve unexpected collisions in unstructured environments. Legged robots require enduring autonomy to traverse distances without human intervention. Such advances in robot loco-manipulation skills have led to pressing requirements such as agility, energy-efficiency, power-density, interaction safety, locomotion stability, etc. A key concept that potentially helps attain the aforesaid qualities is *compliance* or *elasticity*.

Simultaneously, robotics actuation technology has seen a radical departure from stiff to compliant actuators ever since Pratt [1] first conceptualized series-elastic actuation by embedding elastic element in the drive-train. Series-elastic actuators (SEA) exhibit several key characteristics including energy-efficiency [2, 3], high-power density [4], high-fidelity torque regulation and disturbance rejection [5, 6], interaction-safety [7, 8] and peak performance behaviours [9, 10] that render them suitable for service robotics. Consequently, SEAs have evolved to be the core drive-units of several contemporary high-performance articulated robots [4, 11–14]. Rapid advances in robotic actuation technology have also spawned actuators with specialized functionalities such as variable stiffness [15, 16], variable damping [17] and variable transmission drives [18]. Clutched actuators employ locks/clutches in the drive-train to endow safety and backdrivability [19, 20].

Another broad class of elastic actuators that have attracted attention only recently is parallel elastic actuation (PEA). Owing to their enormous energy-efficiency benefits, they have found usage in lab-setups [21], walking or jumping monopedes and bipedes [22–24], prostheses [25, 26] and rehabilitation robots [27]. Moreover, in biological systems, muscle units can span multiple joints and enable energy-efficiency and power transfer benefits between the spanned joints [28–30]. Such biomimetic inspiration has led to multi-articulation of parallel elastic actuators [31, 32] resulting in higher energy savings during locomotory tasks.

Previous research including [33, 34] have investigated suitable combinations of different actuator types to extract their synergetic benefits. In general, choice of a combination of distinct actuator types depends on the particular application. Fig. 1.1 illustrates a non-exhaustive classification of the family of electrical actuators.

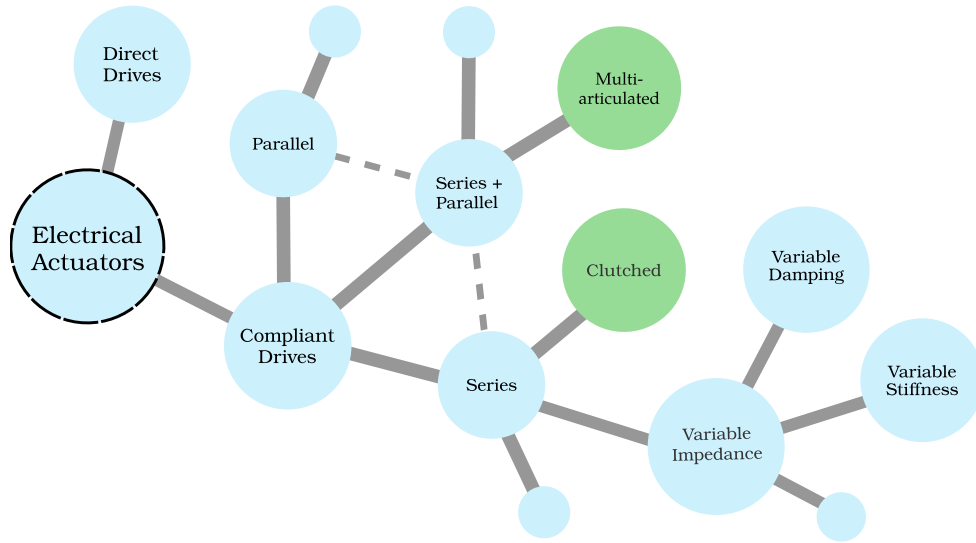


Figure 1.1 Non-exhaustive taxonomy of a family of robotic electrical actuators.

1.1 Motivation

While categorizing robotic tasks or motion phases, it is often meaningful to think in terms of mechanical and electrical power flows. A phase diagram between electrical and mechanical powers can help identify potential areas of improvement for actuators' performance. Such a phase diagram is visualized in Fig. 1.2.

In the first quadrant, energy supplied through external power sources can be stored and released within compliant elements. Such energy recycling not only brings about energy-savings but also leads to agile and highly dynamic motions. Energy recycling has been significantly exploited with SEAs and PEAs in a lot of works [2, 22]. However, their combined usage along with multi-articulation poses several open questions and has not been thoroughly explored. In the second quadrant, when electrical power is employed for braking, the gravity potential driving the link can be actively regulated by means of engaging or disengaging the link from the motor [20]. This functionality is enabled by continuous clutching action and is yet to be explored.

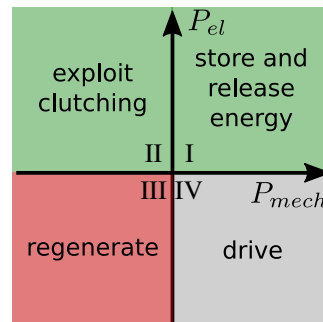


Figure 1.2 A phase diagram of mechanical-electrical power flows with potential use-case scenarios.

The focus of this thesis is on improving the employability of actuators in the aforementioned scenarios, see green shaded areas in Fig. 1.1. In particular, energy-efficiency and explosive motion

benefits of continuously clutchable actuation, series-parallel compliance and active multi-articulation are investigated.

1.2 Contributions

Clutched actuation

Conventional actuators have to overcome intrinsic friction during all motion phases. However, link energy is fully or partly dissipated by actuator friction during motion phases that can benefit from gravity potential, prohibiting the exploitation of natural dynamics. Continuous series clutching action provides the possibility of blending both forced (actively driven) and natural (decoupled motor and link) dynamics.

To this end, port-Hamiltonian framework is used to model a series clutched actuator. A Dirac structure for the clutch component is derived that connects with the various elements - actuator(s) (control port), a dynamic friction model (dissipation port) and a link (interaction port). An iterative Linear Quadratic controller is proposed for torque control action of driving and clutching actuators within the series clutched actuation mechanism. Model-predictive trajectory tracking experiments with randomized parametric variations verify the efficacy of the controller in simulation. Further, a design optimization method is proposed for choosing the various components of the particular actuation mechanism. The corresponding electrical energy savings with the chosen components confirm that the mechanism is energetically more efficient than a conventional non-clutched actuation unit.

Series-parallel compliant multiarticulated actuation

The combination of series and parallel compliant actuators allow for synergetically combining their individual benefits. When combined in asymmetric multiarticulated branches, substantial energy savings were obtained in a monopod robot for various squat motions by allocating the maximum load to parallel actuators [35, 36]. However, their energetic benefits can be further maximized by resolving the joint torques among the redundant actuators based on criteria that aim to minimize electrical energy expenditure.

To that end, a controller is proposed in the quadratic programming framework. The controller is modular such that it can accommodate any quadratic criteria, linear system constraints and actuator hardware specifications. For the same monopod, joint torque references are computed online for series and parallel actuators with distinct articulation topologies. Through an energetic analysis, the controller predicts existence of distinct torque resolution strategies for differing periodic motions. These results are derived from electrical energy expenditure calculations. Over a range of frequencies for a particular periodic motion, the control framework also predicts progressively lesser utilization of parallel actuators to be a better strategy than maximum torque allocation to the higher-efficiency parallel actuators. These results indicate the influence of motion profiles (gravity-driven phases) and motion frequency over choice of a torque resolution strategy.

The explosive motion benefits of combining series and parallel actuators on robots of sizeable mass have been sparsely investigated. Parallel actuators in general, possess higher energy storage capacity which can be exploited for providing short bursts of energy. At the same time, gearbox reductions and limited operating voltage restrict velocities of series actuators and hence the joint output. A sizeable

relaxation of the voltage constraints is provided by field weakening thereby allowing for higher joint velocities. When coupled with parallel articulation, the extent of this relaxation can be enhanced further.

Accordingly, first demonstrations of multi-joint field weakening lead to enhanced jumping motions on a monoped robot weighing 28 kg. Hand-tuned jumping motions confirm that *parallel actuation*, *biarticulation* and *field weakening* can enhance dynamic motion performance significantly. In order to concurrently realize energetic benefits, motion optimization is conducted with a joint-space dynamics model to maximize jump height. Experiments confirm improved energetics and peak performance benefits when utilizing the aforesaid techniques over purely series elastic actuated jumping.

In order to assess the benefits of the aforesaid three methods towards enhancing robot acceleration capabilities, the concept of centroidal dynamic manipulability is utilized. Definition of this manipulability measure is first extended to parallel actuators and their unique ability is identified. An analysis of the robots' explosive capabilities using this measure indicates incremental enhancements with the three methods. Further, an exemplification of applying this measure to optimize for biarticulation design parameters, supplements the main contributions from this thesis.

Publications

The resulting publications from this thesis are included below.

- **V.D. Amara**, J. Malzahn and N. Tsagarakis, "Enhanced energy-efficient optimal jumping motions of a series-elastic monoped with parallel multi-articulation and field-weakening" in *Transactions on Mechatronics* (under review), IEEE.
- **V.D. Amara**, J. Malzahn, W. Roozing and N. Tsagarakis, "Blending of series-parallel compliant actuation with field-weakening control for explosive motion generation" in *Robotics and Automation Letters (RA-L)*, IEEE, 2021.
- **V.D. Amara**, J. Malzahn, Z. Ren, W. Roozing and N. Tsagarakis, "On the efficient control of series-parallel compliant multiarticulated robots" in *Robotics and Automation (ICRA), IEEE Conference on*, IEEE, 2020.
- J. Malzahn, **V.D. Amara** and N. Tsagarakis, "Continuously controllable series clutches for efficient robot actuation", in *Robotics and Automation (ICRA), IEEE Conference on*, IEEE, 2018.

1.3 Organization

This thesis is organized as follows.

- Ch. 2 presents a survey of the state of the art methods and techniques and identifies potential open questions to be probed further.
- Ch. 3 introduces the principle of continuous clutching and derives an energy-based model for a generalized series clutched actuation methodology. An optimal controller for the same is obtained and considerations for various series-clutching implementations are provided.
- Ch. 4 focuses on the problem of joint torque distribution among dissimilar actuators in redundantly actuated series-parallel multiarticulated robots.

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- Ch. 5 demonstrates the blending of parallel multiarticulation branches and field weakening control to substantially enhance explosive motion potential of series-elastic actuated robots.
 - Ch. 6 further enhances the explosive potential of series-parallel multiarticulated robots by integrating motion optimization with field weakening operation. This particular improvement leads to simultaneous energetic and dynamic performance benefits.
 - Ch. 7 quantifies the benefits obtained through field weakening and parallel (multi)-articulation towards enhancing explosive motions. An extension of the concept of dynamic centroidal manipulability is provided to parallel actuators. Further, their implications are discussed for biarticulation design.
 - Ch. 8 concludes by listing out the main contributions from the thesis.

Chapter 2

State of the art

In this chapter, a broad overview of the state of the art is presented for energetic and peak performance aspects of robotic actuation. Subsequently, research gaps are identified which leads to potential thesis objectives.

2.1 Energetic economy of biological systems

Humans and animals are proficient at displaying endurance while performing locomotion tasks over prolonged periods of time. [37] was the first work to propound compliance to be chiefly responsible for energetic economy of humans and animals during terrestrial locomotion. It was also observed that vertebrate locomotion involves change in gaits with varying locomotion speeds [38]. [39] verified through energy consumption measurements that horses change their running gaits based on the locomotion speed such that metabolic cost of transport is maintained at a minimum. Through a series of experiments, [40] predicted that humans change leg stiffness with varying terrains to maintain similar running mechanics such as contact force profiles and stride frequencies. At a higher level, these works suggest a coupling between energy consumption, running speed and gait pattern in vertebrates. At a lower level, principles including energy storage and release [3], variable stiffness [40], multi-articulated muscle-tendon complexes [30] and natural dynamics exploitation [41] have all been cited to contribute to energetic economy of biological systems.

2.2 Energetic economy of robots with compliant actuation

With a view towards obtaining enduring untethered robot operation, several bio-inspired insights have been applied to robots. [42] presents a comprehensive review of various efficiency inducing principles of robot locomotion including series or parallel compliance, impedance regulation, limb segmentation and natural dynamics utilization.

2.2.1 Fixed series and parallel compliance

Research efforts have focused on developing SEAs to bridge the gaps between human and robot endurance [43]. Compliance in the drive-train protects the system from shock impacts as well as safely decouple heavy actuator inertia from the link-side. On the other hand, in spite of their usually higher

energy-storage capacity, PEAs have garnered comparably lesser attention. [44] theoretically explains that parallel compliance with backdriveable actuators can lead to highly energy-efficient locomotion. [45] was the first to demonstrate significant joint torque reduction while employing parallel passive springs at the knees of a humanoid robot. In the same vein, [46, 22] employed springs in parallel to improve efficiency during bipedal walking at various limb joints such as hip, knee and ankle. These parallel springs can compensate for part of the load torques reducing peak torque requirements at the primary motors. [21] augmented the torque capacity of primary motors through passive springs for executing certain motion trajectories. [47] explored the same principle in prostheses to achieve reduction in metabolic energy while walking. [27] went a step further and designed cam-based nonlinear transmission elements to mimic human joint-torque profiles for ankle prostheses.

[48] compared the relative benefits of SEA, PEA and their combination (SEA+PEA) at the ankle joint for walking motions. [49, 50] have performed the same comparison for cyclic motions under gravity. [49] optimizes for energetic benefits analytically considering both motion trajectories and actuation parameters. [50] optimizes for distinct cost functions to present an extensive simulation study for 1-D and 2-D hopping with both series and parallel compliance. Through a simulation study, [51] explains which compliance topology is more effective during cyclic tasks with a single joint. In general, energetic benefits of series compliance is higher when compliance is properly tuned while parallel compliance can provide added benefits during changes in operating point when its equilibrium position is properly tuned. The work of [52] suggests that the relative benefits of PEAs and SEAs are a consequence of the application.

It is still unclear on how to combine the two actuator classes for better energy-efficiency. Only recently, have a few works explored the symbiosis of SEAs and PEAs to draw out their combined benefits. [34] variably recruited several series-parallel elastic actuators (SPEA) placed in parallel based on the application. [53] placed additional locking mechanisms and switches to attain energy reduction for distinct motion profiles. To prevent the counter-action of serial and parallel actuators during certain motion phases, [47, 25] employed unidirectional parallel elements. For the same purpose, [20, 54, 55] employ clutches to disengage the parallel elements at certain motion phases. In [56], the authors demonstrated a method to synergistically employ SPEA to obtain energy savings. A highly asymmetric joint unit that comprises of a high power SEA and an energy-efficient, high energy-storage capacity PEA was conceived. During periodic motions, SEA acts as a primary torque supply while PEA cancels steady-state gravitational loads [33]. The same joint concept on powering a 3-DoF monopod led to significant energetic benefits [57].

2.2.2 Variable compliance

In general, it is difficult to execute distinct motion profiles that are all energy-efficient with fixed compliance. Variable compliance provides an additional actuation degree of freedom to not only achieve higher efficiency than fixed compliance systems but also impart a sufficient level of versatility to legged robots [58]. [59] analytically optimizes link excitation profiles in a variable stiffness joint in order to obtain maximum potential energy-recycling. The principle is analytically extended to variable visco-elastic joints in [60]. The authors of [61, 62] model variable-stiffness actuators (VSA) using the port-Hamiltonian framework and infer kinematic properties that render VSAs energy-efficient among the various designs. [63] derives a measure that determines how efficiently power is transmitted from the controller to the link motion through the internal mechanisms in a VSA. [64] employed VSA at the knee

joint of a 2-DoF hopping monopod to increase its energy-efficiency. In addition, efficiency is ensured through stiffness variations even across a range of hopping stride lengths and frequencies. To sum up, variably stiff actuators can go beyond actuators with fixed compliance in terms of energy-efficiency. However, the added design complexity and increase of control variables lead to added difficulties when extending to multi-DoF systems.

2.2.3 Fixed compliance with clutches

Backdrivability is a key component of energy-economy for legged robot locomotion [65]. While SEAs can be exploited to make robots energy-efficient [66], high torque-capacity SEAs are typically not backdrivable. The friction torque stemming from SEA gearboxes of huge transmission ratios remain, not physically decoupled from the link, impeding comfortable backdrivability. A possible solution is to decouple the link from the actuator temporarily by embedding clutches between them.

In addition to backdrivability, clutchable actuators can actively regulate the amount of actuator torque transmitted to the load and improve system safety. Besides, clutches can lead to energy savings by enabling purely gravity-driven motion phases. During unforced phases of periodic motions, joints can profit from gravity potential if decoupled. During forced motion phases, actuators can remain engaged to drive the links. In general, clutches are appealing due to their unique capability of blending natural and forced dynamics.

Accordingly, [19] implemented adjustable torque limiters at the joints of a 3 degrees-of-freedom (DoF) robot. In [67, 68], a 3-DoF manipulator was designed and developed with magneto-rheological (MR) fluid based clutchable joints. In general, MR clutches are complex to design and operate and have typically lower torque-weight ratios. [20] demonstrated bimodal dynamics by employing a clutch in parallel with the actuator of an SEA. When applied to a knee-prosthesis, the mechanism led to electrical energy savings of about 70%. [69] evaluated safety aspects of a series-clutched actuator during motions involving collisions.

[54] employed a clutchable parallel spring demonstrating substantial reductions in peak torque and electrical energy consumption. [24] utilized a binary clutch in a 2-DoF monopodal robot during its hopping motions to conserve energy. Through bidirectional clutching of a parallel spring in a hydraulic differential mechanism [55] demonstrated significant energy savings during simple joint motion tasks. [70] presents a taxonomy and an almost exhaustive review of clutched actuator prototype designs.

Clutched actuators that could serve as torque limiters can also display variable-damping characteristics. Variable damping mechanisms/actuators can mechanically exhibit a range of damping coefficients [17, 71–73]. A detailed review of variable damping actuators can be found in [74].

2.3 Explosive motions of biological systems

Several day-to-day human/animal tasks involve ballistic or athletic motions such as running, jumping, throwing, kicking, etc. Extreme examples of mammalian motions involve joint velocities far higher than contemporary peak SEA speeds. For instance, the shoulder joint of a baseball pitcher exceeds 170 rad/s [75] and the peak value of a cheetah's hip joint velocity averaged over a single stride exceeds 53 rad/s [76]. Such highly dynamic motions are typically accomplished by exploiting the natural dynamics of limb segments. Specifically, these excitations are brought about by short cycles of energy storage and

power bursts. During such cycles in multi-DoF systems, centroidal or end-effector powers momentarily peak to maximum values.

Furthermore, during explosive human motions, one can observe a sequential extension motion from proximal to distal joints, termed as joint coordination in literature [77]. This synchronization is accompanied by sequential transfer of joint/link power from proximal to distal joints through these multiarticulated muscle-tendon structures [30, 78]. Therefore, they play a major role in obtaining these short bursts of energy/power at the end-effectors. To summarize, principles including parallel muscle mechanisms, varying joint angle-stiffness profiles and proximal-distal joint power transfer play a major role in maximizing dynamic performance in biological systems.

2.4 Explosive robot motions with compliant actuation

Joint/link velocity maximization is a scarcely researched topic in robotics. While gearbox transmissions help scale up torque output, they also scale down velocity output considerably. In addition, substantial gearbox friction further impairs obtaining peak velocities. These effects result in typical peak SEA output velocity values to range from 5-15 rad/s depending on the gear ratio. On analysing the actuator hardware, two constraints that limit peak joint velocities include *torque constraints* at the mechanical level and *voltage limits* at the electrical level.

2.4.1 Exploiting mechanical dynamics

In addition to energetic benefits, compliance can also bring about enhancements to explosive motion capabilities. Through energy recycling, it has been shown that link velocity can far exceed the motor velocity by appropriate excitation of the elastic spring. To this extent, in constant stiffness joints, it has been shown that such high velocities are achievable by spring-exciting limit cycles which are related to the eigenfrequencies of the link. Variable stiffness further enhances this speed amplification behaviour by changing spring stiffness during motion execution [79, 9]. The same principles have been applied to obtain optimal input switching conditions for actuators with variable damping [60] under a variety of inputs/control schemes - position, velocity and torque. In [80] and [81], the optimality principles that lead to a fixed-time and fixed-position performance maximization were studied numerically. A synchronization is optimally obtained between spring stiffness profile, spring equilibrium position and link velocity. Thus by varying the spring stiffness during a swift motion of the link, the tip velocity is substantially enhanced.

2.4.2 Exploiting electrical dynamics

Another approach, chiefly used to increase velocity of speed-controlled industrial drives is called field-weakening or flux-weakening (FW). FW exploits the electrical motor dynamics to decrease the back-EMF and subsequently the voltage requirements for a given velocity, therefore higher velocities can be attained with the same supply voltage constraints [82]. Industrial FW control has garnered a lot of attention with researchers resorting to a wide range of techniques including model-free and model-based methods.

[83] proposes to use a look-up table for sets of electromagnetic torque and velocity based on operating regions to determine field weakening current references. [84] uses reactive power feedback

control to account for parametric variations in rotor flux linkage. Online estimates of reactive power feedback can be used to estimate inductance variations and plan for feedforward current references.

[85] applies optimal control to extract the maximum power from motors of varying rotor topologies through flux-weakening. Further, this is done while the control technique is independent of system parameter variations. [86] combines model knowledge with a disturbance observer to self-tune electrical parameters and maximize FW performance. [87] involves a voltage constraint-tracking method to perform FW operation while maximizing efficiency. An analysis for FW performance-motivated design selection is presented in [88]. The authors also apply overestimated inductance values to obtain simultaneous maximum possible torque and FW operation.

[89] discusses about the thermal effects of field-weakening including lower torque output at elevated temperatures for different rotor topologies. [90] presents a broad review of several state-of-the-art FW control methods. A fairly simple inverse electrical dynamics approach is presented in [91] for torque controlled SEAs. The particular methodology led up to 33% peak velocity enhancement on a high torque capacity SEA.

2.5 Role of multi-articulation in robot actuation

Vertebrate muscle architectures often exhibit parallel biarticulation and more generally, multiarticulation. [28–30] have all probed the role of biarticulation towards energetic and peak-power benefits during mammalian motions. Inspired by the principle of biarticulation, many researchers have demonstrated the added benefits of biarticulating robot joints towards balancing, force regulation, joint synchronization and energy reduction during squatting and bipedal walking. [92] explained that biarticulation significantly improves the end-effector's force ellipsoid. In legged locomotion, this amounts to better contact force regulation capabilities which has implications towards balancing and other dynamic motions.

[93] used monoarticular and biarticular muscles to obtain inter-link power transfer that facilitates better control during swing-leg function. [32] employed biarticular springs during walking and jogging motions of a miniature bipedal robot. Biarticulation was able to better explain contact force profiles observed during human walking motions. [94] expounded the design principles of a highly parallel articulated biped and experimentally verified improved foot contact force regulation properties owing to biarticulation. Both works [32, 94] observe better self-stabilization properties useful for balancing. [35, 36] elaborated the series-parallel asymmetric principle with biarticulation to obtain higher energy-savings and joint tracking properties. When biarticulated, the parallel actuators could better compensate for gravity torques as a result of inherent mechanical coordination.

In addition, [95, 96, 78] particularly suggested the phenomenal role of power transfer brought about by biarticulated muscles during explosive leg extensions. [95] explained that while monoarticular muscles usually acts as power sources during explosive motions, biarticular muscles are chiefly responsible for routing joint powers from proximal to distal elements. Through human jumping experiments, [96] mathematically expounds the unique role of biarticulation. Biarticulated muscles amplify the end-effector velocity several-fold when compared to only monoarticular muscles.

[97, 98] verified these observations in robotic prototypes during vertical jumping motions. The biarticulated knee joint of a pneumatically actuated jumping robot helped achieve power-transfer in a hopping robot [97]. [98] used a serially driven robot with an additional biarticular spring that can be activated at will to gauge the performance enhancement over only serial actuation. An extensive

experimental study in a mini monopod reveals optimal stiffness, activation timing and pretension values for the biarticular spring during jumping. [23] developed a robot solely actuated by parallel monoarticular and biarticular actuators to verify their aforesaid distinct roles.

A detailed study that discusses the role of various biarticular structures towards joint coordination during monopod jumping can be found in [99]. On the particular robot, biarticular muscle that leads to ankle extension contributed lesser to joint coordination than the one that leads to knee extension. This particular observation can be attributed to the higher amount of transferred power between heavier links (hip-knee) than lighter links (knee-ankle).

All these works suggest that multiarticulated PEAs can lead to higher energy savings than when merely employing them in monoarticulation. To conclude, significant energetic benefits of up to 70% have been reported in literature while using PEAs (in biarticulation) either in isolation or in combination with SEAs. Concurrently, the same principle leads to improved peak performance resulting from enhanced joint coordination/power-transfer.

2.6 Motion optimization for energy-economy and peak-power generation

Methods for numerically optimizing robot movement abound in literature. Popular approaches include i) *shooting methods* [100, 101] - optimization of control input profiles, ii) *collocation methods* [102, 103] - optimal state and input solutions through trajectory parametrizations and iii) *hybrid dynamics* approaches [104], [105] - repeated recursions of forward and inverse dynamics.

On one hand, shooting and hybrid dynamics methods aim to perform an accurate integration of the system dynamics at every time-step. Composing motion optimization problems with contact transition scenarios in either of the approaches may require a linear complementarity problem (LCP) formulation [106] or time-stepping schemes [107] to accurately describe contact mechanics. In general, they make optimization problem formulation as well as its numerical convergence difficult. Applying hard configurational constraints in shooting methods are either impossible or not straightforward [108].

On the other hand, collocation methods trade-off dynamics accuracy for computational efficiency [109] since they do not require numerical integration. Optimality is achieved by minimizing difference between parametrized state trajectories and dynamic constraints. The same allows for imposing contact variations with different phases, which makes it well-suited for locomotion tasks. For problems where obtaining state trajectories beforehand are difficult, collocation methods are generally attractive. Moreover, configurational constraints can be posed fairly easily with collocation methods.

Appropriate choice of model complexity for optimization is equally important as the choice of an optimization method. Template-based models such as [110, 13] have been very effective in obtaining running and hopping motions on multi-DoF systems. Lumped mass models with full kinematics such as [111, 112] are frequently used in online optimization owing to faster convergence when compared to whole-body models. However, these reduced models assume negligible limb inertia/dynamics and operate with apriori unknown dynamic constraints on center of mass or trunk. In general, configurational limits, velocity, torque constraints can only be posed at the whole body level. Consequently, the full performance potential usually goes untapped when reduced models are used due to suppression of natural link dynamics. This is especially important when task objectives demand peak performance behaviours

or optimally spring-exciting motions. [41] discusses the importance of considering segmented limb dynamics and their implications to improved energetics and peak-performance.

[113] conducts an optimization-based simulation study on a 3-DoF robot powered by serial actuators and a biarticular spring. The results lead up to a sensitivity analysis of jumping height to various biarticulation parameters. [105] extends the same work to a biarticular muscle that can be engaged or disengaged at will. A hybrid-dynamics approach [104] that involves repeated iterations of forward and inverse dynamics routines is adopted. Numerical motion optimization is conducted for both vertical jumping and forward hopping. [98] applies these optimal motions on a miniature monopod to verify the simulation results as well as the sensitivity study. Results indicate that poor timing of biarticular spring activation may even lead to poorer jumping performance than when jumping without the biarticular spring. [114] identified distinct optimal jumping motions for various actuation configurations on an asymmetric series-parallel multi-articulated monopod. The joint motion trajectories are parametrized as B-splines and a parameter optimization routine is used to maximize jumping height using hybrid-dynamics approach in simulation.

Exploitation of intrinsic resonance modes in compliant systems is integral for energetic economy and peak power performance. A body of works that indirectly amounts to motion optimization for peak power generation exploit the variable stiffness property in VSAs. [9] analytically obtain optimal motion profiles with respect to practical constraints on joint position, velocity and torques. Elastic energy stored in variably stiff springs are maximized through oscillatory motions which then culminate into a peak tip velocity. The analytical study is supported through ball throwing experiments on an multi-DoF arm while employing the controller on 2-DoFs. The elastic robot turns out to be 50% faster than a equivalent stiff robot. [115] uses an iterative-Linear Quadratic Regulator (iLQR) to obtain optimal impedance and motion profiles for a 2-DoF variable impedance robot. The method is able to exploit impedance DoF for multiple throwing task objectives.

The authors of [116] involve human subjects to excite extrinsic variably-stiff springs to study nonlinear excitation motion profiles. From the inferred switching-based optimal control laws, [117] employed a controller on a variable-stiffness arm to excite its nonlinear modes and obtain limit cycles. Limit cycles typically involve exciting the link to peak velocities in alternating directions. The theory of eigenmodes that lead to optimal limit cycle excitations extended to nonlinear systems can be found in [118].

2.7 Research gaps

A summary of the literature survey is presented in Tab. 2.1 along with open questions that concern the various topics. The open questions that are dealt with, in this thesis, are detailed below.

Despite their design and control complexities, energy-efficiency benefits of clutched actuator cannot be overlooked. Although, aforementioned works use clutches as torque-limiters, they all do so only while actively driving the link. Some of the works use clutches only as binary locking mechanisms. However, one could also use the actuator's Coulomb friction to actively dissipate the link energy when necessary. A clutched actuation methodology based on this concept was theoretically proven to display substantial energy savings of up to 60% in [119]. Research gaps prevail for such a clutching concept with regards to their implementation, modelling and control.

Open questions remain in understanding how to optimally distribute the desired torques among the primary and the secondary actuator units in SPEA robots. The authors of [92] compared various

Table 2.1 Summary of literature survey

	Summary	Open questions
Fixed compliance actuators	SEAs and PEAs have their own relative merits and drawbacks. A single actuator type cannot be the best suitable type for every robot joint.	The combined usage of SEAs and PEAs can engender several benefits, but poses questions about their relative recruitment and capacities.
Variable compliance actuators	VSAs are aimed at outperforming fixed compliance actuators in terms of energy-efficiency well as peak-power output, in principle.	Their complicated design and control currently discourage their use in large multi-DoF robots.
Clutched actuation	Through disengagement, clutched actuators provide unique benefits over their ever-engaged counterparts.	The <i>partial</i> disengagement of clutches during gravity-driven motions can lead to energy savings; yet this has never been explored.
Multiarticulation	Articulation of multiple adjacent joints serve the unique roles of inter-joint power transfer and enhanced joint coordination.	It is surprising to note that none of the heavy, high-performance robots utilize the principle in their actuation schemes.
Explosive motions	The exploitation of mechanical dynamics towards improving explosive robot performance has been sufficiently probed. Elastic element can in general amplify link velocity to a value higher than its motor velocity.	Exploitation of electrical dynamics in industrial drives is widely in practice. Yet their application in robotics for peak-power tasks (in multi-DoF robots) has not been explored.
Motion optimization	Several methods of varying levels of complexity exist for obtaining optimization-based reference motions aimed at energy-economy or dynamic performance.	Integration of specific actuator features/capabilities into the optimization method without affecting problem convergence remains the major challenge.

torque resolution approaches with *identical* biarticular actuators powering multiple joints in a redundant manipulator. The question still remains open when *dissimilar* actuators of differing efficiencies and torque/power capacities are employed on distinct articulation topologies. The method of torque resolution in [120] to allocate maximum torque demand to the higher-efficiency PEAs led to massive energetic benefits in a 3-DoF monopod [36]. Yet, a question of whether this is the best torque resolution strategy at all times, remains open.

Furthermore, the benefits of integrating serial and parallel actuators (such as SPEAs) towards peak power tasks has been only scarcely investigated. Although the energy efficiency characteristics of PEAs have been amply addressed, their advantage in improving the peak power characteristics of SEA actuated robotic systems has not been thoroughly studied. To date, only [98] has explored the potential of direct-driven series motors and parallel elements towards performance enhancements during highly dynamic motions of a monopod. Moreover, the particular monopod weighed less than 10 kg. Demonstrations of highly explosive motions on SPEA robots that share similar mass, scale and torque-capacity characteristics with contemporary high-performance legged robots is still pending.

Chapter 3

Continuously Controllable Series Clutched Actuation for Energy Efficiency

A brief survey of state of the art from the previous chapter indicates that substantial energy savings can be obtained with series clutched actuation during periodic link motions that exploit natural dynamics. Specifically, a continuously controllable clutch that can exploit the static friction of high torque-capacity actuators is yet to be explored. The author's own work [119] established theoretical energy savings of up to 60% for such a clutched methodology. To realize the same physically, a series clutch that benefits from Coulomb friction *sink*¹ must

- i continuously (dis)-engage the link from the motor without a positive mechanical power-flow² to obtain a damped link motion, during unforced motion phases,
- ii completely disengage the link from the motor for gravity-driven natural link motions and
- iii can maintain an appropriate amount of friction at the clutching interface that allows to transmit a desired amount of torque from actuator to the link and vice-versa.

For instance, when the link is undergoing free, gravity-driven motions, the Coulomb friction of the actuator can be used to dissipate the kinetic energy of the link to obtain a damped motion.

Realization of the described clutched methodology requires an energy-based model to understand the various power-flows between the different system elements. An investigation must be made about the physical implementation of the series-clutched actuator (SCA). The friction torque at the clutch interface must be modulated to exactly transmit the desired amount of torque to the load or dissipate a desired amount of link energy with actuator friction. In other words, continuous clutching aims to actively exploit slipping between the load and actuator. For this purpose, a clutch-friction modulation controller must be derived.

¹ An element/phenomenon that can bring about power dissipation.

² Power-flow is considered positive when power flows from motor to link.

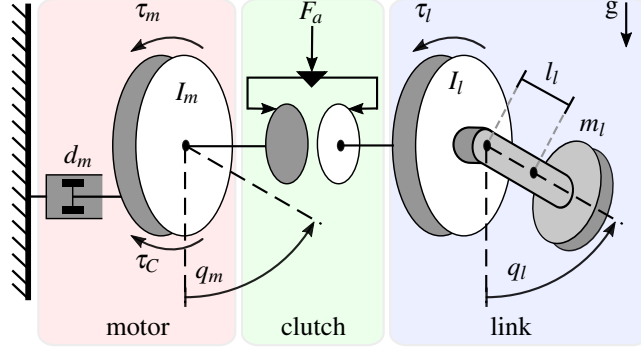


Figure 3.1 Coupled antagonistic compliant joint actuation concept with controllable friction clutch.

3.1 Organization

The chapter is organized as follows. Mechanical dynamics of a generalized series clutched actuator is first introduced in Sec. 3.2. A brief overview of port-Hamiltonian theory and friction modelling framework are presented in Sec. 3.3 and Sec. 3.4. An energy-based model description for the SCA is derived in Sec. 3.5. In Sec. 3.6, an optimal controller is formulated for clutch friction regulation followed by joint-tracking simulations. Sec. 3.7 reviews an antagonistic series-clutched implementation. Sec. 3.8 presents optimization and simulation results for design selection of a compact serial clutched joint methodology. Finally, the chapter ends with a summary in Sec. 3.9.

3.2 Mechanical dynamics

In this section, the dynamics of a SCA that is capable of continuous clutching action is introduced. SCA consists of an actuator that can transmit torque to the load through a clutch which is operated by an additional actuator. A primary torque source is connected to a load through a continuously controllable clutch in between them, see Fig. 3.1. The dynamics of such a clutched actuator are,

$$I_1 \ddot{q}_1 + \underbrace{C(q_1) + d_1 \dot{q}_1}_{\tau_{d,1}} + \tau_c = \tau_{m,1}$$

$$I_l \ddot{q}_l + d_l \dot{q}_l + \tau_g = \tau_c + \tau_{ext} \quad (3.1)$$

where q_1 and q_l are the driving motor and link coordinates. $\tau_{d,1}$ is the actuator friction comprising of Coulomb $C(q_1)$ and viscous components d_1 , τ_c is the load torque transmitted through the clutch which is also equivalent to the clutch friction, $\tau_{m,1}$ is the motor electromotive torque. d_l and τ_g are the damping coefficient and gravity torque of the link, respectively. I_1 and I_l are the motor and link inertiae, respectively. τ_{ext} accounts for any external torque that acts on the link.

Thus by modulating the clutch friction τ_c , the load can be continuously/partially (dis)-engaged from the actuator. Such an operation is energy-efficient, especially during gravity-driven periodic link motions and has shown substantial theoretical energy-savings [119]. Clutch friction modulation is brought about by an additional actuator whose dynamics is described by,

$$I_2 \ddot{q}_2 + d_2 \dot{q}_2 + F_a = \tau_{m,2}, \quad (3.2)$$

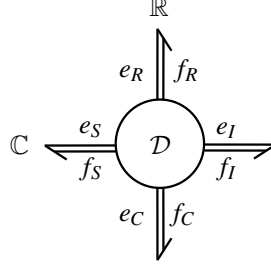


Figure 3.2 General representation of a Dirac structure. $\mathcal{D}$ represents the Dirac structure. $\mathbb{C}$ and $\mathbb{R}$ represent the energy storage and energy dissipation elements. Subscripts S , C , I and R denote storage, control, interaction and dissipation ports, respectively.

where q_2 is the clutching motor coordinate. I_2 and d_2 are the inertia and damping coefficient of the clutching actuator. $\tau_{m,2}$ is the input torque to the actuator. The clutching actuator applies either a force/torque (linear/rotary clutch) F_a through an elastic element k_s to perform the clutching action (bringing the clutching surfaces together).

3.3 Port-Hamiltonian modelling

The port-Hamiltonian framework [121, 122] is the most appropriate for describing series-clutched actuation methodology. The framework has been previously used to model a large variety of physical systems, more importantly, variable stiffness actuators [62]. Moreover, modeling in this framework can provide important guidelines and insights for design improvements and control. The basic theoretical concepts of the port-Hamiltonian framework are first described.

The port-Hamiltonian framework, in general, is based on two concepts: port-based modeling and the Hamiltonian function. Firstly, given any physical system, its network topology can be made explicit by means of a Dirac structure which defines the interconnection between the four fundamental ports of the system [123]. In other words, the power flows through the system's various ports are described by this structure. Fig. 3.2 shows a generic network topology of a Dirac structure. Every port is associated with its power conjugate variables, flow f and effort e whose dual product $\langle e|f \rangle$ is the power flow through the port. The Dirac structure defines the direction and routing of power flows through the various ports, i.e. their interconnection. Sum of all power flows through a power-preserving interconnection must equal zero [124]. Secondly, for every physical system, a Hamiltonian energy function can be defined which is a general measure of the energy stored in the system.

Using the above two concepts, the port-Hamiltonian framework allows for an energy-consistent description of any physical system [121, 122]. This in turn enables the user to monitor the energy exchange between the the various elements of the system and also analyze the passivity property. Power-preserving interconnection of two port-Hamiltonian systems again yields a port Hamiltonian system and this convenient property renders this framework suitable for modeling complex physical systems. An input-state-output form of a linear control-affine port-Hamiltonian system is given by:

$$\begin{aligned} \dot{\mathbf{x}} &= [\mathbf{J}(\mathbf{x}) - \mathbf{R}(\mathbf{x})] \frac{\partial H}{\partial \mathbf{x}} + \mathbf{G}(\mathbf{x}) \mathbf{u} \\ \mathbf{y} &= \mathbf{G}^T(\mathbf{x}) \frac{\partial H}{\partial \mathbf{x}}, \end{aligned} \quad (3.3)$$

where $\mathbf{x}$, $\mathbf{u}$ and $\mathbf{y}$ are the state, input and output vectors of the systems. $\mathbf{J}$ is a skew-symmetric interconnection matrix, $\mathbf{R}$ is a symmetric dissipation matrix and $\mathbf{G}$ is the input mapping matrix. H is the positive, semi-definite Hamiltonian function $H \geq 0$ representing the total energy of the system. The rate of energy change of the Hamiltonian for (3.3) is given by:

$$\dot{H} = \underbrace{\mathbf{y}^T \mathbf{u}}_{\text{Supplied power}} - \underbrace{\frac{\partial^T H}{\partial \mathbf{x}} \mathbf{R}(\mathbf{x}) \frac{\partial H}{\partial \mathbf{x}}}_{\text{Dissipated power}}. \quad (3.4)$$

A more general description of a port-Hamiltonian system also used in this chapter allows for direct-feedthrough of the input at the output and is given by:

$$\begin{aligned} \dot{\mathbf{x}} &= [\mathbf{J}(\mathbf{x}) - \mathbf{R}(\mathbf{x})] \frac{\partial H}{\partial \mathbf{x}} + [\mathbf{G}(\mathbf{x}) - \mathbf{P}(\mathbf{x})] \mathbf{u} \\ \mathbf{y} &= [\mathbf{G}(\mathbf{x}) + \mathbf{P}(\mathbf{x})]^T \frac{\partial H}{\partial \mathbf{x}} + [\mathbf{M}(\mathbf{x}) + \mathbf{S}(\mathbf{x})] \mathbf{u}, \end{aligned} \quad (3.5)$$

where matrices $\mathbf{M}$ is skew-symmetric and $\mathbf{S}$ is symmetric. The interconnection structure is now given by $\mathbf{J}$, $\mathbf{M}$ and $\mathbf{G}$ and the dissipation structure by $\mathbf{R}$, $\mathbf{P}$ and $\mathbf{S}$. The rate of change of Hamiltonian H for this system description is given by:

$$\dot{H} = \underbrace{\mathbf{y}^T \mathbf{u}}_{\text{Supplied power}} - \underbrace{\left[\frac{\partial^T H}{\partial \mathbf{x}} \quad \mathbf{u}^T \right] \begin{bmatrix} \mathbf{R} & \mathbf{P} \\ \mathbf{P}^T & \mathbf{S} \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \mathbf{x}} \\ \mathbf{u} \end{bmatrix}}_{\text{Dissipated power}}. \quad (3.6)$$

As long as the dissipated power in (3.4) and (3.6) is non-negative, the corresponding systems remains passive. One can refer to [121–123] for more details on port-based modeling and the Hamiltonian function.

The various elements of the actuator presented in the previous section are modelled in the port-Hamiltonian framework. For better understanding of this section, a schematic depicting the network topology of the actuator, specifically the ports of the various components and their associated power flows is shown in Fig.3.3.

3.4 Friction modelling

One of the hardest natural phenomena to model is friction. While friction is the tangential opposition between two contact surfaces in motion, it is not yet fully understood. Although, fundamentally a force-velocity relationship, the causality is normal force and friction determines the relative velocity between surfaces. Several approaches to approximate its principal characteristics have resulted in multiple friction models. They can be broadly classified into *static* and *dynamic* friction models.

On one hand, different *static* friction models such as Coulomb [125], viscous, Stribeck [126] and Karnopp [127] models consider an increasing level of complexity that are appropriate for the particular application. While these models can well predict steady-state behaviour, many of them fail to accurately predict stick-slip motions. Fundamentally, these models tend to be empirical constructs. Further, due to inversion of causality, zero-crossover points cannot be detected. Hence these models tend to be

discontinuous and cannot be used to simulate stick-slip scenarios [128]. [129] uses a hybrid modelling approach to overcome the problem of causality in simulation.

On the other hand, dynamic friction models such as Dahl [130], LuGre [131], generalized Maxwell-slip [132] incorporate additional model states to predict effects such as frictional lag, memory, spring-like stiction behaviour and path-dependence (velocity-independence). A detailed review of friction models can be found in [133].

Of the several available models, LuGre and generalized Maxwell-slip (GMS) models are the most attractive for the continuous clutching application. A sensible trade-off is to choose LuGre model owing to the level of fidelity and lesser complexity when compared to the GMS model.

3.4.1 LuGre friction model

LuGre model makes use of deflection force of virtual elastic springs called bristles between the contact surfaces to compute friction. The generalized LuGre model of friction is given by,

$$\begin{aligned}\dot{z} &= \dot{\phi} - \frac{\sigma_0}{g(\dot{\phi})} |\dot{\phi}| z \\ \tau_f &= (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{\phi})\end{aligned}\quad (3.7)$$

where τ_f is the friction force/torque, z is the average deflection of bristles and $\dot{\phi}$ is the relative or slip velocity between the contact surfaces. σ_0 , σ_1 and σ_2 are the bristle stiffness, bristle damping and viscous friction coefficients, respectively. g is a nonlinear conductance function that accounts for the Stribeck effect and is given by,

$$g(\dot{\phi}) = \mu_c + (\mu_s - \mu_c) e^{-\left(\frac{\dot{\phi}}{v_s}\right)^2} \quad (3.8)$$

where μ_c and μ_s are the Coulomb and static friction terms at the clutch interface. v_s is the Stribeck velocity. It is essential to note that the LuGre is dissipative [124] only under the condition,

$$\sigma_1 \leq \frac{4g(\dot{\phi})}{\dot{\phi}}. \quad (3.9)$$

Quite often, σ_1 is taken to be $\sigma_1 e^{-\left(\frac{\dot{\phi}}{v_s}\right)^2}$ to render the friction model (3.7) dissipative. Additionally, in [134], it was shown that the LuGre model remains passive only under certain inequality conditions.

For the application under consideration (SCA), the generalized model shown in (3.7) is slightly extended:

$$\begin{aligned}\dot{z} &= \dot{\phi} - \frac{\sigma_0}{g(F_a, \dot{\phi})} |\dot{\phi}| z \\ \tau_f &= \sigma_0(F_a) z + \sigma_1(F_a) \dot{z} + \sigma_2(F_a) \dot{\phi}\end{aligned}\quad (3.10)$$

where F_a is the normal force acting between the contact surfaces. Assumption of monotonically increasing linear relationship between normal force and coefficients in (3.10) is reasonable to make since higher normal force generally leads to higher friction force/torque. An assumption is made that g can never be zero and varies with contact force,

$$g(F_a, \dot{\phi}) = \mu_c(F_a) + (\mu_s(F_a) - \mu_c(F_a)) e^{-\left(\frac{\dot{\phi}}{v_s}\right)^2}, \quad g > 0, \quad (3.11)$$

where the inequality assumption is made similar to [135]. Again, linear relationship is assumed between coefficients μ_c , μ_s and normal contact force F_a . It is noteworthy that the particular model (3.10) assumes that the bristle deflection z is not sensitive to variations in normal force F_a in a direct fashion. Changes in z due to normal force variations are indirectly brought about by the conductance function g and linear stiffness σ_0 .

3.5 Energy-based model of Series Clutched Actuator

It is often necessary to understand the various power-flows in actuation systems. Control of slip-exploiting SCA can benefit from energy/power flow monitoring between various actuator elements. Therefore, a port-Hamiltonian model of a continuously controllable series clutched actuator is derived. The individual components are first described followed by their interconnection.

Dissipation port

The Hamiltonian for the LuGre friction model described in (3.13) and (3.11) includes the virtual elastic energy stored in the bristles,

$$H(z) = \frac{1}{2} \sigma_0 (F_a) z^2 \quad (3.12)$$

It is important to realize that F_a is not an input variable but only a signal variable that modulates the system's state z and output τ_c . This is because the modulating signal is the result of a constraint in the system and is orthogonal to the friction force/torque developed between the two contact surfaces. Such a modulation by power-free variables external to the port is called a hybrid-resistive port [136]. With regards to the LuGre friction branch, power flow associated with the change of F_a is zero. To model LuGre friction, the general port-Hamiltonian representation with feed-through terms (3.6) is used:

$$\begin{aligned} \dot{z} &= \underbrace{\begin{bmatrix} -\frac{|\dot{\phi}|}{g} \end{bmatrix}}_{-\mathbf{R}_z} \frac{\partial H_z}{\partial z} + \underbrace{\begin{bmatrix} 1 \end{bmatrix}}_{\mathbf{G}_z - \mathbf{P}_z} u_z \\ y_z &= \underbrace{\begin{bmatrix} 1 - \frac{|\dot{\phi}|}{g} \sigma_1 \end{bmatrix}}_{\mathbf{G}_z^T + \mathbf{P}_z^T} \frac{\partial H_z}{\partial z} + \underbrace{\begin{bmatrix} (\sigma_1 + \sigma_2) \end{bmatrix}}_{\mathbf{S}_z} u_z. \end{aligned} \quad (3.13)$$

where u_z is the input $\dot{\phi}$ and y_z is the friction torque τ_c . Note that the model is similar to the one presented in [137] except accounting for the influence of the varying normal force F_a . Passivity and dissipativity are again verified for the extended model without any additional assumptions as shown in Appendix A. Now while the system under consideration is infact associated with dissipation branch, its Hamiltonian is non-zero. The implication amounts to the bristle energy being stored and released from within the dissipation branch. Although, in the framework of Port-Hamiltonian systems, it is unusual to find a storage element within a dissipation branch, the model aims to faithfully capture all characteristics of the LuGre friction model.

It is noteworthy that in [138], the authors model a modulated resistive element as memristor. Memristors have been used for port-Hamiltonian systems to model dissipative elements with memory effects - for variable dampers / resistors. The idea stems from the non-existence of a bidirectional one-valued mapping between force and velocity in hysteretic elements, eg. friction. The advantage of using a memristive model can be seen in the idea of a null-Hamiltonian which allows to include (3.12)

in an augmented Hamiltonian formalism. However, memristive modelling approach to the modulated resistive port is not adopted because,

1. passivity conditions for such a modulated resistor invokes several apriori unverifiable assumptions some of which can become unreasonable. This primarily occurs due to the memristors modelled in [138] being position-dependent (integral of the flow variable), but in this case, friction is dependent on an external effort variable F_a .
2. such a modulated resistor must feature a pinched hysteresis curve as described in [139], however, LuGre friction model does not feature a pinched hysteresis curve [131].

Actuator System

The first equation of (3.1) directly lends to the Port-Hamiltonian system,

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\mathbf{J}_1} \frac{\partial H_1}{\partial \mathbf{x}_1} - \mathbf{R}_1 \left(\frac{\partial H_1}{\partial \mathbf{x}_1} \right) + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\mathbf{G}_1} \mathbf{u}_1 \\ y_l &= \mathbf{G}_1^T \frac{\partial H_1}{\partial \mathbf{x}_1}. \end{aligned} \quad (3.14)$$

where a nonlinear version of (3.3) is used as described in [124]. The clutching actuator dynamics from (3.2) can be similarly written with the Port-Hamiltonian description:

$$\begin{aligned} \dot{\mathbf{x}}_2 &= \underbrace{\begin{bmatrix} -d_2 & -1 \\ 1 & 0 \end{bmatrix}}_{\mathbf{J}_2 - \mathbf{R}_2} \frac{\partial H_2}{\partial \mathbf{x}_2} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\mathbf{G}_2} \mathbf{u}_2 \\ y_2 &= \mathbf{G}_2^T \frac{\partial H_2}{\partial \mathbf{x}_2}. \end{aligned} \quad (3.15)$$

For $i = \{1, 2\}$, $\mathbf{x}_i = [p_i, q_i]^T$ and y_i is $\dot{q}_i$. p_i is the generalized momentum coordinate. The Hamiltonians for the two actuator systems are,

$$H_1 = \frac{p_1^2}{2I_1}, \quad H_2 = \frac{p_2^2}{2I_2}. \quad (3.16)$$

Link system

From the second equation of (3.1), the Hamiltonian energy function of the link is given by,

$$H_l(p_l, q_l) = \frac{p_l^2}{2I_l} + V(q_l), \quad (3.17)$$

where the first term relates to inertial energy storage. The second term, $V(q_l)$ is the gravitational potential energy stored in the link and is given by:

$$V(q_l) = \int \tau_g(q_l) dq_l = mgl(1 - \cos(q_l)). \quad (3.18)$$

With $\mathbf{x}_l = [p_l \quad q_l]^T$, the port Hamiltonian description for the link can then be formulated as:

$$\begin{aligned} \dot{\mathbf{x}}_l &= \underbrace{\begin{bmatrix} -d_l & -1 \\ 1 & 0 \end{bmatrix}}_{\mathbf{J}_l - \mathbf{R}_l} \frac{\partial H_l}{\partial \mathbf{x}_l} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\mathbf{G}_l} \mathbf{u}_l \\ y_l &= \mathbf{G}_l^T \frac{\partial H_l}{\partial \mathbf{x}_l}. \end{aligned} \quad (3.19)$$

where y_l is the velocity of the link $\dot{q}_l$ and $\mathbf{u}_l$ is the sum of the torque transmitted through the clutch τ_c and the external torques τ_{ext} .

Clutch Dirac

The most integral component of SCA is the clutch which serves to interconnect all afore-presented elements. Hence, a power-continuous interconnection for such a clutch is derived. The control ports described by $(f_{c,1}, e_{c,1})$ and $(f_{c,2}, e_{c,2})$ relates to the driving actuator (q_1, τ_c) and the clutching actuator (q_2, F_a) of the driving and the clutching actuators. Next the interaction port of the clutch is defined by the power conjugate variables (f_l, e_l) to which the link will be connected through the variables $(\dot{q}_l, \tau_c)$. The storage port variables (f_s, e_s) are related to the power flow through an elastic storage element. The deflection of the storage element is captured by q_2 and the associated effort is F_a . Finally, the dissipation port described by (f_R, e_R) relates to the power dissipation at the clutch due to slipping. The corresponding flow and effort are $(\dot{\phi}, \tau_f)$. Note that $\dot{\phi}$ relates to the slip or the relative velocity between driving motor and link or the two clutching surfaces. Consequently, the Dirac structure of the clutch can be written,

$$\begin{bmatrix} e_{c1} \\ e_{c2} \\ f_s \\ e_l \\ f_R \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \end{bmatrix}}_{D_c} \begin{bmatrix} f_{c1} \\ f_{c2} \\ e_s \\ f_l \\ e_R \end{bmatrix}. \quad (3.20)$$

Since the Dirac structure D_c is skew-symmetric, the corresponding interconnection between the various flows and efforts is power-continuous:

$$\begin{bmatrix} e_{c1} & e_{c2} & f_s & e_l & f_R \end{bmatrix} \begin{bmatrix} f_{c1} & f_{c2} & e_s & f_l & e_R \end{bmatrix}^T = 0. \quad (3.21)$$

Understandably, the power dissipated in the dissipation port is given by $\tau_f \dot{\phi}$. Infact, the power dissipation is modulated by the constraining force between the clutching surfaces F_a which is related to the effort of the storage and clutching actuator port. Note that the power-flow associated with generating F_a which modulated friction in the dissipation branch is still captured by the storage port. Thus, clutch friction modulation is a not a power-free operation although F_a does not explicitly appear in the dissipated power.

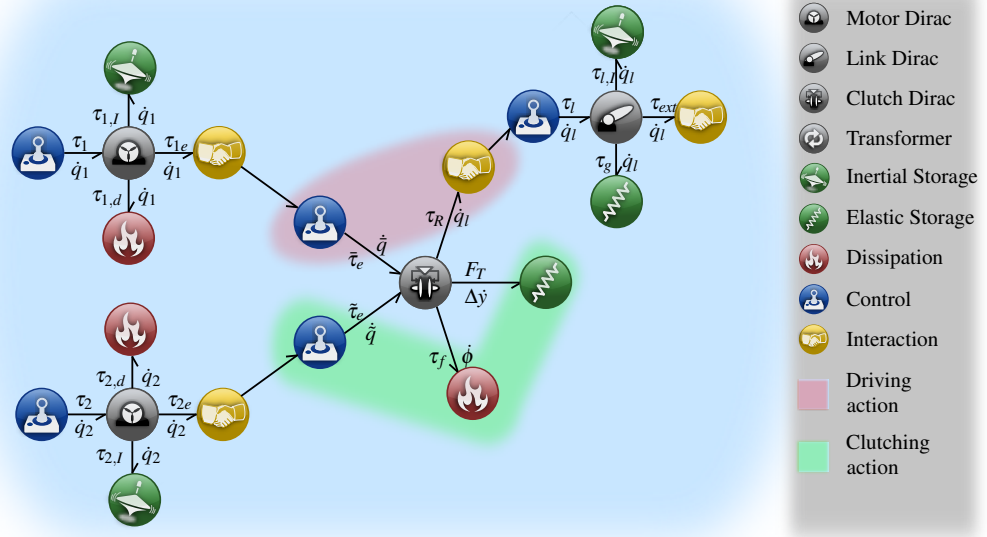


Figure 3.3 Port-based representation of the series clutched actuator considered in the paper. The gray ports signify Dirac structures while the remaining ports indicate one of the four ports associated with those Dirac structures. The power conjugate variables associated with each power exchange are shown. Further, the half arrows only indicate the desired direction of power flow between the ports.

Complete model

Since the spring between the clutching surfaces is the only energy storage element in this system, the augmented Hamiltonian for the SCA can be written as,

$$H_c = H_1 + H_2 + H_l + H_z + \frac{1}{2}k_s q_s^2. \quad q_s = q_2 \quad (3.22)$$

Then by using negative feedback interconnection of the various ports as in Fig. 3.3, port-Hamiltonian description of an SCA can be written,

$$\begin{aligned} \underbrace{\begin{bmatrix} \dot{q}_s \\ \dot{p}_l \\ \dot{q}_l \end{bmatrix}}_{\dot{\mathbf{x}}_c} &= \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}}_{\mathbf{J}_c - \mathbf{R}_c} \underbrace{\begin{bmatrix} F_a \\ q_l \\ \tau_g \end{bmatrix}}_{\mathbf{y}_c} + \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}}_{\mathbf{G}_c - \mathbf{P}_c} \underbrace{\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ -\tau_f \end{bmatrix}}_{\mathbf{u}_c} \\ \underbrace{\begin{bmatrix} \tau_1 \\ \tau_2 \\ \dot{\phi} \end{bmatrix}}_{\mathbf{y}_c} &= \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}}_{\mathbf{G}_c^T + \mathbf{P}_c^T} \underbrace{\begin{bmatrix} F_a \\ q_l \\ \tau_g \end{bmatrix}}_{\mathbf{y}_c} + \underbrace{\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{\mathbf{M}_c + \mathbf{S}_c} \underbrace{\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ -\tau_f \end{bmatrix}}_{\mathbf{u}_c} \end{aligned} \quad (3.23)$$

Passivity of (3.23) can easily be proven using (3.6). The desired level of clutching (engagement ratio as described in [119]) can be directly related to the direction of power flows between the actuator

and the load. However, deriving a control law with the port-Hamiltonian formalism presents some difficulties:

- i the inputs in (3.23) reduce the problem to a velocity control problem which might be unsuitable while deriving a controller for the actuators,
- ii an inversion of the LuGre friction mapping between slip velocity and friction torque is necessary which is cumbersome,
- iii the augmented Hamiltonian in (3.22) contains a term associated with the virtual energy stored in the bristles from LuGre friction model. Although, bristle deflection z does not appear in the states or control terms in (3.23), physical consistency of the formulation is arguable.

Therefore, an alternate approach is sought after for friction regulation control problem under consideration.

3.6 Optimal controller for friction modulation

While researchers have succeeded in actively estimating and compensating for LuGre friction [140], active modulation of friction at the macroscopic level is a largely unresearched topic. Very few works such as [141] and [142] have demonstrated friction modulation in mechanical systems at the macroscopic level, however, only with generalized static friction models. In order to derive a continuous, online controller for regulation of the highly nonlinear LuGre friction, an iterative Linear Quadratic Regulator (iLQR)-based controller [143] is used.

iLQR method belongs to the family of shooting-based optimization approaches [100] that provide for a locally-optimal feedback control law for nonlinear discrete-time systems. The controller iteratively linearizes the system dynamics and quadratizes the cost function, yielding an incremental update to the control law until convergence is obtained. Additional details of iLQR method can be found in [143]. The state vector for the SCA control problem at time-instant t is written as,

$$\mathbf{x} = \begin{bmatrix} q_m, & \dot{q}_m, & q_l, & \dot{q}_l, & z \end{bmatrix}^T,$$

and the input vector is,

$$\mathbf{u} = \begin{bmatrix} \tau_m, & F_a \end{bmatrix}^T,$$

Then from (3.1) and (3.10), the discretized system dynamics can be written,

$$\dot{\mathbf{x}}_{n+1} = f_n(\mathbf{x}_n, \mathbf{u}_n), \quad \mathbf{x}(0) = \mathbf{x}_0, \quad n \in \{0, 1, \dots, N-1\},$$

where N denotes the number of time-steps and $\mathbf{x}_0$ is the initial time-step. Note that the actuation mechanism for obtaining the axial/normal force F_a for the clutching operation is not considered for optimal control problem. It is necessary to provide a stabilizing control law $\mathbf{u}^*$ for the iLQR controller which is initialized to a matrix of zeros of appropriate dimensions. The cost function to be minimized for the optimization :

$$J_c = (\mathbf{x}_f - \mathbf{x}_f^*)^T \mathbf{Q}_f (\mathbf{x}_f - \mathbf{x}_f^*) + \sum_{n=0}^{N-1} \left((\mathbf{x}_n - \mathbf{x}_n^*)^T \mathbf{Q}_t (\mathbf{x}_n - \mathbf{x}_n^*) + \mathbf{u}^T \mathbf{R}_t \mathbf{u} \right) \quad (3.24)$$

where $\mathbf{Q}_f = 1e-3 \text{ diag}([0, 0, 1, 1, 0])$, $\mathbf{Q}_t = 1e-3 \text{ diag}([0, 0, 1, 1, 0])$ and $\mathbf{R}_t = 1e-2 \text{ diag}([1, 1])$. In essence, (3.24) describes a criterion that penalizes link trajectory tracking errors and inputs. Worth noting is that a previous work involved using iLQR controller for optimizing stiffness design within the port Hamiltonian framework [144].

3.6.1 Simulation results

A custom MATLAB implementation of iLQR controller is used during simulations. A dynamic model of SCA with LuGre friction is used for formulating the optimal control problem. A link-mass wherein the maximum gravity torque is twice the Coulomb friction coefficient of the driving motor is used. Euler-step method is used for the forward integration with a time-step duration of 0.5 ms. Further, the controller is used in a model-predictive control (MPC) fashion with a 25 ms prediction horizon and 5 ms control horizon. The actual control loop is executed at 1 kHz. LuGre friction parameters were varied for up to 10% randomly. Desired reference trajectories $\mathbf{x}_n^*$ that mimic virtual joint damping coefficients of 1 - 9 Nm/s were provided to the iLQR solver. The optimal control inputs were computed in all cases in less than the total time duration of the problem asserting that the MPC-based controller can be used on the physical hardware.

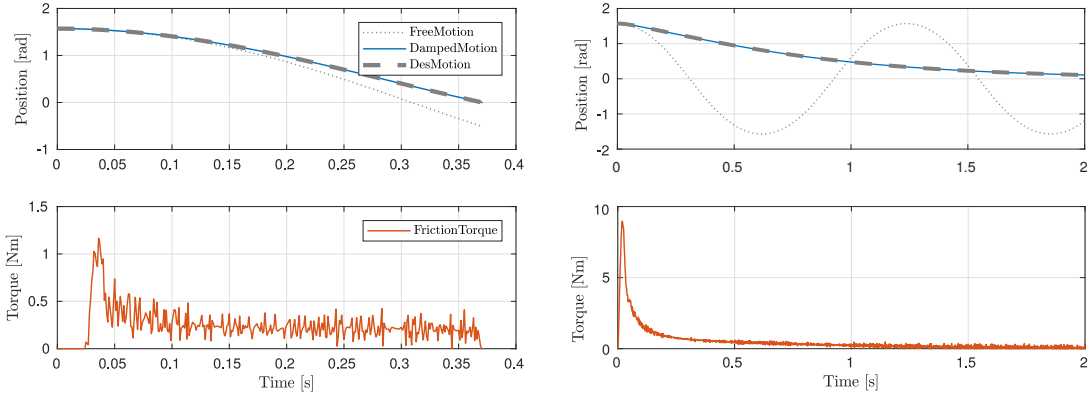


Figure 3.4 iLQR-based tracking controller results (link position tracking and friction torque) obtained for a range of damping coefficients

Fig. 3.4 shows the main results from MPC-simulations for two distinct damping coefficients. The link is able to track the reference motions with virtual link damping for 1 Nm/s and 9 Nm/s. The gravity-driven free motion of the link is additionally shown for comparison. Moreover, the friction torque developed in the clutch is also illustrated.

3.7 Antagonistic implementation of Series Clutched Actuator

Fig. 3.5 illustrates schematic of a cable-based bi-layered antagonistic continuous clutching actuator, originally conceived by Jörn Malzahn. A friction clutch is implemented at the link pulley through steel cables that are routed in two different layers over the pulleys connected to the actuators. In the common torque layer, the steel cable is a continuous loop providing for force closure. The differential action of the actuators tensions the spring (see differential torque layer) and also tensions the cable at the link

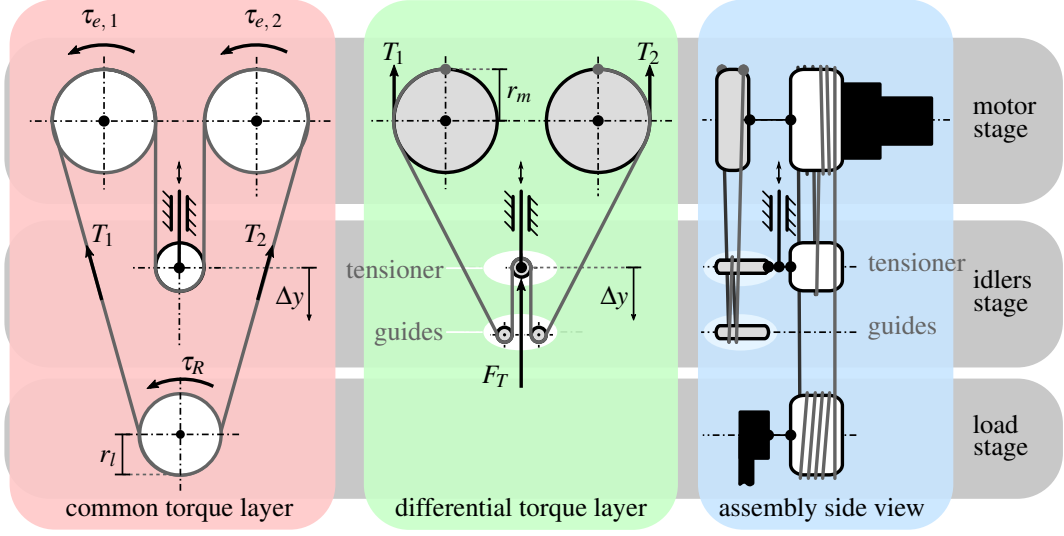


Figure 3.5 Antagonistic series clutched joint actuation concept with a controllable friction clutch.

pulley in the common torque layer. When in unison, the two actuators drive the link through the cable routing in the common torque layer provided the friction at the clutching interface just balances the torque transmitted to the common torque layer. The advantage of such a setup is that the common torque layer doubles the torque capacity at the link by summing the identical actuator torques.

The mechanism can be easily integrated into the presented port-Hamiltonian formalism (3.23) by invoking a transformer element $\mathbf{TF}$ at the interaction ports of the actuator. Accordingly, the transformation matrix is,

$$\mathbf{A} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

such that the driving and clutching torques become

$$\begin{aligned} \tau_c &= \tau_{e,1} + \tau_{e,2}, \\ F_a &= \tau_{e,1} - \tau_{e,2}, \end{aligned} \quad (3.25)$$

where $\tau_{e,i}$ represents the load torque at the i^{th} actuator.

Such an antagonistic implementation is able to replicate clutching action only for torques that do not dissipate Coulomb friction of the actuators but overcome it, i.e. only for torques that require friction compensation at both the actuators. Although at the outset, the common and difference systems seem to be independent, the Coulomb friction from the individual motor dissipation terms do not add up to Coulomb friction on the common and difference velocities as they are nonlinear functions of the motor coordinates q_1 and q_2 . Therefore, the dissipation terms of the common and difference systems remain coupled :

$$\begin{aligned} \bar{R} &= R_1(q_1) + R_2(q_2) \\ \tilde{R} &= R_1(q_1) - R_2(q_2) \end{aligned} \quad (3.26)$$

which means without compensating for the Coulomb friction of both the motors at any instant, the difference variable q_2 cannot be changed. In principle, without compensating for a part (or whole) of the friction of the common system, the difference system cannot be actuated. This violates the necessary condition for exploiting the efficiency of series clutched actuation concept under consideration. In order for the Coulomb friction in the common torque layer to be available for dissipating the link kinetic energy, it must not be compensated. However, tension adjustment for friction regulation in the difference torque layer cannot be brought about without moving the motors which in turn demands compensation of actuators' friction. Equivalently, this amounts to the two systems being coupled and hence they are not independent anymore. However, note that the antagonistic friction clutch can still function as a torque limiter as well as execute variably damped motions similar to the various clutch designs reviewed in Sec. 2.2.3.

3.8 Clutch Motor Specifications

Having identified a potential pitfall of an antagonistic SCA implementation, a series-based clutching approach is resorted to. In order to obtain requirements and specifications for the transmission units involved in operating the clutch, simulations are performed where the series clutched actuator tracks periodic motion trajectories on the link side. Given, the intermittent (127 Nm) and rated torque (41 Nm) of the Orange80 motor [145] which is considered to drive the link, in simulations it is observed that it can achieve link tracking up to the normalized gravity torque and normalized frequency for the motion amplitudes tabulated in Table 3.1.

These motions entail continuous and peak torques of 36 Nm and 40 Nm to be transmitted through the clutch to the link, respectively. Since it is assumed in simulations, that the clutch only exhibits sticking friction, therefore, it is sufficient to produce friction torque that exactly balances these torques. In order to compute the contact force required to develop the above friction torque, a clutch design must be selected. An ideal clutch is expected to develop the friction torque in shortest possible duration and minimize the energy spent in its actuation. As described below, an optimization is performed to arrive at a suitable clutch design.

3.8.1 Clutch actuation mechanism design

Before optimizing for the clutch design, an actuation-transmission mechanism for the clutch is selected. Firstly, since the clutch operation is expected to demand low power, a highly-efficient low power motor will be a suitable torque source. Secondly, since a serial design of SCA is considered, translational/linear movement is expected to comprise the clutching action. The requirement entails a rotary to linear transmission element. Given its high efficiency in rotary-linear power conversion, a ballscrew is chosen to transmit power from rotary motor to the clutch mechanism.

Table 3.1 Motion amplitudes for SCA design selection

Motion Amplitude	maximum τ_g / τ_C	maximum (ω / ω_0)
$\pi/2$	0.5	2
$\pi/3$	0.8	2
$\pi/6$	1.6	2

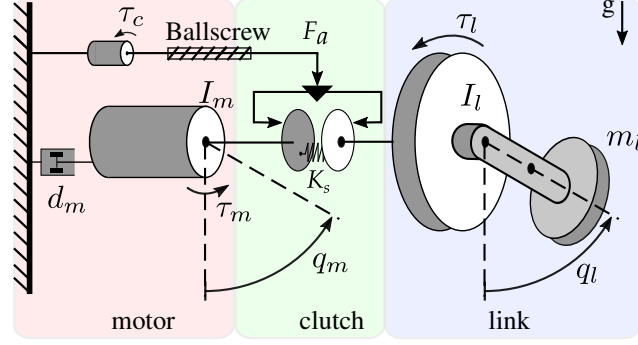


Figure 3.6 Series clutched joint actuation concept with controllable friction clutch modulated by high-efficiency actuator and ballscrew transmission.

Finally, a clutch design that supports energy-efficient operation must be selected. Additionally, the clutch must not be back-drivable. Among all prominent clutch designs, conical clutches offer the highest friction torque to normal force ratio with minimum design complexities, thus making it the prime candidate for SCA design [146]. Therefore, a conical design is chosen for the clutch in SCA. A ballscrew mechanism with high transmission ratio would be sufficient to prevent the clutch from back-driving the clutching motor. Furthermore, a clutch endowed with compliant elements can bring about higher energy-savings and reduce the necessary axial motion between fully engaged and disengaged states. Therefore, the clutch is chosen to have a spring in between the two clutching surfaces. Additionally, the spring passively keeps the two clutching surfaces apart as a safety mechanism. In addition, the spring can provide a deflection-based torque measure if sensorized. Fig. 3.6 visualizes a schematic of the SCA with clutch operation elements.

Screw drive actuated series clutch mechanism

Firstly, the following design considerations are to be made :

1. coefficient of friction at the conical section and the maximum torque to be transmitted,
2. range of axial clutch motion for zero to full engagement,
3. time duration for zero to full engagement.

For simplicity, a Coulomb friction model [125] at the clutching interface. Thus, the three sets of parameters that are to be optimized for minimizing the energy spent in operating the clutch are indicated, as in Fig. 3.7,

1. spring stiffness K_s ,
2. motor parameters including torque constant k_τ , ballscrew reduction ratio N_c and armature resistance R of the motor.
3. clutch parameters including coefficient of static friction μ_{st} at the clutching surface.

The axial force F_a exerted by the screw drive causes deflection of the coil spring which in turn exerts normal force on the clutch plates,

$$F_a = K_s q_s \quad (3.27)$$

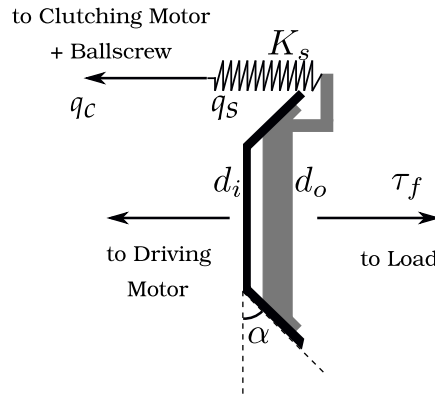


Figure 3.7 Series clutched joint actuation concept with controllable friction clutch modulated by high-efficiency actuator and ballscrew transmission.

where q_s is the deflection of the coil spring and K its stiffness. The natural length of the spring is assumed zero thereby q_l is also the ballscrew coordinate. The resulting normal force between the clutch plates determines the maximum amount of stick friction available in the clutch surface which are related using the Uniform pressure/normal force theory as,

$$F_a = \frac{3\tau_f \sin(\alpha)}{\mu_{st} d_i K_r} \quad (3.28)$$

where K_r is the clutch geometry coefficient, d_i is minor diameter of the conical clutch, α is the cone angle. For a conical clutch,

$$K_r = \frac{1 - (d/D)^3}{1 - (d/D)^2},$$

where D is the major diameter of the conical clutch. Theory of conical clutch design [146] suggests ideal values for cone angle (α of 12° - 15°) and diameter ratio (d/D of 0.6 - 0.9).

From (3.27) and (3.28), the ballscrew coordinate q_s can be computed:

$$q_s = \frac{3\tau_f \sin(\alpha)}{K \mu_{st} d_i K_r}. \quad (3.29)$$

The load torque to the clutching motor necessary to create the required axial force is

$$\tau_e = \frac{F_a p}{2\pi \eta N_c} \quad (3.30)$$

where N_c and p are the reduction ratio and pitch of the screw drive, respectively. The clutching motor coordinate as a function of the screw coordinate q_s or the clutch friction torque τ_f is

$$q_c = q_s 2\pi N_c / p = \frac{6\pi \tau_f \sin(\alpha) N_c}{K \mu_{st} d_i K_r p}. \quad (3.31)$$

The geometry parameters for a cone clutch, tabulated in 3.2 are considered as per design guidelines in [147] as well as the housing/size constraints with respect to the driving actuator.

The best possible rated and peak axial forces are calculated to be as 772 N and 857 N, respectively using Table 3.1 and the relations between (3.27)-(3.31). In order to compute the rated and peak axial clutch velocities, a spring must be first selected.

Table 3.2 Cone clutch parameters

Parameter	Symbol	Unit	Value
Minor cone diameter	d	55	mm
Major cone diameter	D	63	mm
Cone angle	α	13	deg
Coefficient of static friction	μ_{st}	0.5	[.]

Diaphragm spring

A diaphragm spring is chosen over a coil spring since they are compact, light and offer higher axial forces even when the clutch wears out [146]. Moreover, diaphragm springs can be coaxially arranged with the clutch plates within the housing of the clutch assembly. The *Schnorr* 119950 diaphragm spring [148] best matches the space and stiffness requirements within the clutching surfaces. As it has a nonlinear stiffness curve, its stiffness is calculated through linear regression in the operating range of the spring and this amounts to 1.75 kN/mm. Thus, a single spring allows for a favourably maximum deflection of 0.6 mm. Beyond this deflection, the stiffness curve exhibits a progressive charactersitic which may not be suitable for control purposes. In order to further reduce the stiffness and allow for a higher travel distance to obtain higher clutch friction resolution, four identical diaphragm springs are arranged in series. The combined serial stiffness is 0.44 kN/mm. The time derivative of ballscrew axial force trajectories divided by the series diaphragm stiffness determines the desired axial velocities $\dot{q}_s$ for the ballscrew: 4 mm/s (rated) and 7.9 mm/s (peak). Further the maximum axial travel distance for the ballscrew is computed from (3.28) to be just above 1.7 mm which is within the combined deflection of the four springs in series.

Grid optimization for motor, gearbox and ballscrew

Given the desired ballscrew axial velocities $\dot{q}_b$ and the axial forces F_b , a grid optimization routine computes energy calculations while periodic motion simulations for all feasible combinations of motor + gearbox + ballscrew. In particular, while the link performs gravity-driven motions, the clutch is operated such that the motion amplitudes and frequencies mentioned in Table 3.1 are obtained. The energy required to operate the clutch including the candidate motor - gearbox - ballscrew combination is computed. It takes into account all parameters from the datasheets including efficiency, rated and peak torques and velocities, rotor inertia, transmission ratio etc. and chooses the most efficient motor - gearbox combination. The maximum ballscrew efficiency is taken to be 0.8 as provided in the datasheets. Relations from (3.27)-(3.31) are used at various stages of the optimization routine.

The optimization routine predicts two solutions that leads to the least energy consumption. Interestingly, one of the solutions include a *frameless* motor while the other involves an *ECx* motor. Understandably, among the Maxon motors, the *frameless* and *ECx* series have the highest power density and motor constant k_m [65], which is an efficiency metric defined as:

$$k_m = \sqrt{\frac{\tau_m^2}{P_L}} \quad (3.32)$$

where τ_m is the electromotive torque developed in the motor windings and P_L is the resistive power loss which is then given by

$$P_L = i^2 R_L$$

where i is the supplied current and R_L is the motor winding resistance. Maxon *frameless series* are flat motors with generally huge diameters and short lengths whereas the conventional *ECx* motors have small diameters but generally longer lengths. As this secondary clutching motor has to be arranged near the driving motor, *ECx* series is favoured over the *frameless* series.

3.8.2 Simulation results

The optimization routines chooses ECx22L motor and GPX22 gearbox with a transmission ratio of 16 which are then considered for efficiency simulations. Thomson ballscrew PRM0801 whose operating range best suit the above requirements is predicted by the optimizer. The particular ballscrew has a dynamic and static load capacities of 800 N and 1700 N with a pitch of 1 mm. Further, the ballscrew critical speed is far higher than the peak velocity requirement set by the diaphragm spring.

Accordingly, efficiency gain based on electrical energy spent in operating the clutched actuator are computed without considering the actuator's mechanical and thermal limits. Results of the same are plotted at two distinct viewing perspectives in Fig.3.8. The efficiency gains are computed based on the energy savings with a clutched series actuator over a non-clutched series actuator. Simulations predict substantial energy savings (up to 57%) for load torques that are lesser than the Coulomb friction coefficient of the driving motor for a range of motion frequencies. For load torques greater than the driving Colomb friction coefficient such as heavy links, there are sizable energy savings especially for slow periodic motions.

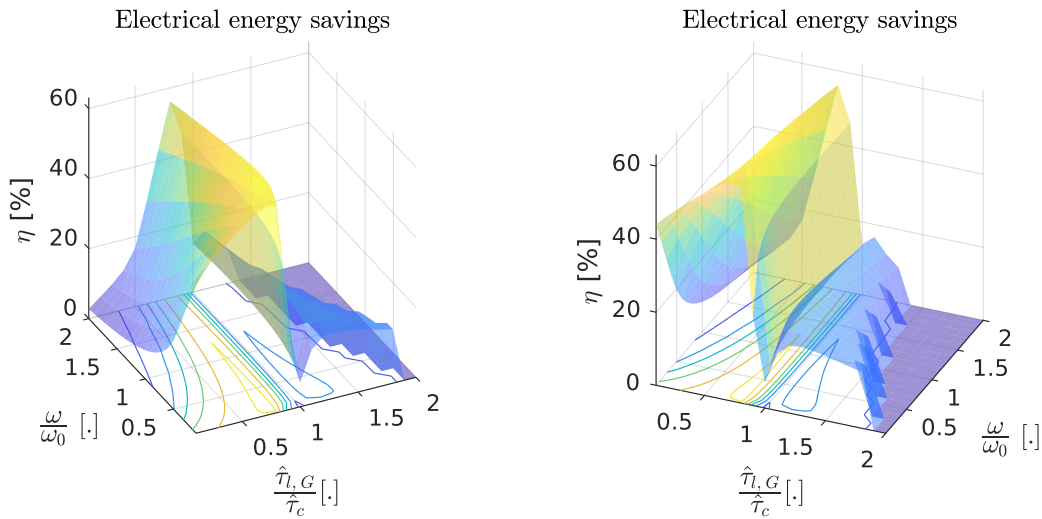


Figure 3.8 Electrical energy savings obtained with the optimized motor, gearbox, ballscrew combination.

Based on the design choices made from the optimization - simulations, a concept design is drawn up by Zeyu Ren, for a compactly housed SCA, see Fig. 3.9. It features a conical clutch in series with the Orange80 motor [145] and is coupled to the link at the other end. A ballscrew mechanism powered by a highly-efficient low-power (in comparison to SEA) motor drives the inner clutch cone through a linear rod and a series of multiple diaphragm springs. The concept design is pending fabrication.

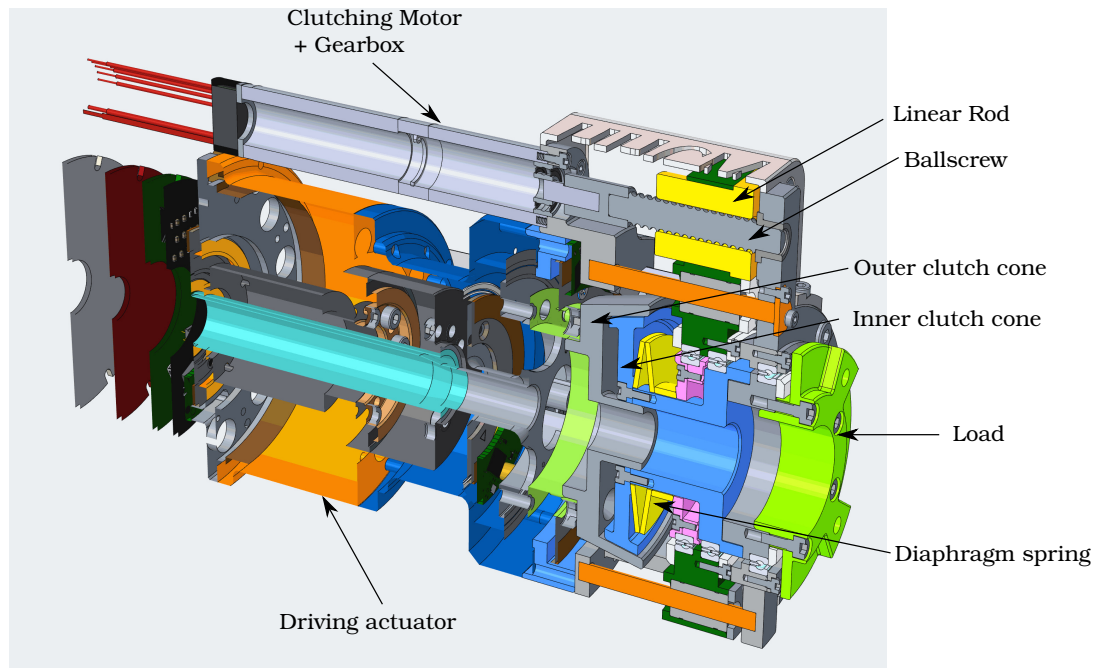


Figure 3.9 Section view of the designed series-clutched actuator where a high-power torque source serves as the driving actuator and a highly-efficient actuates a conical friction clutch.

3.9 Summary

In this chapter, an port-based model of a series clutched actuator is presented. The same is derived by formulating a clutch interconnection structure and integrating a LuGre friction model with SCA. Deriving a friction modulation controller is not trivial owing to the nonlinearities in the energy-based model. Given such complexities, an iLQR-based optimal controller is derived and verified in simulations through an MPC approach for link motions for a range of damping coefficients.

Next, the drawback of an antagonistic implementation of SCA is identified. Although, the cable-based setup decouples significant actuator inertia from the link-side, it is incapable of dissipating link kinetic energy by means of the actuators static friction. Thereby, a design optimization is performed for an alternate serial implementation of SCA wherein the driving and the clutching action are decoupled. Simulations are conducted for the optimized design to gauge the energy savings while operating the clutch for a range of motion frequencies and motion amplitudes. Energy savings of up to 57% during periodic motions motivates for future efforts in pursuing a non-antagonistic SCA design.

Although, theoretical energy savings have been observed and a controller has been derived for SCA, complexities in actuation mechanism and operation cannot be ignored. The disengagement of clutch can become quite problematic when the one end of the clutch is connected to a heavy external load. The additional components necessary in implementing the mechanism in a single joint adds to the link mass and inertia. Integrating the mechanism in a multi-DoF system pose difficulties in ensuring any reasonable amount of energy savings. Hence, given the overhead in hardware, actuation and control, and the shorter than usual PhD timeframe, the idea is not pursued further in this thesis.

Chapter 4

Energetic considerations for Series Parallel Compliant Multiarticulated Actuation

For prolonged operation of legged robots, extending the power autonomy is of paramount importance. Many researchers have promoted the combined use of series and parallel elastic actuators to enhance energy-efficiency in robots, particularly in periodic motion tasks [33, 34, 36]. Nonetheless, both SEAs and PEAs have their specific advantages and disadvantages [52]. For instance, series compliance is generally preferred during motion frequencies abutting the robot's natural frequency. Given their gravitational load compensation characteristics, parallel compliance is preferred during slow motion tasks at lower frequencies. However, these observations may drastically change with variations in their relative actuator dynamics. These series and parallel actuators may feature dissimilarities that arise from differences in energy-storage capacity, power capacity, torque constant, transmission ratio, electrical winding resistance and torque-generation efficiency owing to frictional and mechanical losses. Besides, in redundant robots that feature asymmetric series-parallel actuated branches along with multi-articulation, joint torque distribution is a non-trivial problem. Hence the problem of joint torque distribution among the various (dis)similar actuators powering redundant joints is still unclear.

[92] compared various such approaches for torque distribution on a manipulator that was only powered by monoarticular and biarticular PEAs. [120] proposed to use a gradient descent-based control scheme on a 3 DoF monoped robot powered by SEAs and PEAs, in order to allocate maximum possible torque demand to the PEAs, owing to their higher efficiency when compared with SEAs. Allocation of maximum torque demand to PEAs might not be the best strategy at all times.

To probe this further, a novel quadratic programming-based controller is proposed. The controller can optimize for various quadratic criteria to distribute torques among the dissimilar series and parallel elastic actuators. Torque resolution taking into account the hardware specifications can lead to valuable considerations for control design. Therefore, this chapter investigates the efficiency of various torque-distribution strategies in compliant articulated robots powered by hybrid series and parallel actuation principles.

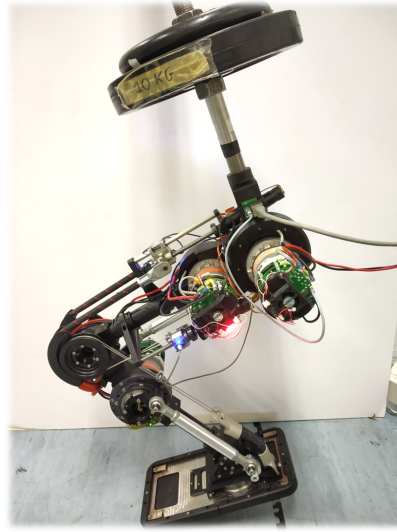


Figure 4.1 The articulated leg prototype used in this work.

4.1 Organization

The chapter is organized as follows¹. A background is first presented on the concept of multiarticulated asymmetric series-parallel compliant actuation in Sec. 4.2. Various quadratic cost functions are composed and their equivalent derivation in the quadratic programming framework are then presented in Sec. 4.3. An experimental analysis is conducted with experiments in Sec. 4.4 followed by an insightful discussion in Sec. 4.5. The chapter concludes in Sec. 4.6 providing valuable recommendations for future work.

4.2 Multi-articulated Asymmetric Compliant Actuation on *eLeg*

Asymmetric compliant actuation (ACA) branches were first conceptualized for a single joint monoped in [56]. The same idea was extended to a 3-DoF monoped along with multi-articulation in [35]. An ACA branch comprises of a primary torque source, usually a high-torque high power actuator such as a SEA and a secondary torque source, a low-power but high energy-storage capacity PEA. In this case, SEA forms the Power Branch (PB). SEAs display substantial friction owing to their large gearbox transmissions. Highly-efficient small DC motors are used for PEAs along with a ballscrew mechanism to transmit power from rotary motors into translational extension of stretchable parallel elements called tendons. These parallel tendons possess high linear stiffness and can store large amounts of potential energy. PEAs along with the transmission elements comprise the Energy Storage Branch (ESB). Dissimilarities in efficiency, power, torque, energy capacities as well as actuators' compliance between PB and ESB leads to the asymmetry of these branches.

When the ESB tendons (bungees) are wrapped over a single joint, the PEAs are *mono-articulated* and contribute to torque only at that particular joint. When the same tendon spans over two or more joints, PEAs are *bi-* or *multi-articulated*. In this case, PEAs lead to torques at all the wrapped joints

¹A part of the work included in this chapter has been presented at International Conference on Robotics and Automation, ICRA 2020 with the publication titled *On the efficient control of series-parallel compliant articulated robots* by Vishnu Dev Amara, Jörn Malzahn, Zeyu Ren, Wesley Roozing and Nikos Tsagarakis.

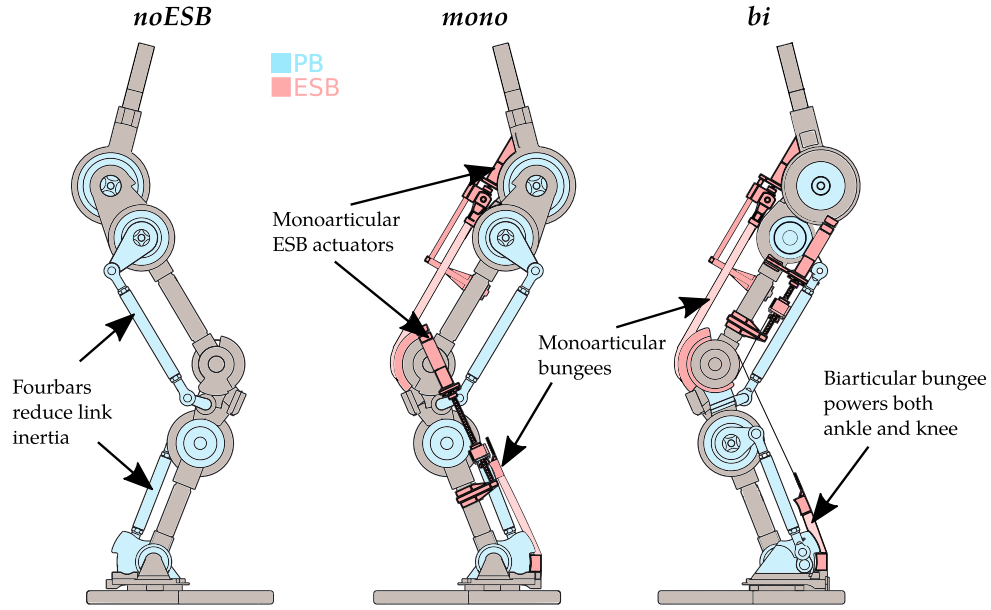


Figure 4.2 The *noESB*, *mono* and *bi* configurations of the anthropomorphic *eLeg* with ACA arrangements.

and the individual joint torques depend on the joint configuration of all spanned joints. Consequently, multi-articulated PEAs also lead to power transfer between the various spanned joints.

eLeg allows to employ ACA branches with reconfigurable ESBs. For the ESB tendons, bungee cords that possess unidirectional force characteristics are used. Moreover, these bungees also display S-shaped stiffness and hysteresis.

PB torques are measured by torque sensors present in the SEAs and bungee forces are measured by strain gauges at the ESB actuators. Elongations of up to 80% of their natural lengths fall under the normal operating range. Strain gauges attached on the ballscrew nuts provide for tendon force measurement. Stiffness coefficients are computed in the working range of the bungees through linear regression. The robot is loaded with 15 kg at the trunk. Asymmetric hardware specifications are tabulated in Table 4.1. All additional hardware specifications can be found in [149] and [35]. In the following works, three distinct actuation configurations are employed, see also Fig. 4.2:

- i. ***noESB*** : SEAs are the only torque source at all three joints. The robot weighs 26.6 kg.
- ii. ***mono*** : Besides SEAs, monoarticular PEAs are present at ankle and knee joints thereby each of those joints being powered by two torque sources (SEA and mono-PEA). Both mono PEAs lead to extension of the two joints and perform the role of *soleus* and *vastus* muscles. The robot weighs 27.4 kg in this configuration.
- iii. ***bi*** : In addition to SEAs, ankle and knee joints are *biarticulated* with a PEA. The ankle joint is powered by two torque sources (SEA, bi-PEA). The knee joint is additionally *monoarticulated* and is powered by three torque sources (SEA, bi-PEA, mono-PEA) as in Fig. 6.1. The bi-PEA serving the role of *gastrocnemius* muscle leads to power transfer between ankle and knee through ankle extension and knee flexion. mono-PEA at the knee leads to knee extension. The robots weighs 28 kg in this configuration.

4.2.1 Actuation Modelling

The dynamics of a single joint powered by PB and ESB can be written as,

$$I_l \ddot{q} + \tau_g(q) = \underbrace{\tau_{PB} + \tau_p}_{\tau} - \tau_e, \quad (4.1)$$

where q is the joint position. I_l is the joint inertia and K_{PB} is the stiffness of the spring element. While τ_g is the gravity torque, τ_e accounts for the external torques at the joint including the multi-body dynamics (multi-joint case). τ_{PB} and τ_p are the torque contributions from PB and ESB actuators, respectively, at the joints.

τ_{PB} can be determined from the SEA motor-side mechanical dynamics:

$$I_m \ddot{\theta} + \underbrace{K_{PB}(\theta - q)}_{\tau_{PB}} = \tau_m - \tau_d, \quad (4.2)$$

where τ_d groups Coulomb and viscous friction at the rotor and gearbox, I_m is the motor inertia and τ_{PB} is the SEA torque at the joint. τ_m is the electromotive torque.

For the ESBs that might be multiarticulated, τ_p is calculated as shown below. Given an $n + 3$ DoF planar floating-base robot with m ($\leq n$) ACA branches, the unilateral ESB tendon forces due to the tendon deflection Δ can be approximated :

$$\mathbf{F}_p = \begin{cases} \mathbf{K}_p \Delta, & \Delta > 0, \\ 0, & \Delta \leq 0. \end{cases} \quad (4.3)$$

where $\mathbf{K}_p \in \mathbb{R}^{n \times m}$ is a stiffness matrix whose diagonal elements are the linear stiffnesses of the corresponding ESB tendon. Tendon deflections can be related to the joint configuration vector of the actuated joints and PEA pretensions by the articulation topology matrix:

$$\Delta = (\mathbf{S}\mathbf{p} + \mathbf{T}\mathbf{q}). \quad (4.4)$$

$\mathbf{T} \in \mathbb{R}^{n \times n}$ is a topology matrix that is a function of tendon transmission ratio and ESB arrangements. $\mathbf{S} \in \mathbb{R}^{n \times m}$ is a selection matrix that chooses the joints which comprise ESB elements. $\mathbf{q}_a \in \mathbb{R}^n$ denotes the joint configuration that correspond to the actuated DoFs and $\mathbf{p} \in \mathbb{R}^m$ denotes the tendon pretensions.

For instance, the topology matrix for the *bi* configuration is,

$$\mathbf{T}_{bi} = \begin{bmatrix} . & . & 0 \\ 0 & . & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (.) \text{ indicates non-zero element}$$

The two non-zero elements in the first row indicate *biarticulation* at the ESB tendon at the ankle. The biarticulation is with the knee joint. The second row containing a non-zero element in the middle signifies monoarticulation at the knee. An empty third row indicates that no parallel articulation or ESB branches are installed at the hip joint.

Table 4.1 Specifications of the dissimilar actuators on eLeg

Specification	PB	ESB
Stiffness ² (K)	5.8 kNm/rad	5.8 - 7.8 kN/m (Knee monoarticulated), 4 - 4.8 kN/m (Ankle monoarticulated) 24 - 29.5 kN/m (Ankle-knee biarticulated)
Torque capacity (τ_{max})	40 Nm (nominal) 127 Nm (peak)	11.8 kN (Ankle bi) 5.5 kN (Knee)
Torque constant (k_τ)	78-85 mNm/A	14.2 mNm/A

When the bungees are not slack, tendon forces can be mapped to joint torques based on the same topology matrix again,

$$\boldsymbol{\tau}_p = -\mathbf{T}^T \mathbf{F}_p = -\mathbf{T}^T \mathbf{K}_p (\mathbf{S}\mathbf{p} + \mathbf{T}\mathbf{q}). \quad (4.5)$$

Grouping all PB torques in $\boldsymbol{\tau}_{PB}$, joint torques can be written as the sum of PB and ESB torques,

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{PB} + \boldsymbol{\tau}_p. \quad (4.6)$$

Hence, employing parallel compliant (multi-)articulation enhances the torque capacities at the joints. Accordingly, the instantaneous mechanical and electrical joint powers can be computed:

$$\mathbf{P}_m = \underbrace{-\boldsymbol{\tau}_{PB}^T \dot{\mathbf{q}}_a}_{\text{PB power}} + \underbrace{\mathbf{F}_p^T \mathbf{T} \dot{\mathbf{q}}_a}_{\text{ESB power}}. \quad (4.7)$$

4.3 Optimization-based torque resolution

Distribution of joint torque demand by resolving actuator redundancy plays a significant role in torque control of redundant robots. Besides, due to multiarticulated ESB elements, torques at a spanned joint is a function of configuration of all the spanned joints. This makes the process of joint torque partitioning among its various torque sources, non-trivial. In this chapter, optimal control method to resolve the instantaneous joint torque demand among its dissimilar PB and ESB actuators, is used. The choice of an optimization method, its optimization criteria and the resulting considerations and their results are detailed below.

4.3.1 Choice of optimization method

In order to optimally distribute the instantaneous joint torques among its torque sources, an online approach that accounts for various hardware constraints must be chosen. Moreover, the actuators can display some linear inequality constraints - box constraints on ESB pretension limits and their velocities. Frequently, constrained optimization of some quadratic function of certain control variables is carried out for this process [92, 150].

Given the efficacy of quadratic programming (QP) in applications such as inverse dynamics [151], footstep planing [152], trajectory optimization [153], etc. a QP-based method [154] is used to obtain references for the ESBs. Specifically, an online QP algorithm, computes instantaneous pretension

²Linearized stiffness values shown for the ESB vary significantly within the 80% elongation range.

references $\mathbf{p}^*$. Then the net difference between the joint torque demand and ESB actuators can be quenched by the SEAs in the PB. Fig. 4.3 visualizes the control architecture used for this novel torque resolution approach. Noteworthy is the fact that such a torque-partitioning scheme is completely unaware of the higher level controller.

4.3.2 Cost function candidates

While several cost criteria can be considered for optimization, the following physically meaningful candidate cost functions are chosen.

Maximize ESB Load share (MELS)

Quite often in optimal control, minimization of actuator torques is carried out [35, 120]. In this case, the same can be done for the lower-efficiency PB actuators. This can mathematically be related to the squared norm of the difference between the commanded joint torque and the ESB load torque. The higher-efficiency ESB drives cannot solely quench joint torque demands in the eLeg, but they can provide for gravitational load compensation which are the most significant torque components in slow periodic motions. A similar criterion that aims to distribute the maximum possible torque demand to the ESB:

$$J = \boldsymbol{\tau}_{PB}^T \boldsymbol{\tau}_{PB} = (\boldsymbol{\tau}_p - \boldsymbol{\tau}^*)^T (\boldsymbol{\tau}_p - \boldsymbol{\tau}^*). \quad (4.8)$$

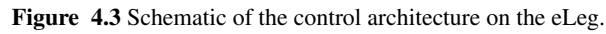
where $\boldsymbol{\tau}^* \in \mathbf{R}^n$ is the joint torque reference vector.

Minimize Sum of squared Load Currents (MSLC)

While minimization of input torques or actuator inputs is frequently used in optimal control, parameters including transmission ratio, motor torque constants, spring compliance, etc. vary significantly, especially when the actuators are dissimilar, see Table 4.2. Accordingly, electrical currents necessary to generate the same amount of torque across the non-homogenous actuators also vary. Therefore, a more general criterion would be to minimize the sum of squared currents that correspond to the load torques:

$$J = \mathbf{i}_{PB}^T \mathbf{i}_{PB} + \mathbf{i}_p^T \mathbf{i}_p \\ = (\mathbf{A}_{PB}(\boldsymbol{\tau}_p - \boldsymbol{\tau}^*))^T (\mathbf{A}_{PB}(\boldsymbol{\tau}_p - \boldsymbol{\tau}^*)) + (\mathbf{A}_p \mathbf{F}_p)^T (\mathbf{A}_p \mathbf{F}_p), \quad (4.9)$$

where $\mathbf{A}_{PB}$ and $\mathbf{A}_p$ are diagonal matrices given by $\text{diag}([({}^{PB}k_{\tau_1} {}^{PB}N_1)^{-2}, \dots, ({}^{PB}k_{\tau_n} {}^{PB}N_n)^{-2}])$ and $\text{diag}([({}^p k_{\tau_1} {}^p N_1)^{-2}, \dots, ({}^p k_{\tau_m} {}^p N_m)^{-2}])$. Here $(\cdot)N_{(\cdot)}$ and $(\cdot)k_{\tau_{(\cdot)}}$ refer to the transmission ratio and the torque constant of the corresponding actuator. Note that the currents considered in (4.9) are computed from estimated actuator torque demand. However, torque-generation efficiency of the actuators can vary due to friction and other mechanical losses. This, in turn can potentially vary the electrical efforts required in powering the actuators to deliver the same amount of torque at the joints. For instance, the friction in the SEAs of PB is quite high which reduces their efficiency when compared with highly-efficient ESB units.



Further, electrical power dissipation across dissimilar actuators can vary due to their distinct electrical dynamics [155]. In particular, PB and ESB display distinct electrical winding resistances leading up to distinct associated electrical losses while generating the same torque output. For instance, from manufacturer specifications, winding resistance of ESB motors are more than twice that of the PB motors. Hence, the squared sum of electrical power losses is considered as a minimization criterion for joint torque distribution.

where $\mathbf{R}_{(\cdot)}$ is a diagonal matrix that contains as its elements the resistances of the electrical windings in the actuators of the corresponding branch. Once again, no actuator friction models are employed in this formulation, to retain the cost function, quadratic. Therefore, the currents used in (4.10) are computed only from their corresponding joint torque demands and not the actuators' electromotive torques.

Another candidate criterion that encompasses both electrical power losses as well as the actuators' mechanical power can be used to penalize the sum of approximated squared electrical powers consumed by the dissimilar actuators.

$$\begin{aligned}
J &= \mathbf{i}_{PB}^T \mathbf{R}_{PB} \mathbf{i}_{PB} + \boldsymbol{\tau}_{PB}^T \dot{\boldsymbol{\theta}} + \mathbf{i}_p^T \mathbf{R}_p \mathbf{i}_p + \mathbf{F}_p^T \dot{\mathbf{p}} \\
&= (\mathbf{A}_{PB}(\boldsymbol{\tau}^* - \boldsymbol{\tau}_p))^T \mathbf{R}_{PB} (\mathbf{A}_{PB}(\boldsymbol{\tau}^* - \boldsymbol{\tau}_p)) + (\boldsymbol{\tau}^* - \boldsymbol{\tau}_p)^T \dot{\boldsymbol{\theta}} + \\
&\quad (\mathbf{A}_p \mathbf{F}_p)^T \mathbf{R}_p (\mathbf{A}_p \mathbf{F}_p) + \mathbf{F}_p^T \dot{\mathbf{p}}.
\end{aligned} \tag{4.11}$$

where $\mathbf{R}_{(\cdot)}$ is a diagonal matrix that contains as its elements the resistances of the electrical windings in the actuators of the corresponding branch. $\dot{\boldsymbol{\theta}}$ corresponds to the vector of SEA motor velocities.

4.3.3 QP Problem formulation

A QP problem can be formulated for each of the above cost candidates which can provide for pretension references $\mathbf{p}^*$. Given the hardware limits, online optimization is performed with the following constraints:

- *ESB pretension constraints* - ESB tendons can be stretched only within its hardware limits.
- *ESB velocity constraints* - owing to the lower actuator power output, ESB actuators have a maximum achievable linear velocity of about 8 mm/s (manufacturer specifications),
- *Unidirectional tendon constraints* - ESB tendons are unidirectionally elastic, slack in them is not preferred. Soft constraints can be enforced with the help of optimization slack variables to always keep the tendons under tension.

Thus the QP problem in discrete form is given:

$$\begin{aligned} & \underset{\mathbf{p}_{i+1}^*, \tilde{\mathbf{p}}_{i+1}}{\text{minimize}} && J_{i+1}(\mathbf{q}_i, \boldsymbol{\tau}_i^*, \mathbf{p}_{i+1}^*, \tilde{\mathbf{p}}_{i+1}) \\ & \text{subject to} && \underline{\dot{\mathbf{p}}} \Delta t \leq \mathbf{p}_{i+1}^* - \hat{\mathbf{p}}_i \leq \bar{\dot{\mathbf{p}}} \Delta t, \\ & && \mathbf{S}(\mathbf{p}_{i+1}^* + \tilde{\mathbf{p}}_{i+1}) + \mathbf{T}\mathbf{q}_i \geq \mathbf{0}, \\ & && \underline{\mathbf{p}} \leq \mathbf{p}_{i+1}^* \leq \bar{\mathbf{p}}, \quad \tilde{\mathbf{p}}_{i+1} \geq \mathbf{0}. \end{aligned} \quad (4.12)$$

where $\tilde{\mathbf{p}} \in \mathbb{R}^m$ denotes the slack variable, Δt is the control step size. $(\underline{\cdot})$ and $(\bar{\cdot})$ denote the minimum and maximum values of the corresponding quantity. $(\cdot)_i$ and $(\cdot)_{i+1}$ denote the quantity at the current and successive time instants. The optimization problem in (4.12) only requires position feedback from ESBs and outputs position references. However, when one prefers force sensing feedback for torque control of the ESB actuators, the same controller can be suitably reformulated:

$$\begin{aligned} & \underset{\mathbf{F}_{\mathbf{p}_{i+1}}^*}{\text{minimize}} && J_{i+1}(\mathbf{q}_i, \boldsymbol{\tau}_i^*, \mathbf{F}_{\mathbf{p}_{i+1}}^*) \\ & \text{subject to} && \dot{\hat{\mathbf{F}}}_{\mathbf{p}} \Delta t \leq \mathbf{F}_{\mathbf{p}_{i+1}}^* - \hat{\mathbf{F}}_{\mathbf{p}} \leq \bar{\dot{\mathbf{F}}}_{\mathbf{p}} \Delta t, \\ & && \mathbf{0} \leq \mathbf{F}_{\mathbf{p}_{i+1}}^* \leq \mathbf{K}_{\mathbf{p}}(\mathbf{S}\tilde{\mathbf{p}} + \mathbf{T}\mathbf{q}_i). \end{aligned} \quad (4.13)$$

Note that the ESB tendons can feature a highly nonlinear stiffness profile with hysteresis. Moreover, $\dot{\hat{\mathbf{F}}}_{\mathbf{p}}$ is a function of all spanned joint velocities which further compounds the problem of determining its upper and lower limits. Therefore, the controller described in (4.12) is used. When torque control of ESBs is feasible, (4.13) can be used.

While formulating the QP problem, it is sufficient to obtain the quadratic and linear terms associated with the states that are solved for. A single template expression in the QP framework can be converted

Table 4.2 QP problem formulation - Weightings

Optimization criteria	C_{PB}	C_p	C_f
Maximize ESB Load Share (MELS) (4.8)	$\mathbf{I}_{n \times n}$	$\mathbf{0}_{m \times m}$	$\mathbf{0}_{m \times m}$
Minimize Sum of squared Load Currents (MSLC) (4.9)	$\mathbf{A}_{PB}^T \mathbf{A}_{PB}$	$\mathbf{A}_p^T \mathbf{S}^T \mathbf{S} \mathbf{A}_p$	$\mathbf{0}_{m \times m}$
Minimize Sum of load Power Losses (MSPL) (4.10)	$\mathbf{A}_{PB}^T \mathbf{R}_{PB} \mathbf{A}_{PB}$	$\mathbf{A}_p^T \mathbf{S}^T \mathbf{R}_p \mathbf{S} \mathbf{A}_p$	$\mathbf{0}_{m \times m}$
Minimize Sum of Total load Powers (MSTP) (4.11)	$\mathbf{A}_{PB}^T \mathbf{R}_{PB} \mathbf{A}_{PB}$	$\mathbf{A}_p^T \mathbf{S}^T \mathbf{R}_p \mathbf{S} \mathbf{A}_p$	$\mathbf{I}_{m \times m}$

into the various aforementioned candidate cost functions:

$$J_{i+1} = \mathbf{p}_{i+1}^{*T} \left(\mathbf{S}^T (\mathbf{C}_{PB} \mathbf{T}_M^T \mathbf{T}_M + \mathbf{C}_p \mathbf{K}^T \mathbf{K}) \mathbf{S} \right) \mathbf{p}_{i+1}^* + \mathbf{p}_{i+1}^{*T} \left(\mathbf{S}^T \left(2\mathbf{C}_{PB} \mathbf{T}_M^T (\mathbf{T}_M \mathbf{T} \mathbf{q}_i + \boldsymbol{\tau}_i^*) + 2\mathbf{C}_p \mathbf{K}^T \mathbf{K} \mathbf{T} \mathbf{q}_i + \mathbf{C}_f (\mathbf{T}_M \dot{\mathbf{q}}_a + \mathbf{K} \dot{\mathbf{p}}) \right) \right) + \tilde{\mathbf{p}}_{i+1}^T \tilde{\mathbf{W}} \tilde{\mathbf{p}}_{i+1}. \quad (4.14)$$

where $\mathbf{T}_M$ is $\mathbf{T}^T \mathbf{K}$. $\mathbf{C}_{PB} \in \mathbf{R}^{n \times n}$ and $\mathbf{C}_p \in \mathbf{R}^{m \times m}$ are diagonal matrices of weighting coefficients associated with PB and ESB actuators. $\tilde{\mathbf{W}} \in \mathbf{R}^{m \times m}$ is a diagonal weighting matrix for the slack variables. Table 4.2 shows how by using (4.5) and merely setting the weighting coefficient matrices $\mathbf{C}_{PB}$ and $\mathbf{C}_p$ in (4.14) appropriately, one can formulate the QP problems with the various aforementioned cost functions.

4.3.4 Implementation

The control experiments were executed with Simulink RealTime interface on an Intel NUC PC with 2GB DDR-3 RAM and a Core i5-3427U processor (3M cache) running at 1.8 Ghz. The control algorithm is implemented and all sensor measurements are obtained at a sampling rate of 1kHz. A custom MATLAB implementation of the Goldfarb-Idnani method [156] is used to solve the strictly convex QP problem. Owing to hardware limits, we restricted the knee ESB tendon extension range and the maximum linear velocity of the ESB actuators in the QP problem to 0.072 m and 0.007 m/s. The elements of the weighting matrix $\tilde{\mathbf{W}}$ for the slack variables $\tilde{\mathbf{p}}$ are set such that they are at least two orders of magnitude higher than the elements of the quadratic term associated with $\mathbf{p}$.

The electrical power is computed using current measurements from the sensors in the actuators and including the back-EMF and the electrical dynamics. They are then integrated to yield the energy expenditure. Further, since untethered robots can be equipped with power sources on-board, typically rechargeable Li-ion batteries that are capable of four quadrant (electrical power - mechanical power) operation, negative power flows are considered in energy calculations. Power-flow is considered positive when power is delivered to the load from the actuators while is negative when power flows from the load and backdrives the actuators, charging the battery.

4.4 Results

Firstly, experiments were conducted with the three candidate costs described in (4.8)- (4.10) whose results are organized below for all three actuation configurations. In all figures, $(\cdot)_1$, $(\cdot)_2$ and $(\cdot)_3$ correspond to quantities associated with the ankle, knee and hip joints respectively.

4.4.1 Deep squat experiments at 0.3 Hz

Cyclic squat experiments were first performed wherein the cartesian position of the trunk is commanded to trace inclined elliptical trajectories of 0.3 Hz with horizontal and vertical amplitudes of 3 and 15 cm respectively. Data corresponding to the *noESB* configuration including the joint tracking errors, torques, measured currents and powers along with the total energy expenditure are presented in Fig.4.4. Data from hip joint is not shown since it is not powered by additional ESBs. Since *eLeg* in this particular configuration is only powered by SEAs, it does not require any torque resolution control. The *noESB* configuration consumes 599 J for the three full motion periods of this squat motion.

Similarly, experimental results from *mono* and *bi* configurations are presented in Fig.4.5 and 4.6. Across configurations, for all candidate criteria, ESBs were initialized such that their pretensions compensate for gravity torques at the initial squat pose of the motion cycle. All reported data correspond to the measured values from after the pretension references from the QP-based controller have converged.

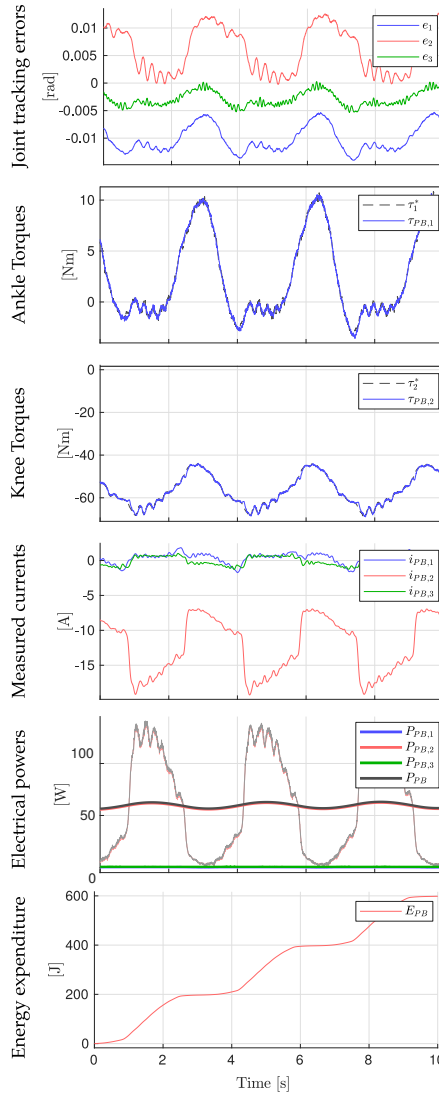


Figure 4.4 Data from **deep squat** experiments (x-amp: 3 cm, y-amp: 15 cm) at 0.3 Hz for the *noESB* configuration.

From ankle and knee torque plots in Fig.4.5 for the *mono* configuration, it is readily seen that the joint torque distribution among the PB and ESB actuators varies with the three criteria. In particular, MSLC and MSPL criteria demand slightly lesser ESB torques at the joints than MELS. This leads to differences in currents and powers, however, effectively leading to negligible differences in the final energy expenditure computed for a 10 s period over three motion cycles. Note that while phases of negative electrical powers (power flows into the actuators) are negligible with the MELS criterion, they bring about some energy savings with the other two criteria. Although PBs are not as efficient as ESBs, during the squat down phase, they counteract the unidirectional ESB forces lesser. This phenomenon leads to all three criteria displaying comparable energy savings of about 58-59% over the *noESB* configuration while executing the same motion.

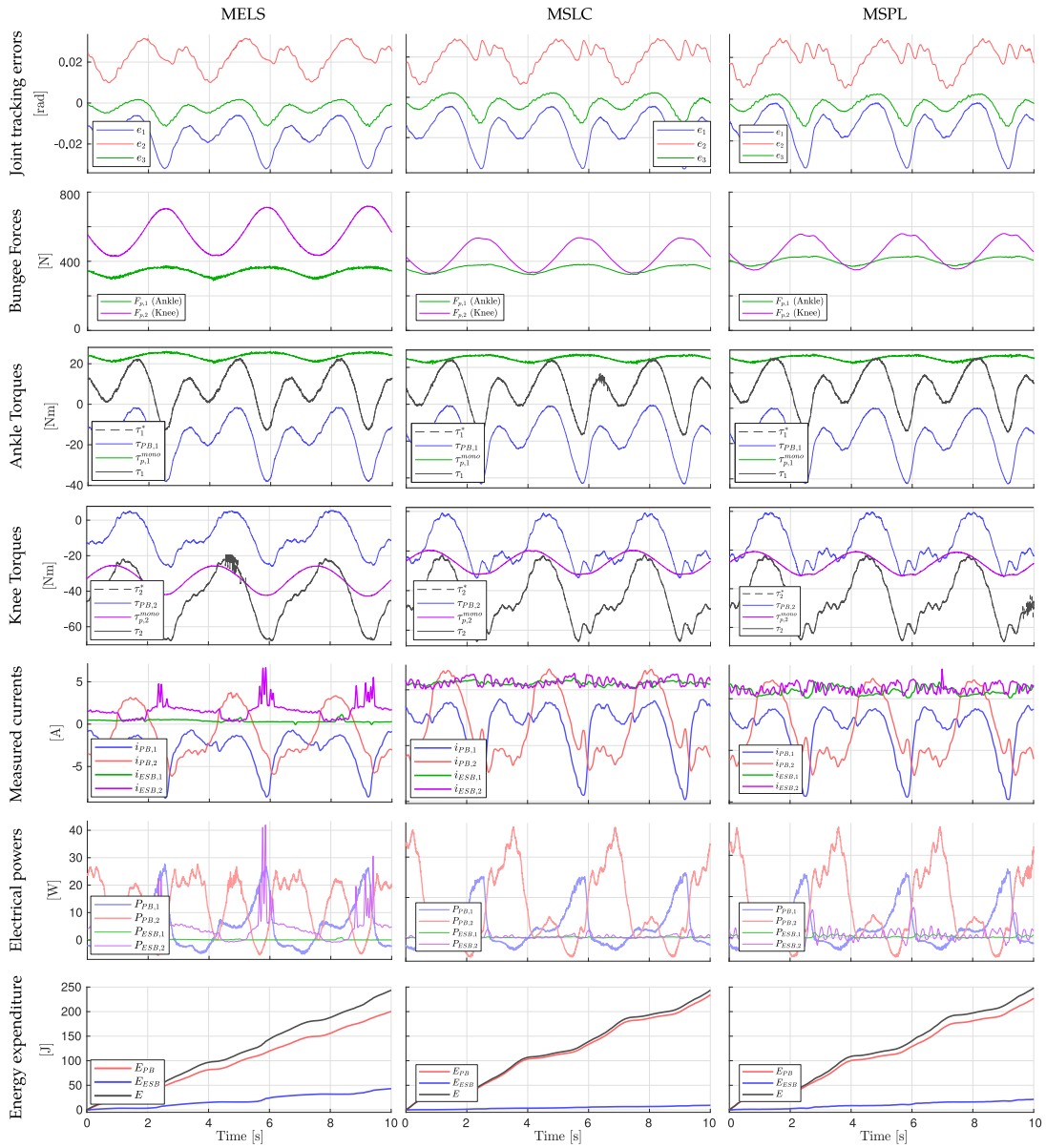


Figure 4.5 Data from **deep squat** experiments (x-amp: 3 cm, y-amp: 15 cm) at 0.3 Hz for *mono* configuration. Energy expenditure while minimizing *Maximize ESB load share* (MELS): 244 J, *Minimize Sum of squared Load Currents* (MSLC): 244 J, *Minimize Sum of Power Losses* (MSPL): 249 J. Associated energy savings are 59% (MELS), 59% (MSLC) and 58% (MSPL).

For the *bi* configuration in 4.6, there are some significant differences in the ESB torques at both ankle and knee joints. While MELS criterion clearly allocates the maximum torque demand to the ESBs, MSLC and MSPL criteria converge to a strategy that demands slightly lesser torque from the ESB actuators.

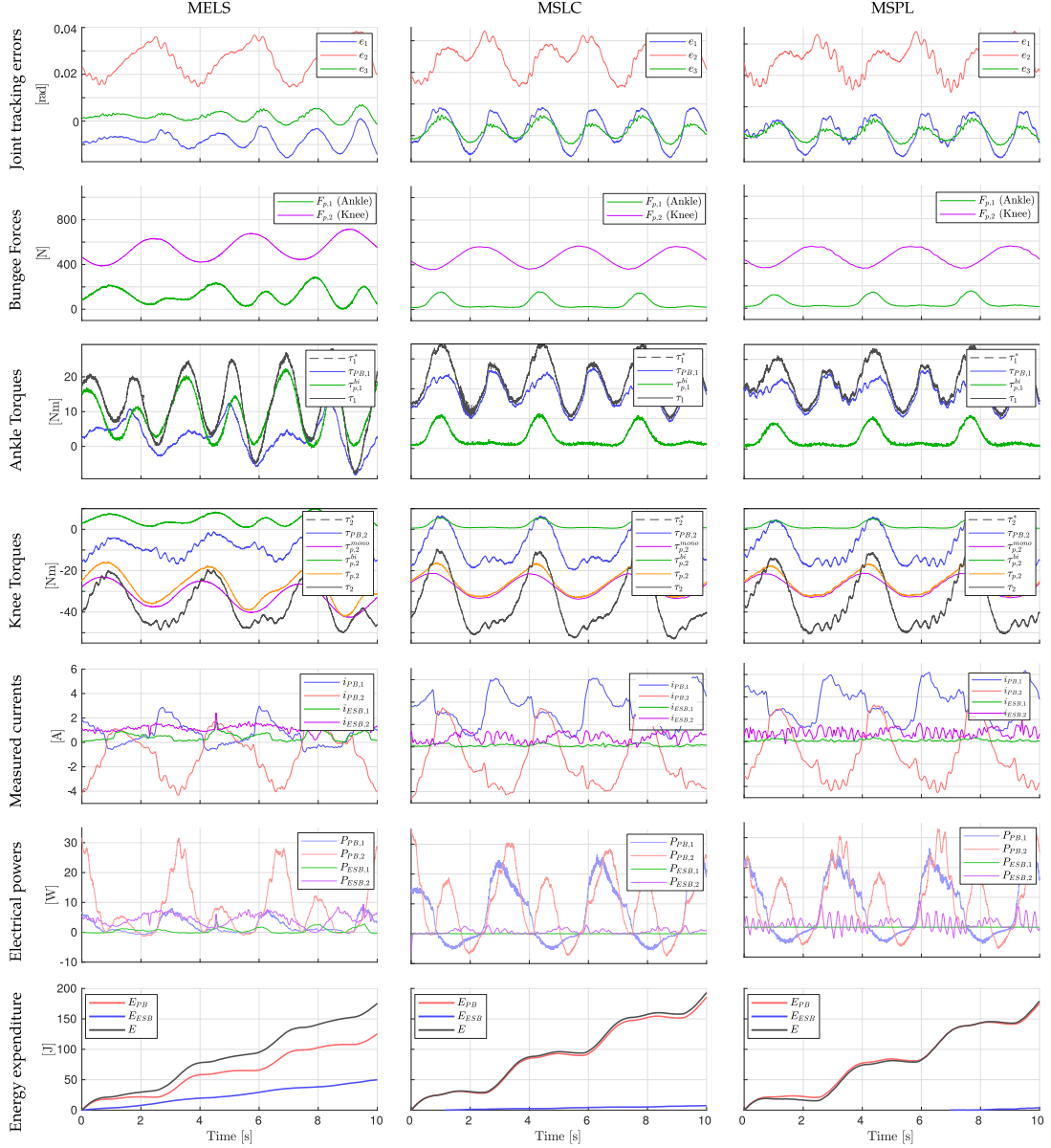


Figure 4.6 Data from **deep squat** experiments (x-amp: 3 cm, y-amp: 15 cm) at 0.3 Hz for *bi* configuration. Energy expenditure while minimizing *Maximize ESB load share* (MELS): 184 J, *Minimize Sum of squared Load Currents* (MSLC): 193 J, *Minimize Sum of Power Losses* (MSPL): 177 J. Associated energy savings are 69% (MELS), 67% (MSLC) and 70% (MSPL).

Ankle ESB currents across all the cost functions are negligible. In comparison to MELS, MSLC and MSPL criteria demand higher positive power from the ankle SEA. It is worth clarifying that while MSLC and MSPL minimize the current and power losses involved with the load torques alone, the plots depict currents and powers that include contributions necessary to overcome friction in the transmission units and drive the actuator inertias to the required accelerations. MSPL once again

performs comparably (slightly better) in terms of energy expenditure than MELS which penalizes PB torques. Significant negative power phases occur with MSLC and MSPL criteria when the robot squats down. This is because ESB tendons are pretensioned lesser than when using the MELS criterion, so the PBs counteract the ESBs lesser to drive the leg in the "squat-down" phase. Even when higher reference torques are demanded when driving against gravity, reference torque demand drops by about 10 Nm during the other phases. Consequently, this phenomenon leads to almost negligible energy consumption during the "squat-down" phase with MSPL. Thus, the criteria that does not directly allocate maximum torque demand to the higher efficiency actuators can still lead to equal or even higher energy-efficiency.

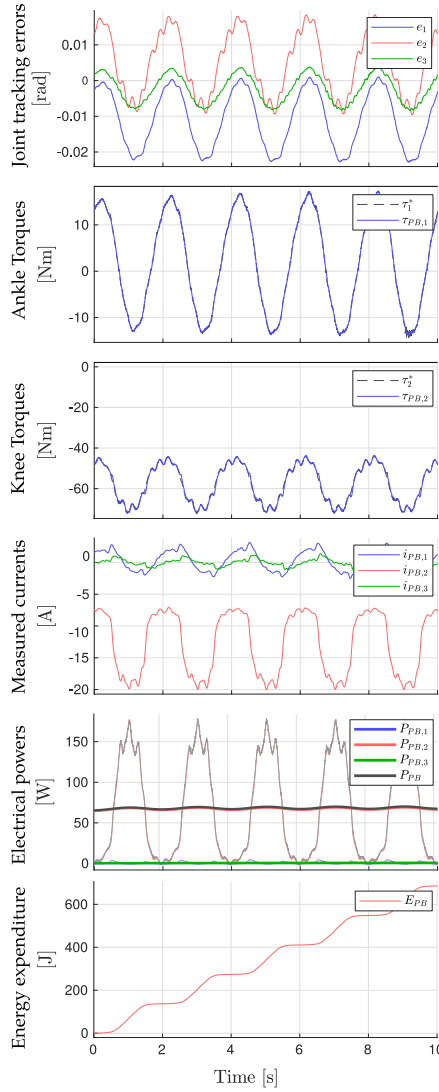


Figure 4.7 Data from **small squat** experiments (x-amp: 3 cm, y-amp: 6 cm) at 0.5 Hz for the *noESB* configuration.

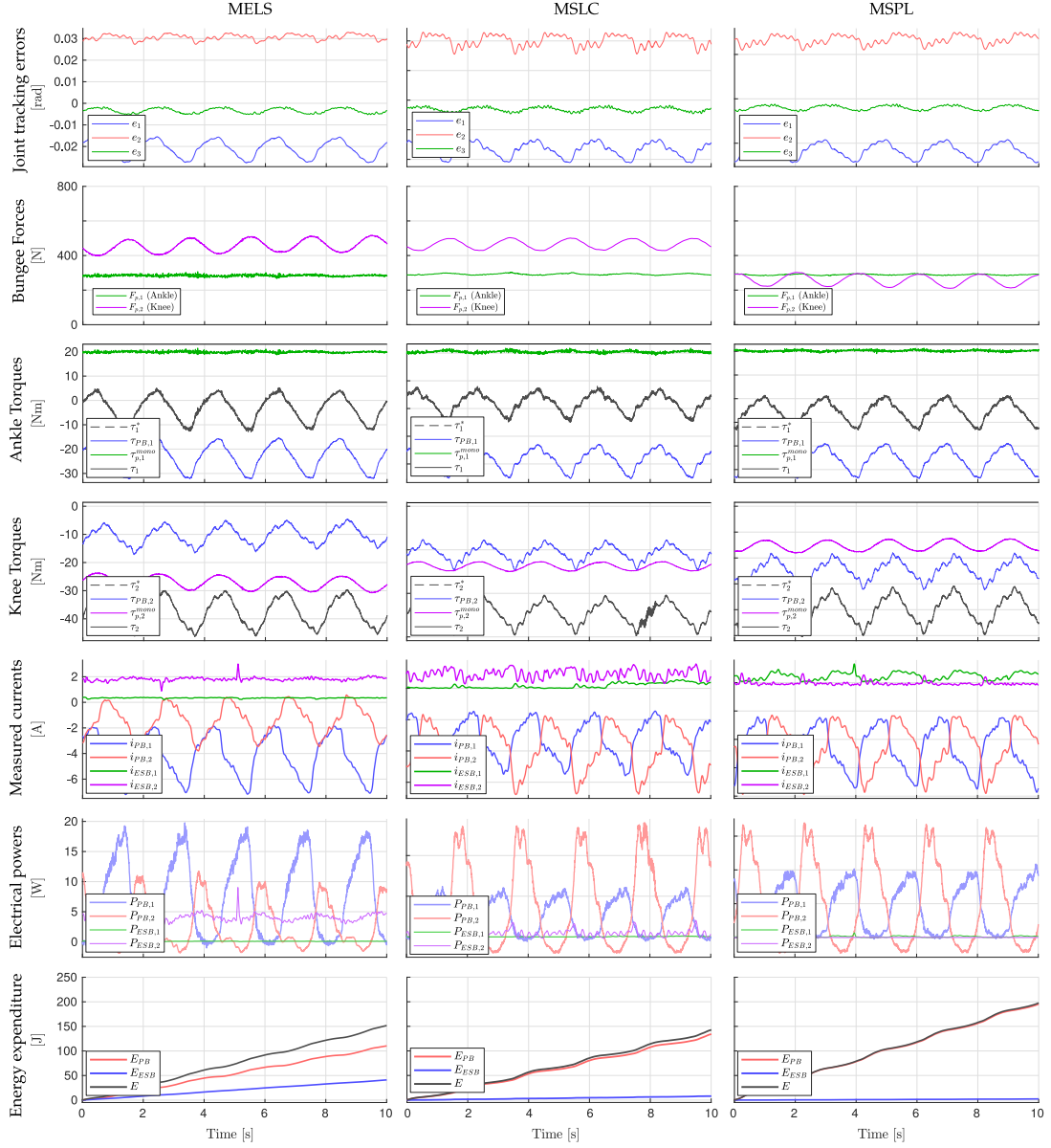


Figure 4.8 Data from **small squat** experiments (x-amp: 3 cm, y-amp: 6 cm) at 0.5 Hz for **mono** configuration. Energy expenditure while minimizing *Maximize ESB load share* (MELS): 152 J, *Minimize Sum of squared Load Currents* (MSLC): 143 J, *Minimize Sum of Power Losses* (MSPL): 198 J. Associated energy savings are 78% (MELS), 79% (MSLC) and 61% (MSPL).

4.4.2 Small squat experiments at 0.25-0.7 Hz

It is further necessary to verify if the energy saved during squat-down phase is higher than the extra energy spent in the squat-up phase with MSLC and MSPL at all times. Therefore, the three candidate criteria are analyzed across a different motion amplitude and frequency. Here, the hip is commanded to track smaller amplitude squat trajectories (x-amplitude: 3 cm, y-amplitude: 6 cm) at 0.5 Hz. Despite the small motion amplitude, it necessitates considerable torques especially at the knee joint, where the torque demand consistently exceeds the maximum continuous torque of the SEA (41 Nm). The noESB configuration consumes about 686 J, see 4.7.

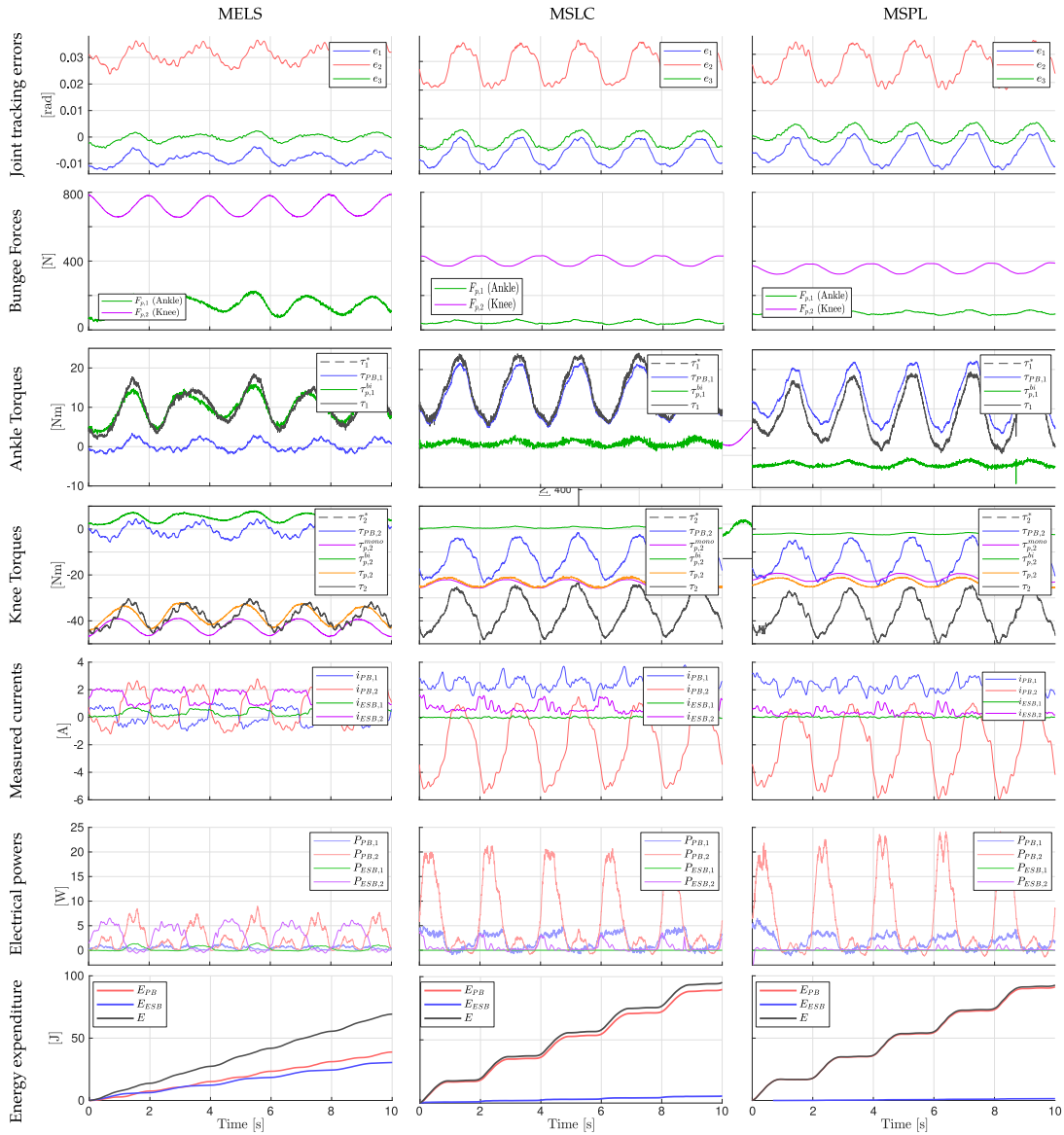


Figure 4.9 Data from small squat experiments (x-amp: 3 cm, y-amp: 6 cm) at 0.5Hz. *Maximize ESB load share* (MELS): 69 J, *Minimize Sum of squared Load Currents* (MSLC): 95 J, *Minimize Sum of Power Losses* (MSPL): 93 J. Associated energy savings are 90% (MELS), 86% (MSLC) and 86% (MSPL).

With the *mono* and *bi* configurations, similar inferences were made about the pretension references and gravity-driven phases. While there was an overall lesser pretensioning of the ESBs with MSLC and MSPL criteria, the MELS criterion performed the same task more efficiently, see Fig. 4.8-4.9 in both configurations. While MSPL fares poorly in comparison with MELS in both cases, MSLC performs comparably equal in the *mono* configuration. Compared to the other two strategies, the excess energy that MELS entails in the "squat-down" phase is lower than the energy saved during the "squat-up" phase. Intuitively, since the torque reference variations are smaller than in the previous case, the knee PB had to oppose the ankle ESB lesser during "squat-down" phase, leading to higher overall efficiency. These results advocate for a case-by-case analysis when choosing the torque distribution strategy with amplitude and frequency variations in trajectories. Further, the same will also depend on the relative dynamics of the dissimilar PB and ESB actuators. In both the previous sets of experiments (including

deep squat experiments in the previous subsection), the *bi* configuration leads to more energy savings than the *mono* configuration, in general emphasizing the need for inter-joint power transfer in legged robots.

Secondly, given the higher savings, it is more interesting to observe the relative performance of the three candidate criteria for the same motion executed at different frequencies with the *bi* configuration. Therefore, a second set of experiments are conducted involving different frequencies. Owing to the limits of actuator performance (poor joint tracking), experiments were conducted only up to 0.7 Hz. Results from this study are plotted in terms of the obtained energy savings over the *noESB* configuration in Fig.4.10. It can be seen that at higher frequencies, the energy savings obtained with the MELS strategy, which relies on the maximum ESB utilization, drops below the savings of other criteria namely, MSPL and MSLC. Consequently, there is lesser opposition to the robot being driven by the gravity potential. This is in line with the ESB characteristics as they cannot be pretensioned at velocities higher than 0.007 m/s and cannot account for the necessary ESB torque variations with the MELS at higher frequencies. Although the conclusion is obtained for the executed trajectory and the same may vary with amplitudes, a similar trend in the variation of these efficiencies for periodic squat motions can be expected. However, the crossover point, the frequency at which MSPL overtakes MELS in efficiency could vary with different motion amplitudes.

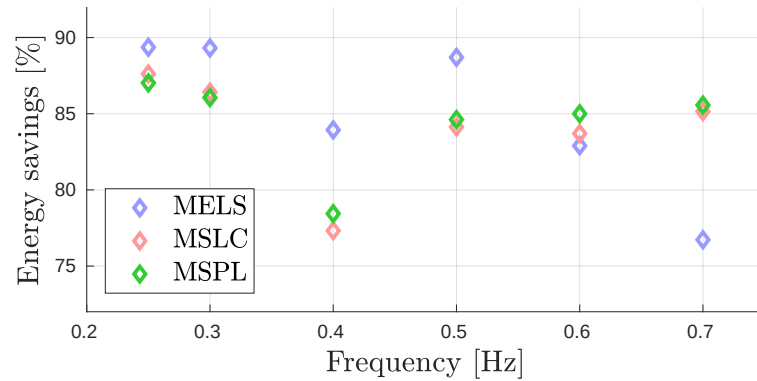


Figure 4.10 Energy savings with the three criteria from squat experiments (x-amp: 3 cm, y-amp: 6 cm) for the *bi* configuration with respect to the *noESB* configuration while eLeg executes squats across a range of frequencies.

4.5 Discussion

4.5.1 Optimization-based joint torque resolution

Torques at robot joints that exhibit multi-articulated couplings depend on the joint configuration of all spanned joints. In such cases, resolving the joint torques among the various actuators is not straightforward. In this work, a QP-based control scheme is used to instantaneously compute reference pretensions for PEAs (ESB) based on the certain energetics-based criteria. A gradient-descent based control law that generates desired references for PEAs was developed in [120]. In this work, the controller can remain completely unaware of the higher level planning strategy and requires only information of the desired joint torque and position feedback from the actuators. Since eLeg features PEAs that have a very poor torque control bandwidth and their tendons feature dominant S-shaped characteristics, the QP-based controller currently uses position feedback to predict desired pretension

references. However, it is also shown that the controller can be reformulated within the QP framework to predict reference force profiles as well.

4.5.2 Experimental results

Since several locomotion tasks such as walking, running, hopping, etc are cyclic motions, an experimental analysis is conducted during periodic squat motions with different motion amplitudes and frequencies on different actuation configurations of *eLeg*. Three distinct candidate criteria are used for this analysis. While MELS criterion is solely inspired by the knowledge of varying actuator efficiencies, the other two criteria are derived due to the differences in actuators' hardware characteristics. Specifically, the analysis is made to verify if the MELS criterion (maximum torque demand allocation of higher-efficiency actuators) is the most energy-efficient owing to the higher ESB efficiencies. The observations made are two-fold. A first set of experiments indicate that MSPL or MSLC can perform equally well at times as the MELS by bringing about motion phases with negative power flows. In a second set of experiments, it is found that the conclusion previously obtained does not hold for a trajectory with a different motion amplitude and frequency. Therefore, a case-by-case analysis should be adopted to determine the best strategy that achieves highest energy savings.

In order to extend the analysis, squat experiments across a range of frequencies for the *bi* configuration are executed. While the MELS results in higher energy savings at lower frequencies, the higher pretensioned biarticulated ESB also hampers the PBs from being back-driven at higher frequencies. Especially, the MSPL criterion brings about negligible energy costs through longer gravity-driven phases. Though not described explicitly, this behaviour naturally emerges out of the controller.

4.5.3 Limitations

The RMS value of the cartesian hip position tracking errors for all reported experiments, across various criteria and configurations of *eLeg* are all below 5 mm. At frequencies above 0.5 Hz, the lower level impedance controller required considerable tuning effort. Moreover, tracking quality was marginally affected at such frequencies entailing higher reference torques, however, the maximum joint tracking error never exceeded 0.05 rad.

It is worth mentioning that the efficiency of torque generation varies between actuators and depends on the actuator inertia, gearbox friction in the transmission units and fatigue due to thermal effects, etc. This transmission efficiency is not a constant and varies with temperature, loading, velocity, component wear, etc. For instance, the efficiency of CPL-20 HD varies between 45-88% [157]. Additionally, the motor torque constant decreases with thermal fatigue. Nevertheless, uniform efficiencies are regarded for all actuators. By linear regression, one can obtain best fits for efficiency profiles under certain operating conditions and incorporate the knowledge in this framework.

4.6 Summary

In this chapter, an analysis is conducted on the *eLeg* that is powered by modular ACA branches to identify energy-efficient joint torque distribution strategies. An optimization based controller is presented which upon using quadratic criteria can provide instantaneous reference pretensions for the parallel actuators. An energy-based analysis is conducted to check if delegating the maximum torque demand to the highly efficient ESBs is always the best strategy.

Observations from one set of cyclic squat experiments indicate that the MSPL by lengthening negative power phases attains comparable energy savings with MELS strategy. From another set of experiments where eLeg executes squats across a range of frequencies, it is found that at lower frequencies although MELS is more effective, MSLC and MSPL can result in better efficiencies at higher frequencies as MELS results in longer actuator-driven motion phases.

Future work would be to account for thermal efficiencies and plan for longer horizons in a model-predictive fashion and obtain optimized pretensions specific to periodic motion amplitudes and frequencies. During extended operations, demanding higher torques from the ESBs once the SEAs are fatigued may hold the key to enduring autonomy. A valuable extension would be to integrate the efficiencies and their thermal variations specifically in (4.9) and (4.10). Moreover, suitably simplifying and incorporating non-quadratic criteria such as joint work minimization into the presented framework might provide additional insights.

Chapter 5

Parallel Compliant Multiarticulation for explosive motion generation

Legged robots are expected to be agile and dynamic to navigate through unstructured terrains. SEA/VSA powered legged robots feature gearboxes with high transmission ratios. High gear ratios reduce the joint output velocities substantially. Since higher speeds can be achieved only through higher voltages, operating voltage is what essentially determines achievable peak joint velocity.

To alleviate these voltage constraints, field weakening can be employed on the fly along with any high-level motion control schemes. The method draws on the nonlinearities in the actuator electrical dynamics to reduce back-EMF. Hence, in this work, the synergistic benefits of coupling field-weakening with biarticulated PEAs are investigated. The proposed concept combines torque and inter-joint power transfer benefits of parallel and biarticulated articulation with the velocity increase permitted by FW to amplify the explosive power output in a monopod robot prototype, *eLeg* shown in Fig. 5.1.

5.1 Organization

The chapter ¹ is organized as follows. Sec. 5.2 provides a brief overview of the adopted field weakening controller. The jump controller is presented in Sec.5.3 and the experimental results are reported in Sec. 5.4. An engaging discussion follows in Sec.5.5. The chapter ends with a summary in Sec. 5.6.

5.2 Field weakening for Series Elastic Actuators

All three joints of *eLeg* are identical and powered by Alberobotics Orange SEAs [145]. These SEAs are driven by Surface Permanent Magnet Synchronous Motors (SPMSM). The steady-state voltage

¹A part of the work included in this chapter has been accepted for publication in *Robotics and Automation Letters* (RA-L), January 2021 under the publication title *Blending of series-parallel compliant actuation with field-weakening control for explosive motion generation* by Vishnu Dev Amara, Jörn Malzahn, Wesley Roozing and Nikos Tsagarakis.

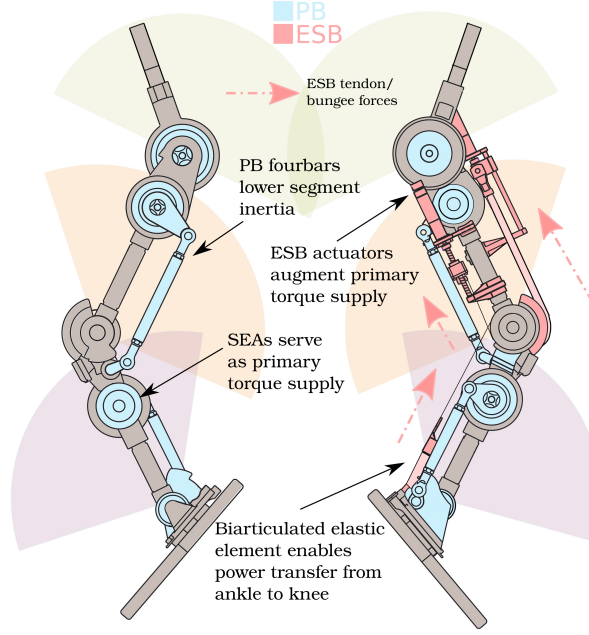


Figure 5.1 Schematics of the *noESB* and *bi* configurations of eLeg. The tinted areas indicate the ranges of joint motion.

equations for a SPMSM as described in [158] are,

$$\begin{aligned} v_q &= R i_q + n_p N L i_d \dot{\theta} + \underbrace{n_p N \psi \dot{\theta}}_{\text{back-EMF}}, \\ v_d &= R i_d - n_p N L i_q \dot{\theta}, \end{aligned} \quad (5.1)$$

L is the inductance along both the current axes (for SPMSM, $L_d = L_q = L$), ψ is the flux linkage of the motor, n_p is the number of poles in the motor and R is its electrical winding resistance. $\dot{\theta}$ and N are the gear output velocity and gear ratio of the particular SEA. v_{max} and i_{max} are the maximum inverter phase-voltage and phase-current. The SEA electromotive torque in (4.2) can be related to the quadrature current as,

$$\tau_m = \underbrace{\frac{3}{2} n_p \psi N}_{k_\tau} i_q. \quad (5.2)$$

where k_τ is the torque constant of the motor. Note that only the quadrature current contributes to torque-generation. The electrical dynamics (5.1) is subject to constraints on the available current and voltage reserves:

$$v_d^2 + v_q^2 \leq v_{max}^2, \quad i_d^2 + i_q^2 \leq i_{max}^2, \quad (5.3)$$

where v_{max} and i_{max} are the maximum values of available inverter DC voltage and supply current. From (5.1), quadrature voltage is unfavourably increased by the back-EMF term while negative direct current can bring about reduction in the same voltage. However, direct current does not lead to any usable work at the output.

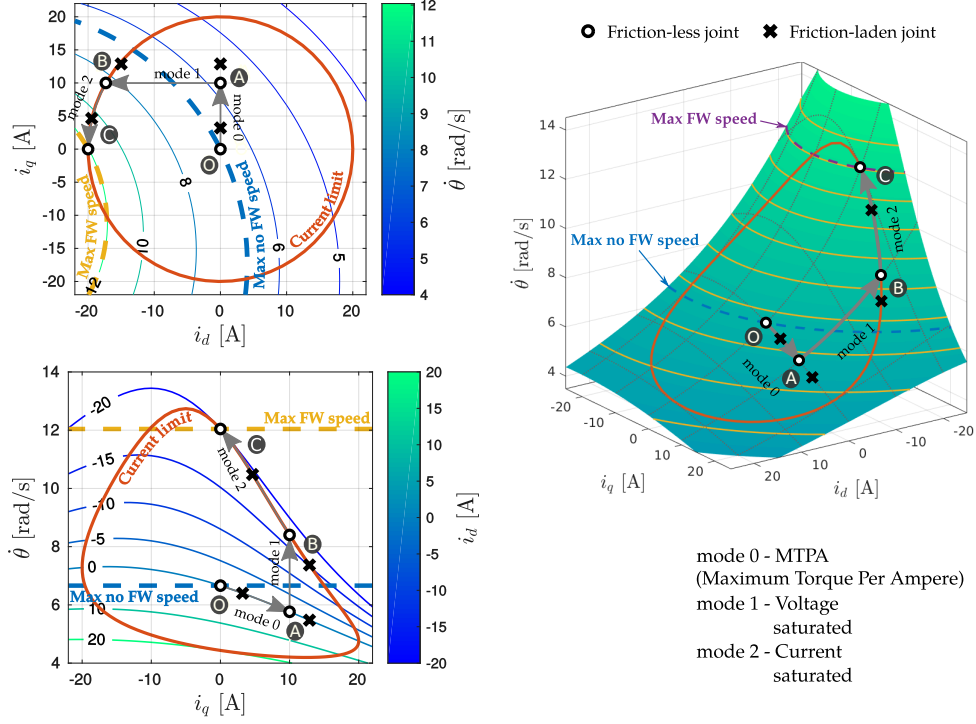


Figure 5.2 FW controller schematics on the upper halves of $i_q - \dot{\theta}$ plane, $i_d - i_q$ planes. Solid isolines indicate voltage limit curves for various i_d currents or speeds. Current limit is depicted by red loci. Dashed horizontal lines indicate maximum no-FW speed and maximum FW speed at the gear output, in the frictionless case. Grey arrows indicate directions along which the current references are generated by the FW controller.

Therefore, it becomes important to modulate currents and voltages within these constraints. Conventional current control operation minimizes direct current magnitude to maximize electrical efficiency. This is called the Maximum Torque per Ampere (MTPA) operation and limits the maximum achievable speed (base speed) as,

$$\dot{\theta}_{base} = \frac{v_{max}}{n p \psi} \quad (5.4)$$

Hence, the maximum achievable motor speed is limited by the maximum available DC voltage. Yet, the quadrature voltage magnitude can be reduced leading to a higher peak motor velocity within the supply voltage. This operation is *field-weakening* action and it also reduces the maximum feasible quadrature current since the supply current is limited as in (5.3). At the expense of usable current reserves (and hence torque capacity), the voltage requirements for a given velocity are brought down. Consequently, during FW, for a given magnitude of the current vector, the maximum motor torque is reduced. A previously feasible task may become unfeasible due to the torque sacrifice. It is also worth noting that in the $d - q$ plane, FW operation allows to avoid voltage saturation for higher current vector magnitudes.

5.2.1 Controller

To realize FW action, the state-less inverse electrical dynamics controller presented in [91] is used. The action of this controller is schematically visualized in Fig. 5.2 with grey arrows for the frictionless case. For controlling the individual current components, a decoupled current controller that includes feedforward terms, decoupling terms and P-I terms on the current errors is utilized.

The FW controller regulates current references for d - q components according to desired torque and current velocity. When torque demand and the current velocity does not saturate the voltage, the controller performs MTPA operation which happens between point O to A (**mode 0**) in Fig. 5.2. Point O represents the maximum speed achievable in the absence of FW, also described by (5.4). Point A is when the torque demand is met - the torque that is just enough to compensate for frictional losses and damping. Point A is also the speed at which voltage reserves are used up. Mode 0 also corresponds to operation along the speed-torque curves usually found in electrical motor datasheets [159].

Beyond A, the FW controller starts increasing the direct current magnitude in the negative direction while maintaining the level of quadrature current. This can happen until when the current reserves are fully depleted, point B. This is the voltage saturated phase (**mode 1**).

Beyond B, both current and voltage reserves are depleted and quadrature current level cannot be maintained anymore. Hence direct current is increased at the expense of quadrature current. Speeds beyond B are achievable as long as the quadrature current (and resulting torque) can compensate for frictional effects. This phase stretches until point C (**mode 2**) called the current-saturated phase.

Finally, although the theoretical maximum attainable FW speed is quite high for the frictionless joint, in practice, this value is reduced due to motor and gearbox friction, inertial effects and external forces. The aforesaid points are also plotted in the presence of friction, see points 'x' adjacent to the corresponding 'o' in Fig. 5.2. Coulomb and viscous friction values used are that of the Orange80 motors. The electrical parameters of the motors utilized in this work are tabulated in Tab. 5.1.

5.3 Jump Controller

In this section, the procedure adopted for obtaining joint motion plans for jumping is presented followed by the control architecture.

5.3.1 Joint motion plans

Jumping motion plans are obtained for both *eLeg* actuation configurations by tuning for position trajectories of the ankle, knee and hip joints. In particular, these jumping plans lead to rapid leg extension. Second order continuous (C^2 -continuous) joint reference trajectories are parametrized by the initial and final positions and velocities and trajectory duration. The particular parametrization is mathematically derived in Appendix C. Given the joint configuration limits, joint positions are initialized such that *eLeg* squats with its Center of Mass (CoM) is at its lowest possible position while maintaining its Center of Pressure (CoP) within the area of the foot. This is done to maximize the feasible range of

Table 5.1 Orange80 electrical specifications

Specification	Symbol	Unit	Value
Current limit	i_{max}	A	20
Voltage limit	v_{max}	V	$48/\sqrt{3}$
Winding resistance	R	Ω	0.318
Number of pole pairs	n_p	-	6
Gear ratio	N	-	80
Torque constant	k_τ	Nm/A	0.078
Inductance	L	mH	0.2

joint motion. The final configuration of the robot at the moment of take-off is tuned based on heuristics such as a near-singularity robot pose and an extended ankle configuration. By heuristically setting for the curve duration, joint position and velocity at final time instant, joint motion plans can be obtained. Ballpark figures for the various parameters were obtained from extensive numerical simulations with [160]. The extension duration is limited to under 0.5 s to obtain a highly dynamic leg extension. Thus, joint position and velocity references are obtained by experimentally tuning for the best possible vertical CoM displacement. The process is repeated for each actuation configurations for both *noESB* and *bi* configurations, see Fig. 5.3. Jump motion references are tuned for the two configurations such that when they are applied on the corresponding configuration, the angular momentum of the robot at the apex is not significant enough to cause large deviations from an upright pose.

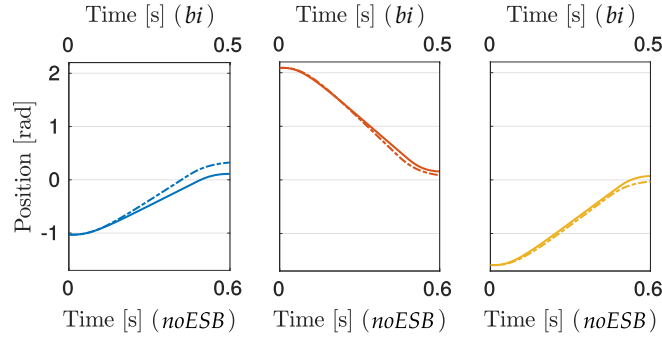


Figure 5.3 Joint motion references used for the *noESB* (solid) and *ESB* (dash-dotted) configurations for ankle (blue), knee (red) and hip (yellow) joints until about the apex are shown.

5.3.2 Control architecture

The control architecture adopted for this work is shown in Fig. 5.4. An inverse dynamics controller with a PD joint impedance loop turned out to be sufficient for performing the jumping motion. Given the lack of ground constraint force estimates, the constraint force terms in the joint-space inverse dynamics controller are neglected:

$$\boldsymbol{\tau}^* = \mathbf{M}_a(\mathbf{q}) \ddot{\mathbf{q}}^* + \mathbf{n}_a(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}^* - \mathbf{q}) + \mathbf{K}_d(\dot{\mathbf{q}}^* - \dot{\mathbf{q}}), \quad (5.5)$$

where $\mathbf{M}_a$ and $\mathbf{n}_a$ comprise the rows of the joint-space inertia matrix and joint space bias forces (Coriolis, damping and gravity torques) that correspond to the actuated DoFs ($\mathbf{q}_a$).

Moreover, the robot lacks real-time estimation of foot position and orientation in the world frame, hence no balance controller upon landing is employed. For the same reason, the bias force terms are inaccurate once the foot is not flat on the ground. Therefore, to ensure reasonable joint position tracking, the following diagonal impedance gain matrices are used, $\mathbf{K}_p = \text{diag}([1500, 1600, 1200])$ Nm/rad and $\mathbf{K}_d = \text{diag}([12, 13, 10])$ Nms/rad. These values are obtained by experimental tuning.

For the ESB (shown in red), a cascaded controller with 1 kHz outer PD position control loop and an inner 20 kHz MTPA based PI current control loop is employed. The pretensions are empirically tuned to 14 mm and 45 mm for the biarticular and monoarticular ESBs for all the *bi* configuration jumps. Although the ESBs can be actively controlled, their maximum translational velocities are lower than 8 mm/s. Due to sub-second jump durations, the maximum change in bungee pretensions is limited to 4 mm. Hence, constant pretensions are maintained during the jumps.

For the PB (shown in blue), multi-rate cascaded controller is adopted. PB torque references are computed by subtracting the ESB tendon contributions from the joint torque references. Subsequently, the FW controller converts them to current references in the d - q plane at 1 kHz following the strategy described in Sec. 5.2. In the absence of FW, the torque references are directly converted to quadrature current references. Two inner d - q current control loops that comprise feedforward, back-EMF, cross-coupling and PI terms are utilized for the current control, see (5.1). While, torque and current references are generated from Simulink-RealTime at 1 kHz, current control is executed in the embedded DSP boards of the actuator drivers at 20 kHz. A current limit of 20 A (~ 124 Nm) is applied on all SEAs across experiments.

Instantaneous electrical joint powers can then be computed as,

$$\mathbf{P}_e = \underbrace{\frac{3}{2}(\mathbf{v}_q^T \mathbf{i}_q + \mathbf{v}_d^T \mathbf{i}_d)}_{\text{PB}} + \underbrace{\mathbf{v}_p^T \mathbf{i}_p}_{\text{ESB}}, \quad (5.6)$$

where the voltages are computed using (5.1) with current measurements from sensors on the embedded electronics.

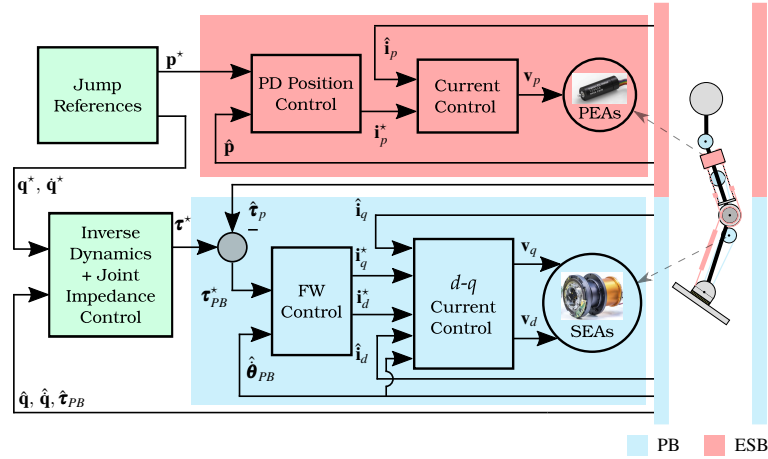


Figure 5.4 Schematic of the controller used in this work. ($\hat{\cdot}$) and ($\cdot^*$) refer to measured and reference signals.

5.4 Experiments

Explosive performance of the jumping motion is rated by measuring the CoM displacement at apex from a standing pose (singularity). This CoM stance displacement is alternatively referred to as the *jump height*. Instantaneous CoM position is computed during post-processing through forward kinematics after obtaining foot angle and position estimates in the inertial frame through motion capture cameras. The jump experiments are executed on an Intel NUC PC with 2GB DDR-3 RAM and a 1.8 GHz Core i5-3427U processor. The hip joint does not require substantial torques as compared to the other joints during the explosive motion. This prevents the onset of voltage saturated phase and the triggering of FW for that joint. In addition, the ankle and knee joints are the major contributors to vertical CoM velocity and are augmented with ESBs. Only experimental details that correspond to ankle and knee joints are presented.

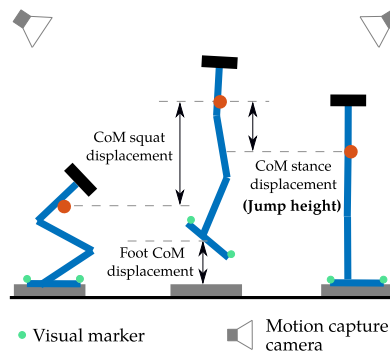


Figure 5.5 Visual description of the various measured CoM displacements. Indicated displacements and velocity are computed for the vertical direction. Red dot indicates CoM of the monopod. *Jump height* is measured as CoM stance displacement

5.4.1 Jumps tuned for *noESB* configuration

Fig. 5.6 shows results of experiments where joint references are tuned for the *noESB* configuration to maximize its vertical CoM displacement. Data is shown from start of the jumping motion to the instant the CoM reaches apex height. Table 5.2 summarises numerical details including CoM and foot displacements, vertical take-off velocity at take-off, peak joint velocities and torques, average mechanical and electrical powers. Measured SEA currents are filtered with a 200 Hz low-pass filter. Average electrical and mechanical powers as computed as the sum of positive electrical and mechanical powers from ankle and knee joints.

Jump #1

In the absence of FW (left column of Fig. 5.6), from the SEA d - q current tracking plots, voltage saturation can be observed on account of high joint velocities. While direct currents are minimized, the quadrature current tracking quality worsens as the jump motion progresses. It is also visible that the peak joint velocities are all below 6.6 rad/s (maximum no FW speed, see Fig. 5.2). As the robot extends and approaches singularity, the torque output reduces. The same can be noticed in the torque plots. It is clear from the power plots that knee is the major contributor to increasing the vertical CoM velocity initially. At the point of take-off, around a maximal extension pose, ankle is the dominant contributor to the jumping motion. The *noESB* configuration achieves a CoM displacement of 9.6 cm at apex from stance pose without FW.

Jump #2

Data is shown for the same joint references with FW enabled on the *noESB* configuration in the middle column of Fig. 5.6. Although the joint motion references were tuned for the *noESB* configuration in the case without FW, the actual joint motions lag behind the references. Therefore, FW modes engage as long as the velocities and quadrature currents are high enough to saturate the voltage. The action of FW controller is visible from SEA d - q current plots - joints avoid voltage saturation maintaining good current tracking quality. Knee current reference tracking suffers slightly at the end, potentially due to inter-motor electrical parameter variations. The achieved CoM displacement from stance is 11 cm, a 15% increase over without FW.

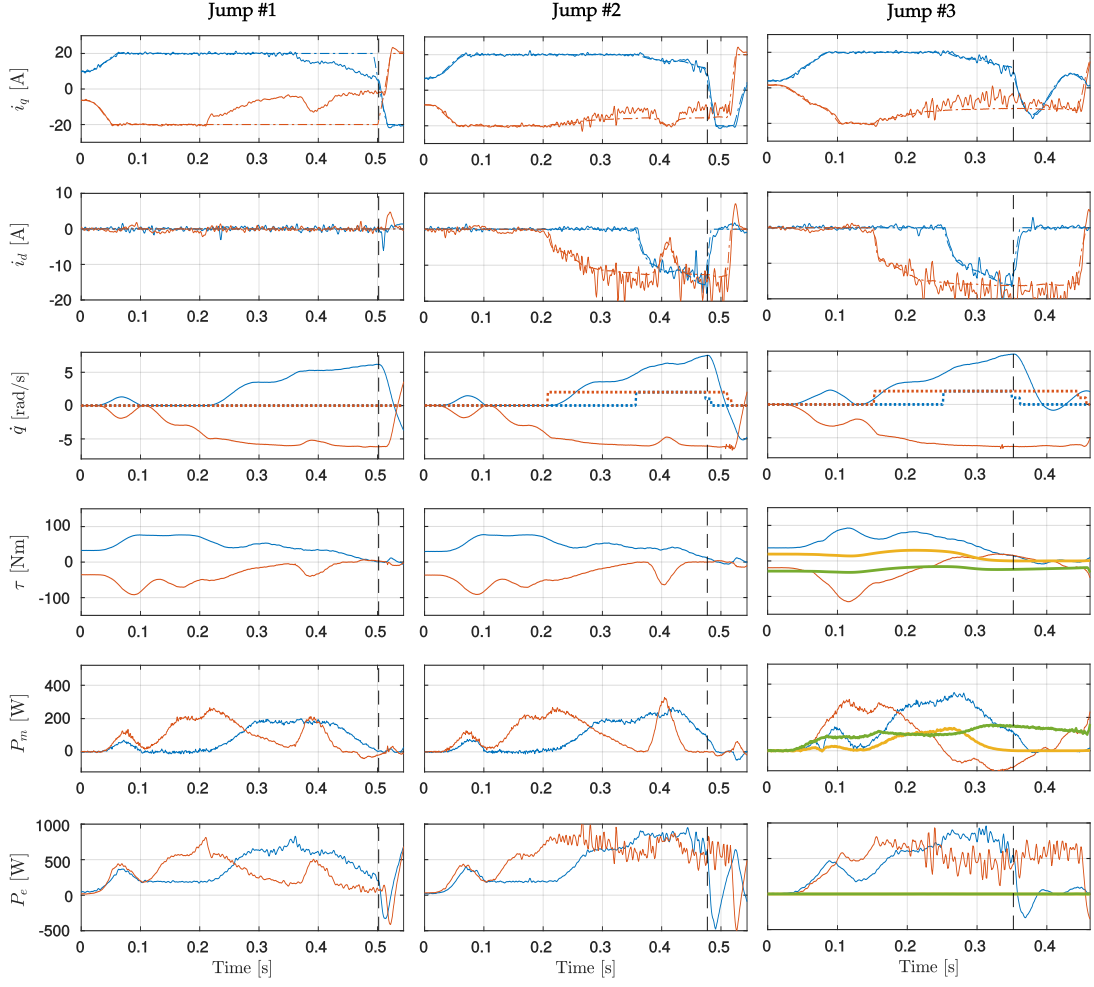


Figure 5.6 Jump experiments with references tuned for *noESB* configuration, for **ankle** (blue) and **knee** (red): left column (*noESB*, **no FW**, **jump #1**), middle column (*noESB*, **with FW**, **jump #2**) and right column (*ESB*, **with FW**, **jump #3**), dash-dotted lines (references), solid lines (measured). SEA d - q current tracking, resulting joint velocities, torques, mechanical and electrical powers are shown. Torques and powers corresponding to ankle-knee biarticular ESB and knee monoarticular ESB are shown with yellow and green solid lines respectively. Dotted lines indicate FW modes. Vertical dashed lines indicate take-off moments. CoM stance displacement: **9.60 cm** (*noESB*), **11.01 cm** (*noESB* with FW) and **13.17 cm** (*ESB* with FW).

Although, the SEA quadrature current and torque seem saturated, they are not enough to move the ankle. As seen in the velocity plot, while FW increases peak velocities, the ankle remains stationary during the initial part of the motion (between 0.1 to 0.2 s). This observation asserts that the ankle can potentially benefit from additional torques from PEAs and lead to improved jump motions.

Jump #3

The same motion references tuned for the *noESB* configuration are applied to the *bi* configuration with FW enabled. Fig. 5.6 illustrates the corresponding data in the right column. SEA current plots indicate that the tracked quadrature currents are noticeably smaller than jump #2 (*noESB* with FW). Yet, the peak torques are higher as reported in table in Table 5.2. This is a consequence of joint torque augmentation by the ESBs. The additional ESB torques contribute to higher joint accelerations which lead to quicker velocity build-up as compared with jump #2. Note that although the peak velocities are comparable

with jump #2, the peak torques are higher than jump #2. Increase in peak velocity at knee (6.6 rad/s) is limited when compared to the ankle (7.5 rad/s). Additionally, notice that the velocities are not any higher than the ones obtained in jump #2.

The knee joint appears to remain saturated at its peak velocity from about 0.27 s throughout the jump. Moreover, although both the torque-generating quadrature current at SEA as well as its current reference are negative, the net joint torque at the knee changes sign and becomes positive. This is due to an power influx from the multi-body robot dynamics as well as the biarticular power transfer from knee to ankle. The phenomenon is exemplified in [98], where timely activation/deactivation of the biarticular spring can help prevent this undesired effect. In particular, biarticulation can remain deactivated during the initial phase of the jump. However, in eLeg, the unilateral ESB tendon forces cannot be activated/deactivated at will during mid-motion. Nevertheless, owing to the ESB tendons, the mechanical power output from the two joints are higher than the *noESB* case leading to a jump height of 13.2 cm, 37% higher than jump #1. Electrical power consumption decreases when compared to jump #2, as a result of the stored elastic energy in the bungees. Furthermore, given the presence of biarticulation enabling power transfer from knee to ankle, the contribution of ankle power to the heel push-off and hence the jump motion performance is substantially increased.

5.4.2 Jumps tuned for *bi* configuration

The *bi* configuration in jump #3 brings about a marked enhancement over the *noESB* configuration. However, mass and inertia distribution across the various links in the *bi* configuration differs from the *noESB* configuration. Due to the differences in actuation, investigation is made to probe if joint motion references specifically tuned for the *bi* configuration can further enhance jumping performance. Corresponding results in the presence of FW (jump #5) and with FW disabled (jump #6) are reported in Fig. 5.7. Since the reported experiments involve hand tuning, to establish a fair comparison, the same motion trajectories are applied on the *noESB* configuration as well (jump #4), also shown in Fig. 5.7. Additional numerical details are displayed in Table 5.2.

Jump #4

From jump #4 (shown in the left column of Fig. 5.7), the robot CoM pitches too much forward owing to significant angular momentum from the extension motion. This is because references suitable for the *bi* configuration are not suitable for *noESB* configuration. Although joint velocities exceed 6 rad/s and joint torques and mechanical power profiles are similar to the previous *noESB* jump (#1), ensuing CoM vertical displacement from a stance pose is only about 2.6 cm, significantly worse than jump #1. Moreover, the robot does not land safely after the jump.

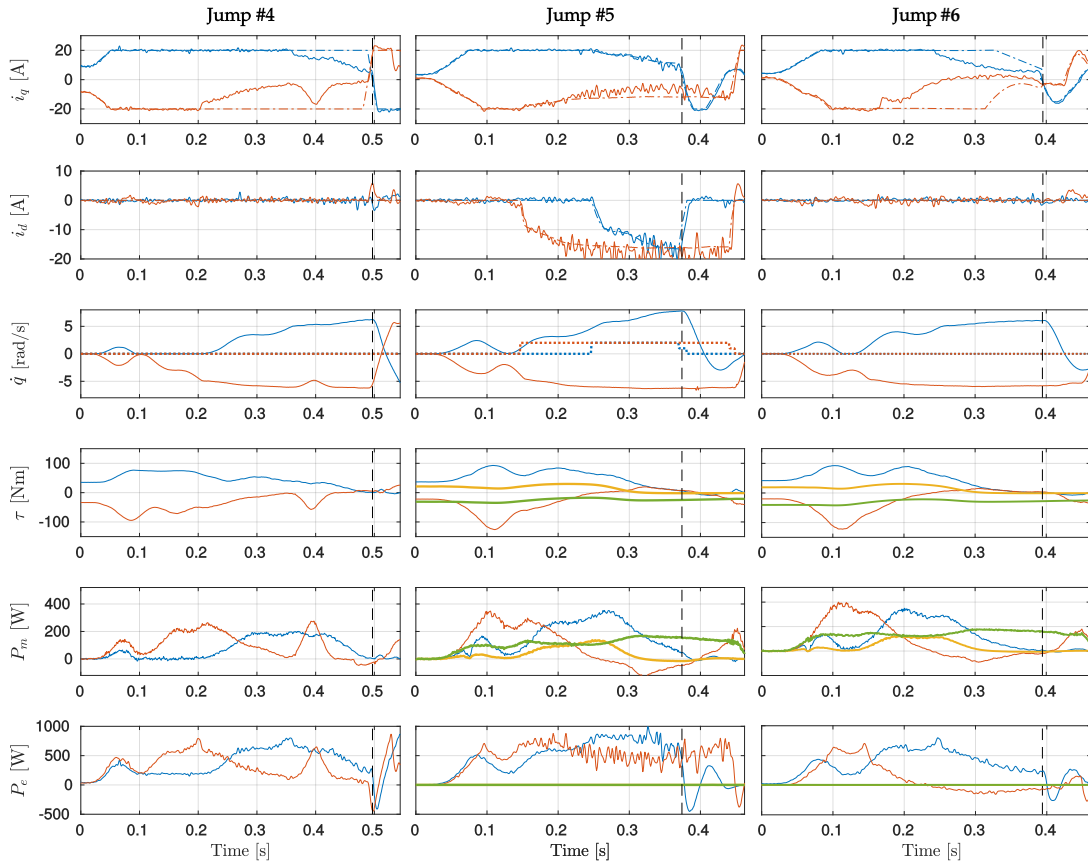


Figure 5.7 Jump experiments with references tuned for *bi* configuration, for **ankle** (blue) and **knee** (red): left column (***noESB* without FW, jump #4**), middle column (***bi*, with FW, jump #5**) and right column (***bi*, without FW, jump #6**), dash-dotted lines (references), solid lines (measured). SEA *d-q* current tracking, resulting net joint velocities, net joint torques, net joint mechanical and electrical powers are shown. Torques and powers corresponding to ankle-knee biarticular ESB and knee monoarticular ESB are shown with yellow and green solid lines respectively. Dotted lines in velocity plots represent FW modes. Vertical dashed lines indicate take-off moments. CoM stance displacement: **2.62 cm (jump #4), 14.78 cm (jump #5), 12.74 cm (jump #6)**

Jump #5

As for the *bi*-tuned motions on the *bi* configuration (jump #5), depicted in Fig. 5.7, the PEAs supply part of the joint torques and hence allow to lower torque generating quadrature current references for SEAs. As a result, higher current reserves are available to be leveraged by the FW controller. Current plots readily indicate tracking of greater direct current magnitudes at both joints for a larger portion of the jump motion than the previous *bi* experiment (jump #3), especially for the ankle. Consequently, the joints achieve the highest velocities and torques with this particular jump, among all the jumps reported in this work (as shown in the table in Fig. 5.5). These observations can be ascribed to the motion references being specifically tuned for the *bi* configuration. Resulting CoM stance displacement is 14.8 cm, an increase of 54% over the *noESB*, no FW case (jump #1).

Jump #6

Finally, to establish and isolate the significance of FW from PEAs, the same motion references are applied on the *bi* configuration with FW disabled, depicted in the right column in Fig. 5.7. Eventhought

PEAs augment torque capacity, notice that SEA quadrature currents remain saturated and untracked by the current controller for a short period of time. This indicates that the inability of SEAs to achieve higher velocities is not a result of torque saturation but voltage saturation. While the peak joint torques are about the same as jump #5, owing to the lower joint velocities, the average mechanical powers (206 W) is about 15% lesser than jump #5 (238 W). Consequently, the jump height is also limited to 12.7 cm which motivates the use of FW in high torque density actuators such as SEAs. Nevertheless, the jump already brings about 33 % improvement over the *noESB* configuration with FW disabled, jump #1 whilst consuming lesser electrical power on average. This observation again emphasizes the benefits of energy-efficient PEAs in robots powered by high torque-density SEAs.

Table 5.2 Numerical results from jump experiments

Specification	tuned for <i>noESB</i>			tuned for <i>bi</i>		
	<i>noESB</i> #1	<i>noESB</i> , FW #2	<i>bi</i> , FW #3	<i>noESB</i> #4	<i>bi</i> , FW #5	<i>bi</i> #6
CoM stance displacement (cm)	9.60	11.01	13.17	2.62	14.78	12.74
CoM squat displacement (cm)	39.03	40.47	42.98	32.48	45.01	43.20
Foot CoM displacement (cm)	8.36	10.05	13.22	11.19	15.00	10.56
CoM take-off velocity (m/s)	1.12	1.31	1.54	0.86	1.60	1.36
Peak ankle velocity (rad/s)	6.20	7.53	7.56	6.26	7.79	6.07
Peak knee velocity (rad/s)	-6.24	-6.71	-6.63	-6.24	-6.74	-6.00
Peak ankle torque (Nm)	76.58	77.81	92.60	76.84	92.79	92.43
Peak knee torque (Nm)	-91.80	-91.26	- 114.66	-94.66	-126.17	124.56
Average mechanical power output (W)	185.49	200.16	228.86	188.52	238.10	206.31
Average electrical power consumed (W)	694.00	993.02	899.15	684.47	947.16	512.58
Efficiency (%)	26.7	20.1	25.4	27.5	25.1	40.2

5.5 Discussion

5.5.1 Field-weakening with parallel articulation enhance explosive motion performance

Simultaneous trajectory tracking with field-weakening in torque controlled actuators is in general, non-trivial. FW requires the reference trajectories and hence the torques and currents to saturate the voltage for its engagement. This makes it apt-suited for explosive motion tasks. Thereby, standing jumps are demonstrated through multi-DoF field weakening control on a monopod from a squat pose.

First experiments are conducted with joint motion references tuned for *noESB* configuration. It can be observed that while FW improves explosive motion performance (jump #2, +15%) compared to jump #1 (see Fig.5.8), addition of ESBs can bring about an even higher performance enhancement (jump #3, +37%). As described in [98, 114] addition of ESBs or biarticular springs leads to a different optimal jump motion.

Therefore, in the next set of experiments, it is shown that by tuning the motion references particularly for *bi* configuration, one can further exploit its explosive motion potential with jump #5, +54% (see Fig.5.8). The energy-efficiency benefits of PEAs are demonstrated with jump #6 (+ 33%) wherein the robot jumps higher with lesser electrical energy consumption. The result also asserts that in a multi-joint robot, while PEAs can enhance both explosive motions and energy-efficiency, FW can further improve jump height by alleviating voltage constraints. Combining FW with parallel articulation can lead to a considerably higher mechanical power output as result of increase in peak velocities and torque augmentation. The additional torque reserves are especially useful during FW when operating at peak output as current constraints limit the available torque from the primary actuators.

5.5.2 Field weakening reduces voltage demand at the expense of energy-efficiency

In comparison with jump #1 (*noESB*, no FW), application of FW #2 leads to i) higher peak joint velocities by alleviating voltage constraints, as can be seen in their velocity plots of Fig. 5.6; ii) consequently, a sizeable increase in the average mechanical power output, as seen in the table in Fig. 5.5; iii) an increase in electrical power consumption due to consuming voltage and current reserves, as shown in power plots in Fig. 5.6. Therefore, FW improves peak joint velocities while sacrificing energy-efficiency. However, as FW increases power consumption only momentarily during explosive motions, this is an acceptable trade-off that allows to reduce the size and weight of system's actuators, thereby making it more efficient overall.

5.5.3 Parallel articulation compensate for the loss in energy-efficiency

Addition of PEAs allows for a significantly higher mechanical power output in *ESB* jumps (#3 and #5) than the *noESB* jumps (#1 and #2), see Fig. 5.6-5.7. It is also interesting to observe that the higher expulsion of mechanical power in *ESB* jumps does not lead to any higher electrical power consumption than the *noESB*, FW case (jump #2). Moreover, power transfer from the knee to ankle explains the sizeable increase in ankle power, when compared to jump #1 (*noESB*, no FW). Further, notice the negligible *ESB* electrical power consumption, since pretensions are held constant throughout the jump. These observations demonstrate the energy-efficiency potential of *ESBs* even while executing explosive motions. The efficacy of coupling PEAs with FW can thus be seen. While FW consumes electrical power to achieve higher joint velocities, PEAs at least partially compensate for the loss in torque capacity by contributing to the mechanical power through the energy stored in its elastic elements. Concurrently, due to additional PEA torques at the joints, the quadrature current required by SEAs is also reduced as a result of their reduced torque demand.

5.5.4 Optimal motion references are configuration-specific

On one hand, in jump #4, where motion references tuned for *bi* configuration are applied to *noESB* configuration, the robot tips forward and falls over. On the other hand, the *bi* configuration achieves the highest jump height while applying the motion references that are tuned particularly for its configuration (jump #5 compared to #3). These results confirm that distinct optimal joint motion reference exist for different actuated configurations. Motion references tuned for a particular configuration, when applied for the other configuration are either not suitable or do not exploit the full potential towards peak performance.

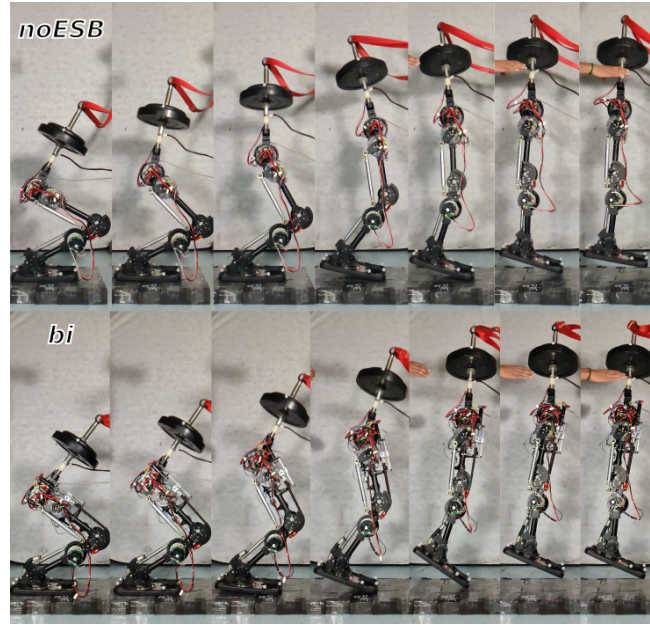


Figure 5.8 Snapshots of jump experiments with *noESB* (above) and *bi* (below) configurations. Notice the visible difference in foot height at apex.

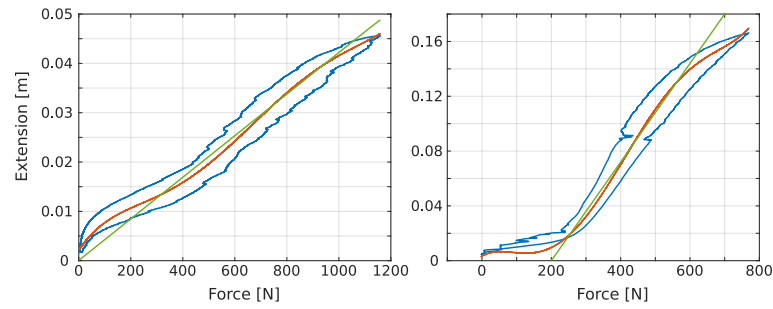


Figure 5.9 Figure depicts measured bungee stiffness profiles for both ankle-knee biarticulated (left) and knee monoarticulated (right) bungees upto 80% of their elongation ranges. Both profiles (blue) display hysteresis and sharp nonlinearities. Fifth order polynomial approximations are shown in red and linearization of 30 - 70% operating ranges are shown in green.

5.5.5 Energy-efficiency improvements

From Table 5.2, it is clear that the *noESB* configuration displays around 27% energy-efficiency (#1). As expected, when FW is enabled, the same decreases to about 20% in #2. However, when the same jump is conducted with the *bi* configuration with FW, the energy-efficiency rises, albeit only slightly to about 25% which is lesser than jump #1. The same efficiency is obtained even with jump #5 when the references are tuned for the *bi* configuration. Finally in the absence of FW, with PEAs (#6), there is a substantial surge in energy-efficiency, to 40%. These results indicate that PEAs enhance energy-efficiency during explosive tasks (#6 compared to #1). Concurrently, they also notify that the hand-tuned jump motions were not aware of the actuation enhancements since efficiency values of *bi* jumps with FW (#3 and #5) are not any higher than *noESB* jump without FW (#1). This is a drawback of adopting a heuristic-based tuning approach to explosive motions. These observations indicate that an *actuation-aware* motion optimization would be valuable to exploit the full potential of *eLeg*.

5.5.6 Hardware limitations

The biarticular ESB contributes to extension of the ankle, but also leads to flexion at the knee. Conversely, the monoarticular ESB aids in knee extension. Moreover, the force-extension profiles of the ESB bungees are S-shaped, see Fig. 5.9. Therefore, a trade-off exists in setting the pretensions, so as to avoid any adverse effects during the extension motion at the knee joint. Empirically, the applied pretensions reported in this work, lead to the best trade-off. Obtaining objectively optimal pretensions is subject to future work.

In addition, fourbar mechanisms in both ankle and knee joints exhibit a small configuration-dependent play at times, which leads to a small difference in the initial joint configuration between the *noESB* and *bi* configurations.

5.5.7 Future work

Given the limited range of joint motion (less than 2.1 rad, see Fig. 5.3) and the dynamic nature of the explosive motion, estimating the best possible outcome from the two configurations, while operating at the boundaries of actuation, is important. The difficulty in obtaining these outcomes is further compounded by nonlinearity and hysteresis in the bungees. These considerations prompt to apply optimization concepts such as in [114], to plan optimal vertical jump motions while taking into account constraints arising from nonlinear mechanical and electrical dynamics and contact transitions.

Moreover, FW controller at the knee joint suffers from current tracking especially towards the end of the jumping motion possibly due to parametric variations. This observation motivates to perform adaptive FW control with online parameter estimation such as in [161]. The same along with torque controller improvements would serve as valuable future work.

5.6 Summary

In this work, the synergistic benefits of coupling field-weakening with biarticulated PEAs are investigated. Results indicate considerable improvements to the explosive motion performance of a SEA-powered monopod are obtained. From a same squatting pose, the monopod robot jumps in two actuation configurations - i) only SEA (*noESB*), ii) SEA + PEA with biarticulation (*bi*). A baseline jump performance is first established with *noESB* configuration with specifically tuned motions. Next, the potential of FW on multiple joints are explored. Although FW increases the mechanical power output sizeably, the robot joints torques can additionally benefit from parallel torque reserves. Therefore, the same take-off motion on the robot augmented with PEAs can result in even higher jump performance. Now, optimal take-off motions can vary with actuation configurations. Hence, jump experiments are conducted again with jump motion tuned for the *bi* configuration. Leveraging FW and parallel actuation together with motions that exploit them, results in a substantial performance improvement of over 50% over the baseline jump. The results suggest that parallel articulation can further be exploited for energy-efficiency reasons by optimizing for joint motion trajectories which will be shown in the next chapter.

Chapter 6

Concurrent energy-economy and explosive motions with Parallel Multiarticulation

Quite often, energy-efficiency and peak performance turns out to be conflicting objectives in optimal control as well as biology. Several robotic tasks are performed while minimizing for joint torques and accelerations. Such an operation not only promotes energy-efficiency but also protects robots from aggressive motions and hard collisions. Even in biology, when a cheetah runs at its fastest speed, it consumes an enormous amount of energy, making it less efficient [162, 38]. Nevertheless, breakthroughs in robot endurance is possible only through sufficiently efficient actuation even during agile motions. Locomotory tasks involving repeated explosive manoeuvres [163], [164] can certainly benefit from the added efficiency benefits.

In the previous chapter, PEAs, biarticulation and FW led to substantial improvements in peak joint velocities, torques, mechanical power output and consequently jump performance. However, energy-efficiency improvements owing to PEAs were quite scarce in the presence of FW. Numerical results infact suggest that energy-efficiency slightly decreased when combining FW and PEAs from that of the *noESB* actuation configuration, see Table 5.2. The particular observation suggests that the proposed symbiosis of FW and PEAs is far from perfect with hand-tuned jumping motions. Therefore in this chapter, jumping motions are optimized for explosive performance while not sacrificing energy-economy.

6.1 Organization

The chapter ¹ is organized as follows. Sec. 6.2 introduces actuation-aware trajectory optimization methodology and tailors the approach for the jumping problem. A short disucssion about the optimized results follows. Sec. 6.3 describes a jumping controller that has a few advances from the previous chapter. Optimized jumping motions are then used as references on *eLeg* whose results are reported in Sec. 6.4.

¹A part of the work included in this chapter is under review in *Transactions on Mechatronics* with the title *Enhanced energy-efficient optimal jumping motions of a series-elastic monopod with parallel multi-articulation and field-weakening* by Vishnu Dev Amara, Jörn Malzahn and Nikos Tsagarakis.

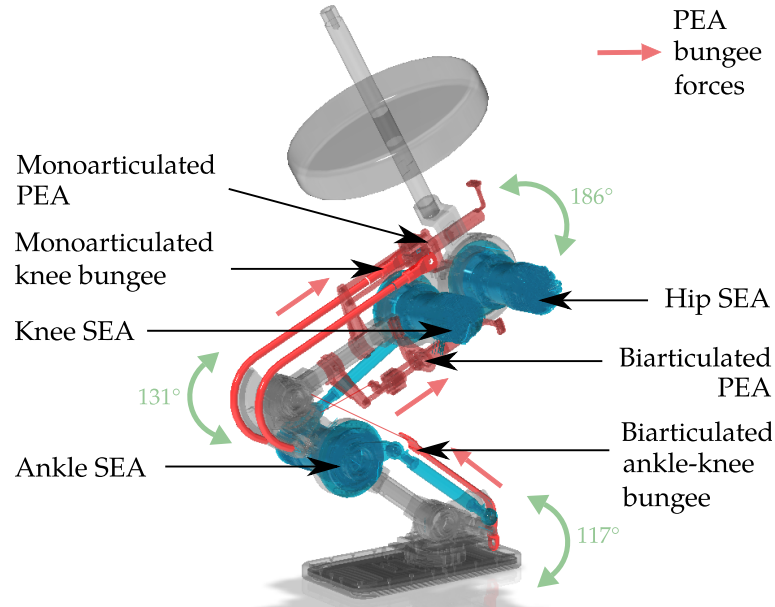


Figure 6.1 Illustration of the 3D model of *eLeg* in its *bi* configuration. The actuators and transmission units of PB (blue) and ESB (red) are highlighted. Ranges of joint motion are indicated by green arrows.

In addition to the previous chapter, jumping experiments are conducted on SEA + monoarticulated PEA (*mono*) configuration. These results are compared and contrasted in Sec. 6.5 and finally summarized in Sec. 6.6.

6.2 Actuation-Aware Jump Optimization

FW and parallel articulation greatly enhance the explosive motion capacity of *eLeg* across its various actuation configurations. Yet, in order to fully exploit this enhanced potential within the electrical actuation boundaries, limited ranges of joint motion, sub-second time durations and contact transitions, an *actuation-aware* trajectory optimization (TO) routine is adopted to plan for the jumping motion references. Accordingly, multi-phase TO is used to obtain reference joint motion trajectories that maximize the vertical displacement of the robot CoM. In this section, reasoning for the choice of optimization method and formulation of the associated optimal control problems (OCP) and their solutions are presented.

6.2.1 Choice of a motion optimization method

Devising joint motion profiles for robots executing explosive motions during locomotion is not trivial. Jumping motions may involve varying contact phases and system constraints on joint angles, actuator torques and dynamics. Maximizing centroidal or end-effector velocity cannot be achieved by maximizing joint velocities and torques.

In the context of *eLeg* jump optimization, it is preferable to obtain optimal state trajectories (in addition to optimal feedforward control trajectories) as their motion tracking can be better achieved through feedback control. Model inaccuracies can hamper the transfer of optimal control profiles obtained through shooting-based methods. Shooting-based methods may require the use of time-

stepping solvers [106] or LCP formulation [165] to account for contact transitions. To avoid the aforesaid difficulties, a collocation-based method is sought after.

To accommodate the contact transitions that would arise during the jumping motion, an NLP consisting of three phases is utilized - *stance*, *push-off* and *flight* phase. While the *stance* phase is characterized by the robot foot in complete contact with the ground, approximated through contacts at heel and toe, the *push-off* phase features only toe contact. The phase durations are not explicitly set beforehand.

6.2.2 Choice of robot model for optimal control problem formulation

Next, it is of prime importance to determine the appropriate level of model complexity to be used in the optimization problem formulation. Reduced models [110, 13, 111, 112] capture significant characteristics of robot locomotion with lesser dimensionality leading to faster computations. But these reduced models lack any information about the robot's multi-joint dynamics. Since the mapping between CoM accelerations and joint torques and velocities depend on the robot's joint configuration, setting constraints on CoM accelerations is non-trivial. Overestimated CoM dynamic limits render the optimization far from reality. For instance, in [112], the authors consider unconstrained torque capabilities, which is grossly unsuitable here since explosive motions aim to leverage the complete joint torque capacity. Conservative approximation of dynamic limits might lead to not exploiting the full performance envelope of the robot. In other words, these aforementioned constraints can all be expressed only at the whole-body/joint-space level [166]. Hence, for explosive motions, optimization with such template models involve i) posing constraints on the unknown centroidal motion, ii) assumption of massless limbs - limb motion must not significantly alter centroidal momentum and iii) obtaining possibly limited motion features.

For *eLeg*, dynamic limits can only be expressed at the joint-space level. Centroidal dynamic limits are apriori unknown. Besides, more than 35% of the robot mass is concentrated below the hip. Given that the *eLeg* has only six DoFs (three actuated DoFs), following the motivation provided in Sec. 6.2.1, TO is performed for the whole body joint-space dynamics of the robot. Thus, the decision variables $\mathbf{x}$ fully describe the generalized coordinates (joint configuration $\mathbf{q}$ and velocities $\dot{\mathbf{q}}$) and inputs (joint torques $\boldsymbol{\tau}$ and ground contact forces $\boldsymbol{\lambda}$) of the robot in addition to the phase durations. Fig. 6.2 visually describes most of the decision variables involved in the optimization. The planar multi-body dynamics of the robot are transcribed into a non-linear programming problem (NLP). To this end, ICLOCS [167] library is utilized which transcribes the NLP through *direct collocation* and solves using IPOPT solver [168]. Hermite-Simpson quadrature method is used for discretizing the OCP [169]. Fig. 6.3 sums up the TO problem in totality.

6.2.3 Constraints

Given the link-side joint-space dynamics of a floating-base robot actuated by ACAs,

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{B}\boldsymbol{\tau} + \mathbf{J}^T(\mathbf{q})\boldsymbol{\lambda}, \quad (6.1)$$

constraints are imposed on the optimization problem that enforce (6.1) at every grid point in all phases. $\mathbf{M}$, $\mathbf{n}$ and $\mathbf{J}$ denote the mass matrix, bias forces (Coriolis, centripetal and gravity terms) and the contact Jacobian, respectively. $\mathbf{B} \in \mathbb{R}^{(n+3) \times n}$ is a selection matrix for the actuated joints. Additionally, box

constraints on the joint configuration, velocities, actuator torques and ground contact forces are imposed at every grid point. Given the initial time t_0 , limits are set for duration of each phase, ΔT_i . The sum of phase durations add up to the final time t_f .

The translational velocities of ESB motors are limited to 8 mm/s. Therefore, constant ESB bungee pretensions are held during jumping. Consequently, the pretensions $\mathbf{p}$ from (4.5) serve as optimization parameters in the TO problem.

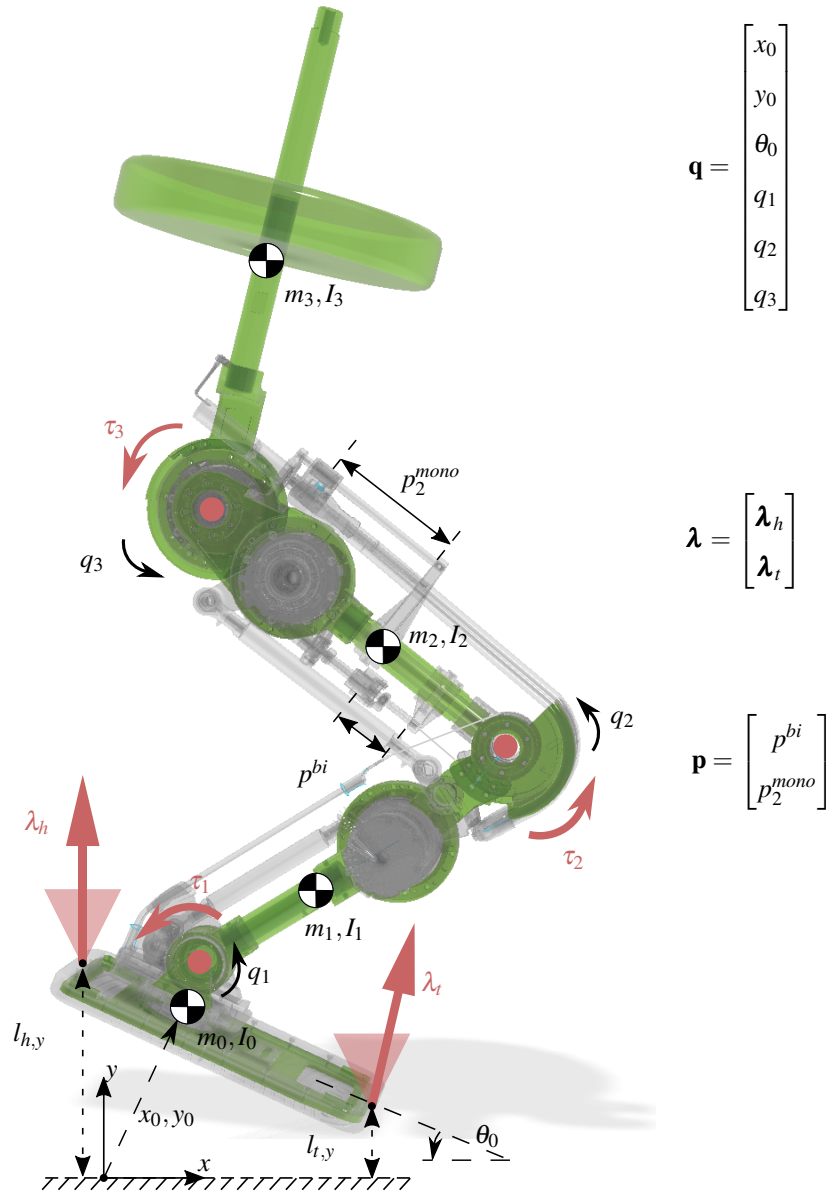


Figure 6.2 Floating base (underactuated) planar model of eLeg in *bi* configuration. The joint-space link-side dynamics model known by the optimizer is shaded in green. SEA dynamics, fourbar mechanisms and parallel actuator dynamics are not known to the optimizer. Various link masses, inertiae and their CoM, pretensions, joint angles and torques used in the TO problem formulation are indicated. Red arrows indicate sample ground constraint forces at heel and toe and their corresponding friction constraints are shown by triangles underneath. For clarity, the configuration coordinate vector and other variables used in the TO problem are expanded.

To describe the unidirectional behaviour of ESB bungees as a continuous function, the following smooth approximation of the bungee extension Δ is adopted:

$$\tilde{\Delta} = \frac{1}{2}(\sqrt{\Delta^2 + \varepsilon^2} + \Delta), \quad \varepsilon = 1e-2. \quad (6.2)$$

To avoid any slippage during the jumps on horizontal terrain, the optimizer should only predict forces that lie within an approximated planar friction cone. Such a set of feasible forces that lie within the cone defined by ground contact Coulomb friction coefficient μ is given:

$$\Lambda(\mu, \boldsymbol{\lambda}_i) := \{ \boldsymbol{\lambda}_i \in \mathbb{R}^2 \mid |\lambda_{i,x}| \leq \mu \lambda_{i,y}, \lambda_{i,y} \geq 0 \}, \quad (6.3)$$

where the subscripts x and y denote the vector components along x and y axes. $i = \{h, t\}$ where h and t signify heel and toe respectively.

$$\begin{aligned}
\min_{\mathbf{x}(\mathbf{t})} \quad & J(\mathbf{x}) = -c_y(t_{2f}) - \frac{1}{2g} \dot{c}_y^2(t_{2f}) + \int_{t_0}^{t_{2f}} \frac{1}{100} k_G dt, \\
\mathbf{x} = \begin{cases} \mathbf{q}(\mathbf{t}) \in \mathbb{R}^{n+3}, & \text{(Joint positions)} \\ \dot{\mathbf{q}}(\mathbf{t}) \in \mathbb{R}^{n+3}, & \text{(Joint velocities)} \\ \boldsymbol{\tau}_{PB}(\mathbf{t}) \in \mathbb{R}^n, & \text{(PB torques)} \\ \boldsymbol{\lambda}(\mathbf{t}) \in \mathbb{R}^{2j}, \quad j=2, & \text{(Contact Forces)} \\ \mathbf{p} \in \mathbb{R}^2, & \text{(Pretensions)} \\ \Delta T_i \in \mathbb{R}^+, \quad i = \{1, 2, 3\}, & \text{(Phase durations)} \end{cases} \\
\text{s.t.} \quad & [\dot{\mathbf{q}}, \ddot{\mathbf{q}}]^T = f(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}, \boldsymbol{\lambda}), \quad \text{(System dynamics)} \\
& \underline{q} \leq \mathbf{q} \leq \bar{q}, \quad \underline{\dot{q}} \leq \dot{\mathbf{q}} \leq \bar{\dot{q}}, \quad \text{(Configuration limits)} \\
& \underline{\tau} \leq \boldsymbol{\tau}_{PB} \leq \bar{\tau}, \quad \underline{\lambda} \leq \boldsymbol{\lambda} \leq \bar{\lambda}, \quad \text{(Input limits)} \\
& \underline{\delta} \leq \Delta T_i \leq \bar{\delta}, \quad \text{(Phase durations)} \\
& \underline{\mathbf{p}} \leq \mathbf{p} \leq \bar{\mathbf{p}}, \quad \text{(Pretension limits)} \\
& \underline{\ddot{q}}_a \leq \ddot{\mathbf{q}}_a \leq \bar{\ddot{q}}_a, \quad \underline{\dot{\tau}} \leq \dot{\boldsymbol{\tau}}_{PB} \leq \bar{\dot{\tau}} \quad \text{(Actuation limits)} \\
& \boldsymbol{\tau}_{PB}^T \cdot \dot{\mathbf{q}}_a \leq \bar{P} \\
& \lambda_h(t_{1f}) = \mathbf{0}, \quad \lambda_t(t_{2f}) = \mathbf{0}, \\
& \lambda \in \Lambda(\mu), \quad l_{h,y} = l_{t,y} = 0. \\
& \lambda_h = \mathbf{0}, \quad l_{h,y} > 0, \quad \lambda_t \in \Lambda(\mu), \quad l_{t,y} = 0. \\
& \lambda = \mathbf{0}, \quad l_{h,y} > 0, \quad l_{t,y} > 0.
\end{aligned}$$

Figure 6.3 Cost function, decision variables and constraints in the jumping TO problem formulation are shown. Constraints shown between the dashed lines are enforced only in the corresponding phases. Constraints shown in green are phase terminal constraints. Other constraints are applied at all phases.

Therefore, (6.3) is used to constrain the contact forces at both heel and toe in the *stance* phase and at the toe in *push-off* phase. The remaining contact forces are all essentially constrained to be zeros. In tandem, the heel and toe at *stance* phase and toe at *push-off* phase are constrained to be in contact with the ground. To ensure continuity for the ground constraint forces across the various phases, phase terminal constraints are enforced, as shown (in green) in Fig. 6.3. Finally, the following boundary constraints (not shown in Fig. 6.3) are imposed: i) equality constraint on the initial joint configuration ii) box constraint on the terminal trunk angle to prevent the robot from performing any untoward motion during *flight*.

6.2.4 Integrating actuation-awareness

Although the complete joint-space robot dynamics is utilized in the optimization problem, the physical hardware exhibits several modelling inaccuracies. The mechanical and electrical dynamics of the SEAs are not captured in the TO formulation. Further, during explosive motions, a sizeable share of motor torque is expended as inertial torques as well as to overcome friction. The joint torque profiles are further affected by actuator nonlinearities, fourbar play, control delays, etc. Besides, during FW operation, the quadrature current references do not scale proportionally with torque references. Capturing all these phenomena in the OCP formulation severely affects its convergence. Therefore, these effects are approximated by accounting for reduced torque ($\tau_{PB} \leq 90$ Nm) constraints. Moreover, to consider the possibility of FW, enhanced velocity limits ($\dot{q}_a \leq 8.5$ rad/s) are used.

Moreover, explosive motions are characterized by rapid build-up of joint torques to accelerate the joints resulting in enormous joint power output. Moreover, PB powers are limited and their torque and velocity cannot assume peak values at the same instant. It is essential to introduce additional *actuation constraints* that render the TO process *actuation-aware* and ensure reasonable consensus between optimized motions in simulation and hardware. Hence, the following box constraints (shown in brown in Fig. 6.3) are set: i) joint accelerations ($\ddot{q}_a \leq 150$ rad/s²), ii) PB torque rates ($\dot{\tau}_{PB} \leq 500$ Nm/s) and iii) PB powers ($P_{PB} \leq 400$ W). These constraints were set based on experiments involving hand-tuned jumping motions. The power constraints are bilinear and cannot be directly posed in shooting methods which also motivates for using a collocation method.

6.2.5 Objective criteria

A composite cost function J is contrived as shown in Fig. 6.3, for the optimization problem. The first part of J maximizes the apex height based on the vertical position c_y and velocity $\dot{c}_y$ of the robot CoM at the *take-off* instant (end of *push-off* phase), t_{2f} . The second part of J penalizes the integral of centroidal angular momentum k_G of the robot [170] in the world frame in a weighted fashion until the end of *push-off* phase. Since maximization of vertical CoM displacement is the paramount objective in this work, no additional criteria for maintaining posture or energy-efficiency (input minimization) such as in [98] are posed. The efficacy of such a simple physics-based heuristic can be seen from not having to levy any additional path constraints on the CoM components nor the necessity for cumbersome tuning of weights for the various terms of the cost function. Further, since no criterion is optimized during *flight*, the optimization problem simplifies to a feasibility problem in the third phase.

6.2.6 Optimal solutions

Accordingly, three OCPs are solved, one each for the three actuation configurations of the robot. Decision variables across the different OCPs vary by pretension parameter limits, topology matrix and the differences in segmental masses and inertiae owing to parallel actuator arrangements. To ensure sufficient solution accuracy, a grid resolution of 600 points and NLP error tolerances of $1e-6$ with a maximum end time t_f of 0.5 s are used. The main results of optimization are conveyed in Fig. 6.4 and numerically reported in Table 6.1. In general, the joint motion references are characterized by differences in the extent of leg extensions, take-off instants and OCP terminal time. Actuation configuration with ESBs undergo higher accelerations and pronounced extensions at the point of take-off. It is interesting to observe that *mono* and *bi* configurations do not undergo full leg extension at take-off.

This is possibly due to non-availability of ESB torques at the joints because bungees are slackened just before full extension. The peak ankle and knee velocities across configurations do not vary significantly. Joint motion profiles, peak torques and mean mechanical power output of the two configurations are reasonably similar.

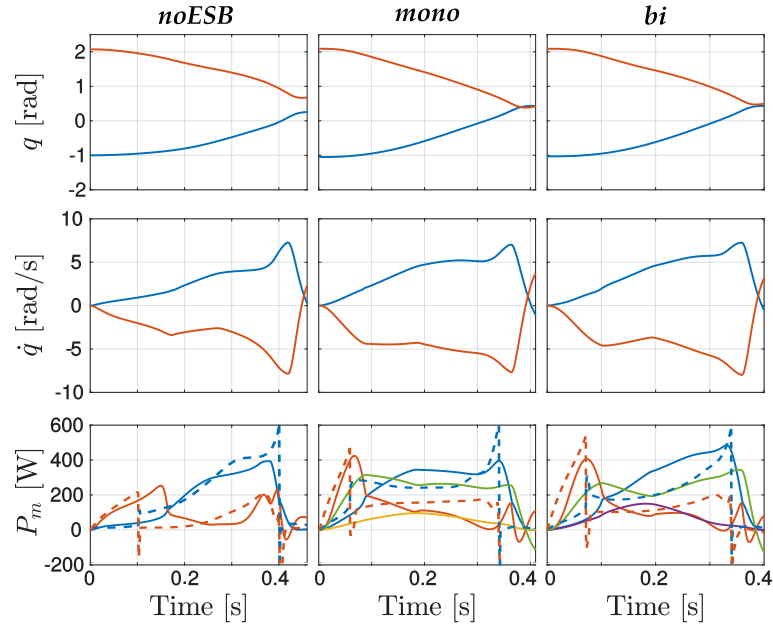


Figure 6.4 Summary of results - joint positions, velocities and joint mechanical powers from optimized jumping motions for various actuation configurations of *eLeg* for ankle (blue) and knee (red) joints shown with solid lines. Mechanical powers corresponding to mono-ankle (yellow), mono-knee (green) and bi-ankle-knee (magenta) PEAs are also shown with solid lines. In addition, joint powers from optimized solutions without actuation constraints are shown by dashed lines.

It is worth mentioning that during several hand-tuned jump motions, higher ankle velocities were obtained particularly in the *noESB* configuration, but their SCD as well as the joint powers were not as high as the aforementioned results which further motivates the need for such an actuation-aware planning.

Table 6.1 Numerical details from optimized solutions

Specification	<i>noESB</i>		<i>mono</i>		<i>bi</i>	
	ankle	knee	ankle	knee	ankle	knee
Peak velocities (rad/s)	7.6	8.2	7.5	8.2	7.7	8.0
Peak torques (Nm)	84.0	77.5	92.8	141.0	95.1	136.8
Peak joint powers (W)	394	214	395	424	485	406
Mean mechanical power (ankle+knee) (W)	234		319		338	
SCD (cm)	13.1		16.0		16.2	

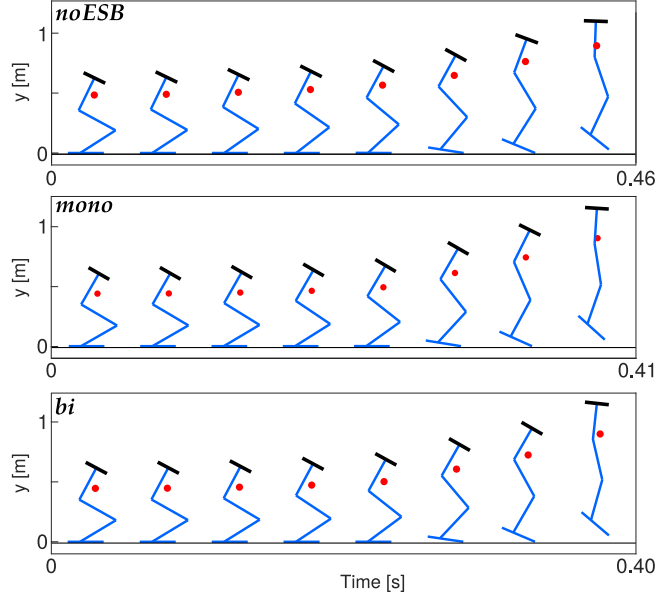


Figure 6.5 Snapshots of the optimized jumping motions for various actuation configurations of *eLeg*.

6.3 Controller

Fig. 6.6 visualizes the control architecture used in this work. TO plans for offline joint motion and torque references as described in the previous section. An inverse-dynamics based feedback controller is used for computing the torque references for the actuated joints. It is worth noting that *eLeg* lacks sensing at the foot link and hence full state estimation cannot be achieved once the foot is not flat on the ground.

PD position control on the reference pretensions $\mathbf{p}^*$ is employed along with PI current control for the ESBs (shown in red). Bungee cords used with ESBs display substantial nonlinearity in their stiffness profiles. Therefore, the pretension references $\mathbf{p}^*$ are adjusted such that their force profiles reasonably match the ones obtained in the optimal solutions. ESB torques computed from the bungee force measurements are subtracted from reference joint torques provided by the aforementioned inverse dynamics controller.

The net torques from above serve as PB (shown in green) torque references. Since PB drives are elastic (SEAs), an impedance control scheme described in [171] is used to compute the motor references from the planned joint motion references. Given the link motion references from the solutions of TO, motion reference for the PB motor can be computed using (4.2).

$$\boldsymbol{\theta}^* = \mathbf{K}_{PB}^{-1} \boldsymbol{\tau}_{PB} + \dot{\mathbf{q}}_a^*. \quad (6.4)$$

where $\mathbf{K}_{PB}$ is a diagonal matrix containing the stiffness values of the PB actuators at the three joints.

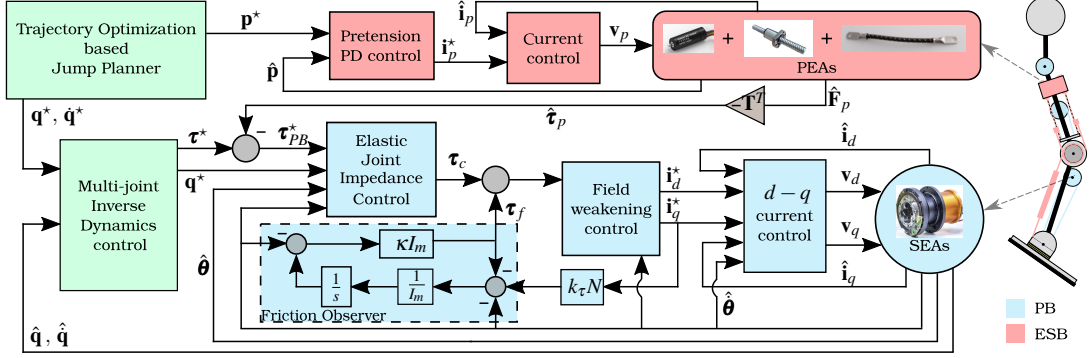


Figure 6.6 Schematic of the controller used in this work. $(\cdot)$ and $(\cdot^*)$ refer to measured and reference signals respectively.

Consequently, the impedance controller references serve as inputs to the FW control layer described in Sec.5.2 which then outputs current references in the $d-q$ plane. For the current references, PI controllers with feedforward, back-EMF and cross-coupling terms are used, see (5.1). Given the non-negligible friction at SEAs, a momentum-based friction observer similar to the one described in [172] is employed in parallel to the torque controller to compensate for the friction in the SEAs. Note that the motor reference torques for the friction observer is obtained from the quadrature current references from the FW controller. Although the torque references to the field-weakening controller are recomputed based on a friction compensation scheme, torque tracking can be perfect only when the voltage is not saturated.

The controller is executed on Simulink-RealTime which provides for joint motion, torque and current references at 1 kHz. Current control is executed at 20 kHz in the firmware. To avoid possible voltage saturation during FW due to imperfect parameters, a conservative approximation of maximum available voltage, 96% of $48/\sqrt{3}$ is used at the knee joint. Furthermore, to avoid any undesired FW in the no-FW case (to operate within MTPA mode), the controller actively computes quadrature current references that does not exceed the inverter voltage [173]. This is necessary in order to regulate the direct current to zero simultaneously. It is only fair to obtain a baseline that uses pure MTPA mode, up on which the velocity enhancement due to FW can be computed. Besides, no sophisticated balance control scheme is employed during landing. since the work is concerned with the jumping motion performance with FW and PEAs,

6.4 Experimental Results

A jumping motion analysis is conducted by applying the planned optimal motion references obtained in Sec. 6.2.6 for the corresponding actuation configurations. To demonstrate the potential of FW, the same motion references are applied both in the presence and absence of FW. This leads to an analysis across six experimental conditions (A-F, 3 actuation configurations without and with FW). Jumping performance is rated across the various conditions by measuring the SCD. Fig. 6.7 and 6.8 summarises the results of jump experiments with and without FW control respectively. Since the hip joint does not undergo voltage saturation, details that correspond only to the ankle and knee joints are presented. Numerical details from the same experiments including peak absolute values of joint torques, velocities, mechanical powers of ankle and knee respectively, ratio of average mechanical powers to average electrical powers and SCD are reported in Table 6.2.

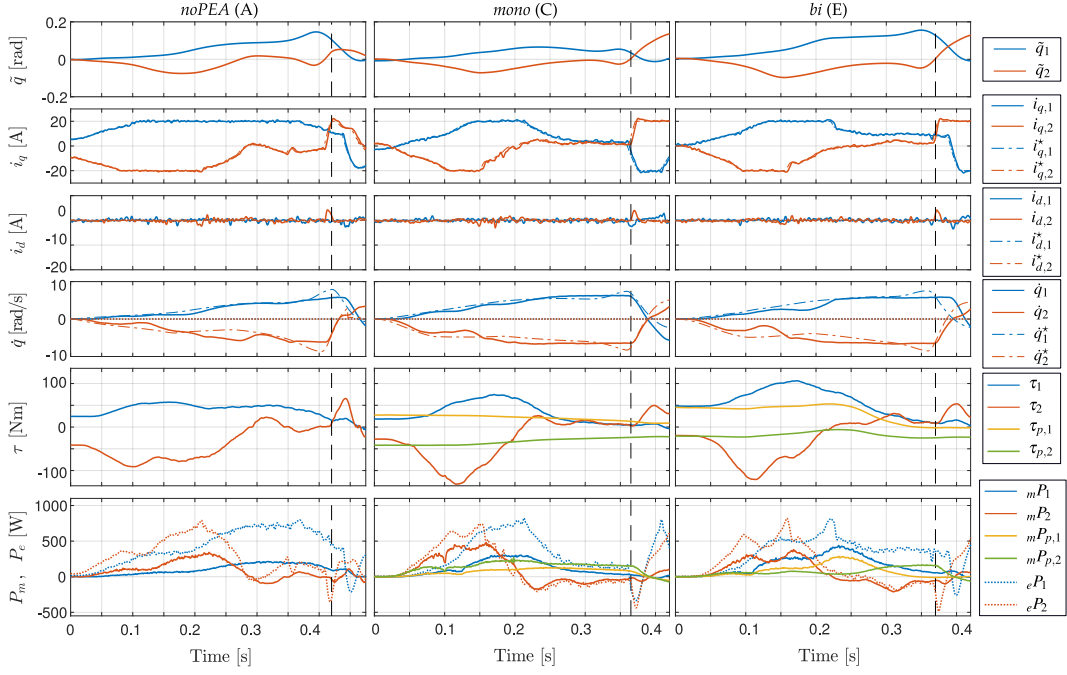


Figure 6.7 Tracking errors, currents, torques, velocities and mechanical powers for ankle and knee joints from jumps without FW across the three actuation configurations are displayed with solid lines. Torques and mechanical powers corresponding to PEAs are also shown. Joint electrical powers ($eP_{(.)}$) are shown in the power plots with dotted lines. Vertical dashes indicate take-off moments. Dotted lines in velocity plots indicate FW modes as depicted in Fig. 5.2. Results are depicted up to the instant at which robot reaches jump apex.

6.4.1 Results - Jumps without field weakening

For the experimental conditions A, C and E in Fig. 6.7, peak joint velocities are either below or about 7 rad/s. In jump A, where the robot is devoid of PEAs, the peak torques are below 90 Nm and the robot attains a SCD of 10.7 cm. Owing to additional torque sources, both the joints in jumps C and E display higher peak torques than the *noESB* configuration. For the same reason, the PB quadrature current demand reduces considerably for jumps C and E when compared to jump A. In particular, peak knee torque is the highest in jump C where the PEAs are only monoarticulated. However, peak ankle torque is the highest in jump E due to biarticular ankle-knee ESB which transfers power from knee to ankle. Accordingly in jump E, peak knee torque is slightly lower than in jump C. Nevertheless in the presence of ESBs, peak joint velocities and torques lead to increased mean mechanical power output and consequently, an increase in SCD by about 8% and 23%. However, it is noteworthy that experiments A, C and E attain only about 75-82% of their optimal SCD values (seen in Table 6.1).

Although PEAs supplement SEA torques for jumps C and E, both the joints approach their peak velocities quite early in the motion and remain almost the same until take-off. This indicates that voltage reserves are depleted and that additional torques cannot boost velocities any further. Therefore, in order to enhance the jumping performance further, FW is applied with the same optimized motion references.

6.4.2 Results - Jumps with field weakening

From Fig. 6.8 for jumps B, D and F, it is immediately visible that the peak joint velocities of ankle and knee are higher than their non-FW counterparts. Especially for jumps D and F with PEAs, the peak joint velocities are higher than 7.5 rad/s, with the *bi* configuration reaching higher than 8 rad/s

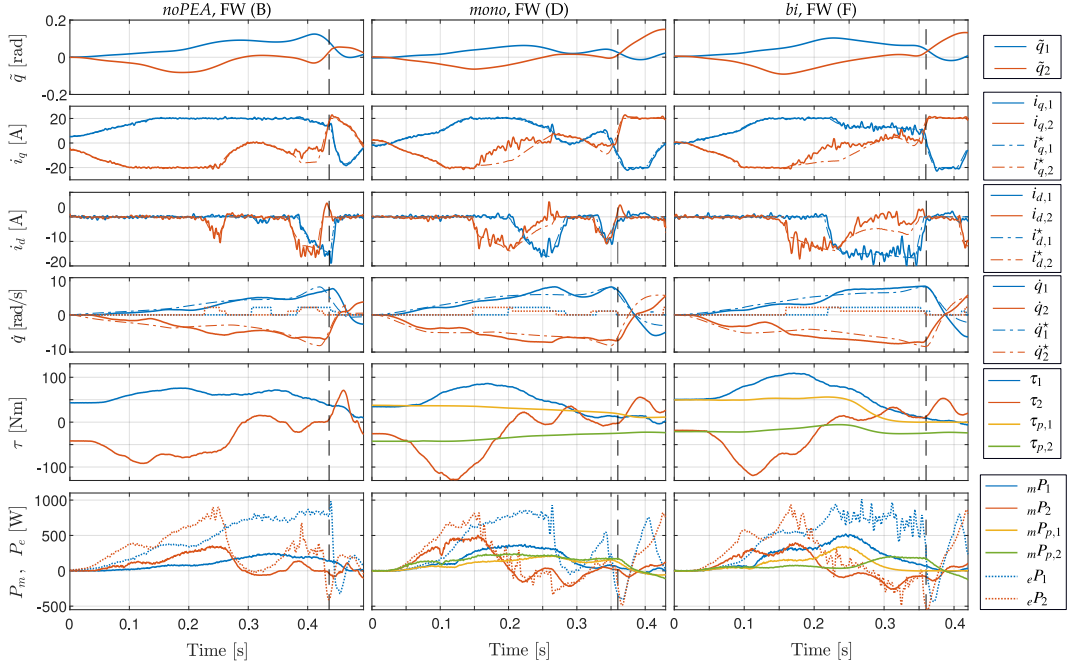


Figure 6.8 Tracking errors, currents, torques, velocities and mechanical powers for ankle and knee joints from jumps with FW across the three actuation configurations are displayed with solid lines. Torques and mechanical powers corresponding to PEAs are also shown. Joint electrical powers ($eP_{(.)}$) are shown in the power plots with dotted lines. Vertical dashes indicate take-off moments. Dotted lines in velocity plots indicate FW modes as described in Fig. 5.2. Results are depicted up to the instant at which robot reaches jump apex.

as a result of longer FW engagement durations. Moreover, it is interesting to observe that although there is power transfer from knee to ankle in jump F, FW remains engaged at the knee joint for a longer duration than in jump D where the PEAs are only monoarticulated (no power transfer). However for jump B, owing to lack of ESBs, the peak velocities are notably lower than jump D and F.

As for the current tracking, the knee joint suffers marginally during FW, possibly due to imperfect electrical parameters across all three jumps. Besides, the quadrature current demand in jump B is saturated for longer durations than in jumps D and F which highlights the significance of ESBs. ESBs act as additional torque reserves, when FW uses up the SEAs' current reserves to alleviate voltage constraints. The peak torques across configurations are similar to the corresponding ones obtained in the absence of FW (shown in Fig. 6.7).

While sole application of FW increases SCD by 11% when compared with jump A, PEAs with FW increase the same by about 34% (*mono*) and 39% (*bi*), reasonably higher than their non-FW counterparts. Furthermore, these three jumps (B, D and F) are able to achieve more than 93% of the predicted SCD values from corresponding optimal motion profiles.

6.5 Discussion

6.5.1 Essence of actuation-aware trajectory optimization

In this work, an actuation-aware TO predicts optimal jumping motion profiles for distinct actuation configurations of a monoped robot heavier than 25 kg with a substantial limb mass. Constraints that render the TO "actuation-aware" facilitates obtaining jumping motions that do not feature simultaneous

Table 6.2 Numerical details from experiments

Specification	<i>noESB</i>		<i>mono</i>		<i>bi</i>	
	-	FW	-	FW	-	FW
	A	B	C	D	E	F
Peak joint velocities (rad/s)	6.2	7.3	6.3	7.8	6.2	8.1
	6.5	6.9	6.7	7.6	6.9	8.0
Peak joint torques (Nm)	79.1	77.2	86.0	85.6	106.3	109.0
	89.0	91.8	131.2	128.0	120.5	118.7
Peak joint mechanical powers (W)	213	255	303	373	438	521
	343	351	474	502	381	396
Mean mechanical power (ankle + knee) output (W)	193	215	253	301	274	312
Efficiency (%)	29.8	28.5	53.5	48.4	50.0	45.2
CoM stance displacement (SCD) (cm)	11.2	12.4	12.1	15.0	13.8	15.6
	-	+11%	+8%	+34%	+23%	+39%

peak values of torque and velocity (for instance PB powers do not exceed 395 W) and physically unfeasible build-up of joint torques and accelerations. As shown in Fig. 6.4, without actuation-aware constraints, the resulting PB powers are very high which were not experimentally replicated by PB actuators (not shown). Similarly, these constraints help obtain motion profiles without abnormal jerks. Their ensuing utility can be seen in the tracking errors obtained across all six experimental conditions until the point of take-off being reasonable for an explosive motion, below 0.15 rad ($\leq 9^\circ$). On comparing the obtained results with those of the hand-tuned jump motions from the previous chapter (5.4), it quite clear that actuation-aware TO based joint motion references were able to lead to lower electrical energy consumption for higher jumps as well as higher energy-efficiency. These observations further motivate the need to plan for explosive motions while rendering the OCP aware of the actuation capabilities of the robot.

6.5.2 Benefits of field weakening

On comparing FW jumps (B, D and F) with non-FW jumps (A, C and E), no significant changes in the joint tracking errors are observed in Fig. 6.7 and 6.8. While FW might allow for better tracking of high-velocity motion trajectories, the optimized solutions showcase joint velocities greater than 6.6 rad/s (point A in Fig. 5.2) for less than 100 ms just before take-off, as seen in Fig. 6.3. This highlights the intended use of FW, to improve peak velocities at the point of take-off. Moreover, in this work, optimal motion profiles (obtained with velocity constraints higher than 6.6 rad/s) are applied on jumps without FW to showcase the speed-enhancing benefits of FW. Furthermore, application of FW results in an increase of more than 10% in mean mechanical power output with small losses in efficiency, which is an acceptable trade-off. FW leads to inefficiency briefly from an electrical to mechanical energy conversion standpoint. However, its application is a sensible trade-off as it helps improve overall efficiency of the robot by reducing size and mass of the actuators.

6.5.3 Enhanced energy-efficiency during explosive motions

Comparing ESB jumps (C-F) with *noESB* jumps (A-B), it is quite clear that parallel actuation can not only enhance explosive capabilities through rapid joint velocity build-up but can also concurrently improve energy-efficiency of the robot. These higher efficiency values are a natural consequence of using ESBs that feature tendons with high energy-storage capacity. Besides, the cost function as shown in Fig. 6.3 does not include any energy-efficiency criterion. This is a valuable consideration while designing robots - one can consider using PEAs in addition, instead of opting for heavier higher power SEAs.

The ankle-knee *bi* ESB results in increased ankle power output in the initial stages of the jump but does not transfer power (and hence does not contribute to ankle torque) in the final 50 ms before take-off. However, the ankle joint in *mono* jumps (C-D) continues to receive torques from ankle-mono ESB even during the final stages of jumping motion. Yet, the SCD values of *bi* jumps (E-F) are higher than *mono* (C-D). This phenomenon highlights the importance of biarticulation in jumping motions which aids in power transfer to distal link segments to improve joint coordination.

Parallel biarticulation has been shown to improve energy-efficiency more than parallel monoarticulation. However, in this work, jumps C-D exhibit higher energy-efficiency than E-F, as seen in Table 6.2. This is possibly due to ESB pretension limits and stiffness being optimized for gravity compensation during squatting rather than jumping motions.

6.5.4 Future Work

Despite having dedicated additional torque sources, both ankle and knee joints undergo some decelerations in jumps B and D before subsequent acceleration. For instance, in jump D, disengagement of FW occurs at both joints due to their applied voltages falling below the available voltage, at around 0.27 s. This is an undesired effect of planning for the jump motions without accounting for motor electrical dynamics while intending to use FW. This observation motivates the need to incorporate FW dynamics in the OCP in future. In order to make TO plan for motions that particularly aim to keep the FW modes engaged while keeping the OCP solvable, electrical constraints in (5.3) can be suitably linearized.

Generalizing the presented actuation-aware optimization to the joint space of a full humanoid robot that exhibits substantial leg inertiae might be computationally inefficient. Yet, to exploit its full performance envelope, one can arrive at a reduced model that incorporates joint-space dynamics for the leg segments (major contributors to the jumping motion) and lumped mass dynamics for the other segments to alleviate problem complexity.

6.6 Summary

Explosive robot motions such as jumping involve high centroidal accelerations. Actuation-aware trajectory optimization is utilized to plan optimal motions that exploit energy-economy and peak-performance aspects concurrently. While limits on torques and velocities can all be expressed in the joint-space, acceleration constraints cannot be expressed at the centroidal level. Therefore, OCP is formulated at the whole-body (joint-space) level. Furthermore, the offline optimization approach is tailored to robot dynamics in order to ensure likeness between simulation and hardware. Towards this goal, actuation-awareness is integrated into the optimization routine by introducing constraints, especially on joint powers.

Over two sets of three experiments each, the potential of FW, PEAs and biarticulation towards explosive motions are demonstrated in isolation as well as in combination. The work serves as a valuable tool in estimating the explosive motion capabilities of *eLeg* when it is, i) solely actuated by SEAs and ii) enhanced through FW, PEAs and biarticulation. Although, convergence time prevents one from applying the planning approach online in an MPC fashion, the work provides sufficient knowledge of robot's capabilities in the centroidal space. In future, this information can be used to optimize with a reduced robot model in centroidal space.

Chapter 7

Centroidal manipulability - analysis and applicability to biarticulation design

In the previous chapters, it has been verified beyond doubt, the efficacy of PEAs in enhancing energy-efficiency as well as explosive motion capabilities. Infact, the dominant role of biarticulated PEAs cannot be overlooked. As a result, the number of legged robots powered (either completely or partially) by multi-articulated PEAs are steadily increasing [174, 175, 94, 176]. Yet, design guidelines for choice of PEA parameters or a suitable articulation topology for the desired application are scarce.

Moreover, during legged robot design, it is oft non-trivial on how to articulate the robot link segments with parallel actuators. Anthropomorphic robot design generally makes use of clues inferred from mammalian musculoskeletal architecture. Yet, given the complexity it is not straightforward to choose PEA-related design parameters. Suppose, PEAs are to be installed as secondary torque sources such as in *eLeg*, quantitative measures of robot agility and explosive capabilities at the centroidal level would be very beneficial. Specifically, the appendages to be articulated in parallel, tendon stiffness, tendon attachment point, etc. can be chosen based on such quantitative measures.

A useful measure for floating-based legged robot applications including balancing, impact/contact-force regulation and whole-body acceleration capabilities is dynamic manipulability of center of mass (DCM). The idea of dynamic manipulability was developed primarily for manipulators by [177] and [178]. The same concept was extended to floating-base humanoids in [179]. The authors of this work, further demonstrated applying DCM measures to optimizing design parameters including link masses and lengths for a miniature humanoid. [180] extended the theory to include constraints arising from friction cone, CoP and single point contact to develop a feasible DCM notion for planar robots.

More recently, [181, 182] generalized the theory to multi-contact constraints and three-dimensional humanoids. In addition, meaningful choices for computing a DCM measure that can be related to actuator capacities were presented. The same tool can also be used to evaluate the explosive motion capabilities of humanoids or quadrupeds for running, hopping, jumping motions. Moreover, DCM-motivated design procedures can be adopted for multiarticulation design. It is interesting to point out that the maximal extent of parallel articulation in robotics is only biarticulation. Contrasting examples

from biology include highly multiarticulated tendons spanning several bone segments in terrestrial animals (ungulates) [183] and triarticulated muscles in ostriches [184].

7.1 Organization

The chapter¹ is organized as follows. Sec. 7.2 mathematically describes a useful decomposition of the translational centroidal accelerations and their interpretation. Sec. 7.3 comments on the benefits of FW and PEA through a brief centroidal dynamic manipulability analysis.

7.2 Decomposition of centroidal accelerations

Recalling the $n + 6$ DoF robot dynamics equations in the joint space from (6.1),

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{B}\boldsymbol{\tau} + \mathbf{J}^T(\mathbf{q})\boldsymbol{\lambda},$$

with l linearly independent kinematic constraints, one can assume contact constraints such that

$$\mathbf{J}(\mathbf{q}) \dot{\mathbf{q}} = 0. \quad (7.1)$$

For convenience, dependencies on $\mathbf{q}$ and $\dot{\mathbf{q}}$ are dropped. Differentiating (7.1),

$$\dot{\mathbf{J}} \dot{\mathbf{q}} + \mathbf{J} \ddot{\mathbf{q}} = 0, \quad (7.2)$$

Substituting for $\ddot{\mathbf{q}}$ from (7.2) in (7.1),

$$(\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^T) \boldsymbol{\lambda} = -\left(\mathbf{J}\mathbf{H}^{-1}(\mathbf{B}\boldsymbol{\tau} - \mathbf{n}) + \dot{\mathbf{J}}\dot{\mathbf{q}}\right). \quad (7.3)$$

Substituting (7.3) in (7.1) yields,

$$\ddot{\mathbf{q}} = \mathbf{P}_M \mathbf{H}^{-1} \left(\underbrace{\mathbf{B}\boldsymbol{\tau}}_{\text{due to joint torques}} - \underbrace{(\mathbf{n} + \mathbf{J}^+ \dot{\mathbf{J}} \dot{\mathbf{q}})}_{\text{due to gravity and joint velocities}} \right), \quad (7.4)$$

where $\mathbf{J}^+$ is the joint-space inertia weighted pseudoinverse of $\mathbf{J}$ given by

$$\mathbf{J}^+ = \mathbf{H}^{-1} \mathbf{J}^T (\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^T)^{-1}. \quad (7.5)$$

$\mathbf{P}_M$ is the null-space projection matrix of $\mathbf{J}$,

$$\mathbf{P}_M = \mathbf{I} - \mathbf{J}^+ \mathbf{J}.$$

¹A part of the work is under review in *Transactions on Mechatronics* with the title *Enhanced energy-efficient optimal jumping motions of a series-elastic monopod with parallel multi-articulation and field-weakening* by Vishnu Dev Amara, Jörn Malzahn and Nikos Tsagarakis.

Now the robot's translational CoM velocity $\dot{\mathbf{c}}$ can be related to the joint velocities using the CoM Jacobian $\mathbf{J}_c$ in the world frame,

$$\dot{\mathbf{c}} = \mathbf{J}_c \dot{\mathbf{q}}.$$

Differentiating the same and applying (7.4) yields,

$$\begin{aligned} \ddot{\mathbf{c}} &= \mathbf{J}_c \ddot{\mathbf{q}} + \dot{\mathbf{J}}_c \dot{\mathbf{q}}, \\ &= \mathbf{J}_c \mathbf{P}_M \mathbf{H}^{-1} \mathbf{B} \boldsymbol{\tau} + \dot{\mathbf{J}}_c \dot{\mathbf{q}} - \mathbf{J}_c \mathbf{P}_M \mathbf{H}^{-1} (\mathbf{n} + \mathbf{J}^+ \dot{\mathbf{J}} \dot{\mathbf{q}}), \\ &= \mathbf{J}_c \mathbf{J}_q \boldsymbol{\tau} + \underbrace{\dot{\mathbf{J}}_c \dot{\mathbf{q}} - \mathbf{J}_c \mathbf{P}_M \mathbf{H}^{-1} (\mathbf{n} + \mathbf{J}^+ \dot{\mathbf{J}} \dot{\mathbf{q}})}_{\ddot{\mathbf{c}}_{vg}}, \end{aligned} \quad (7.6)$$

where $\mathbf{J}_q$ is regarded as a mapping between joint torques and joint accelerations. The translational accelerations of the CoM can be finally decomposed [181],

$$\ddot{\mathbf{c}} = \underbrace{\mathbf{J}_\tau \boldsymbol{\tau}}_{\substack{\text{due to joint} \\ \text{torques}}} + \underbrace{\ddot{\mathbf{c}}_{vg}(\mathbf{q}, \dot{\mathbf{q}})}_{\substack{\text{due to gravity and} \\ \text{joint velocities}}}, \quad (7.7)$$

where $\mathbf{J}_\tau$ is the mapping between joint torques and CoM accelerations given by,

$$\mathbf{J}_\tau = \mathbf{J}_c(\mathbf{q}) \mathbf{J}_q(\mathbf{q}), \quad (7.8)$$

where $\mathbf{J}_c$ is the Jacobian of robot CoM in the world frame [170] and $\mathbf{J}_q$ can be regarded as the mapping between joint torques and joint accelerations. Therefore, CoM accelerations can be decomposed as the sum of components due to i) joint torques and ii) joint velocities and gravity. Note that while $\ddot{\mathbf{c}} \in \mathbb{R}^3$ for a 3D robot, for a planar robot $\ddot{\mathbf{c}} \in \mathbb{R}^2$ and the number of DoFs with underactuation is $n + 3$.

7.2.1 Centroidal dynamic manipulability ellipsoids

During jumping motions, actuators of the robot propel its center of mass (CoM) upward against gravity. Given the robot dynamics and actuator capacities, it is insightful to consider the maximum achievable CoM accelerations. If the unit weighted norm of joint torques can be posed as an inequality,

$$\boldsymbol{\tau}^T \mathbf{W}_\tau \boldsymbol{\tau} \leq 1, \quad (7.9)$$

then by substituting for $\boldsymbol{\tau}$ from (7.7) in (7.9), it can be shown that

$$0 \leq (\ddot{\mathbf{c}} - \ddot{\mathbf{c}}_{vg})^T (\mathbf{J}_\tau \mathbf{W}_\tau^{-1} \mathbf{J}_\tau^T)^{-1} (\ddot{\mathbf{c}} - \ddot{\mathbf{c}}_{vg}) \leq 1, \quad (7.10)$$

where $\mathbf{W}_\tau$ is a symmetric positive definite weighting matrix. The inequality in (7.10) mathematically describes an ellipse in the planar translational CoM acceleration space centered at $\ddot{\mathbf{c}}_{vg}$. The size and shape of the ellipse depend on joint configuration, inertial parameters, kinematic constraints and actuation capabilities (encapsulated by the weighing matrix $\mathbf{W}_\tau$). This ellipse is called the CoM dynamic manipulability ellipse (DME) and can be used to quantify the ability of the robot to accelerate its CoM. Higher the CoM dynamic manipulability, better is the robot's explosive performance. For completeness, derivation of (7.10) is detailed in Appendix B.

7.3 Extension of dynamic centroidal manipulability to parallel actuators

In this section, a short mathematical analysis helps understand the benefits of the methods adopted to enhance jumping performance in *eLeg* reported in the previous chapters. This is the first instance of an application of DCM to redundant joints as well as biarticulated actuators. Therefore, a minimal extension to (7.6) is first presented. The generalized description of torques due to parallel multiarticulation used in (4.5) is reinvoked:

$$\boldsymbol{\tau}_p = -\mathbf{T}^T \mathbf{K}_p (\mathbf{S}\mathbf{p} + \mathbf{T}\mathbf{q}).$$

The effect of added torques, parallel elasticity and inter-joint power-transfer from parallel multiarticulation to centroidal accelerations (7.7) can be written,

$$\ddot{\mathbf{c}} = \mathbf{J}_\tau \left(\underbrace{\boldsymbol{\tau}_{PB} - \underbrace{\mathbf{T}^T \mathbf{K}_p \mathbf{S}\mathbf{p}}_{\substack{\text{active part of parallel} \\ \text{articulation}}}}_{\boldsymbol{\tau}} \right) + \left(\underbrace{\ddot{\mathbf{c}}_{vg}(\mathbf{q}, \dot{\mathbf{q}})}_{\dot{\mathbf{c}}'_{vg}} - \underbrace{\mathbf{T}^T \mathbf{K}_p \mathbf{T}\mathbf{q}}_{\substack{\text{passive part of} \\ \text{parallel articulation}}} \right). \quad (7.11)$$

The joint torques due to the passive part of parallel articulation arise from the robot dynamics. Hence, it is only natural to include all non-control variables with the robot dynamics (bias terms).

In this thesis, $\mathbf{W}_\tau$ in (7.10) is chosen to reflect the corresponding torque limits at the three actuated joints as described in [181],

$$\mathbf{W}_\tau^{-1} = n \left(\text{diag} \left[\tilde{\tau}_1^2 \quad \tilde{\tau}_2^2 \quad \tilde{\tau}_3^2 \right] \right), \quad (7.12)$$

where $\tilde{\tau}_i = \max(|\underline{\tau}_i|, |\bar{\tau}_i|)^2$. In [181], n is considered as the number of actuators and its presence in (7.12) helps account for the entire actuation capabilities of the robot. However, for redundant robots, it is more appropriate to consider n to be the number of actuated joints rather than the number of actuators. Thus, DME is apt suited for evaluating the explosive motion capabilities of any robot beforehand since it visualizes the achievable CoM accelerations given the joint torque capacities. It is worth mentioning that while parallel elasticity results in changes in size and shape of ellipse, PEA torques adds to the joint actuation capacity. Dependence on joint velocities, renders isolating the multi-articulation power transfer benefits/changes to DCM or DME ellipse difficult.

Futhermore, it is interesting to note that the most commonly used actuators (direct-drives or SEAs) can only influence the torque capacity (first term in (7.7)). Only design changes including inertia and kinematic parameters can influence the DME center (second term in (7.7)). However, parallel actuators possess the unique ability to both shift the DME center as well as influence its shape and size.

²($\bar{\cdot}$) and ($\underline{\cdot}$) denote the (a)symmetric upper and lower limits of the corresponding quantity.

Table 7.1 Hip-Knee parallel biarticulation on HyQ

	Stiffness	Hip transmission ratio	Knee transmission ratio	Max. pretension
Unit	N/m	m/rad	m/rad	m
Value	10000	0.06	-0.04	5e2

For instance, this is demonstrated for a planar version of HyQ [185] robot. The hip-knee joints of both the legs are biarticulated in the sagittal plane such that they mimic *rectus femoris* muscle action with the considerations tabulated in Table 7.1³. Accordingly, the corresponding DMEs are shown for *no biarticulation* (blue), passive biarticulation (green) and active biarticulation (red) cases. This unique property provides the designer with more options - when one cannot afford active parallel articulation due to space or mass constraints, parallel springs can provide for obtaining better whole-body/centroidal characteristics.

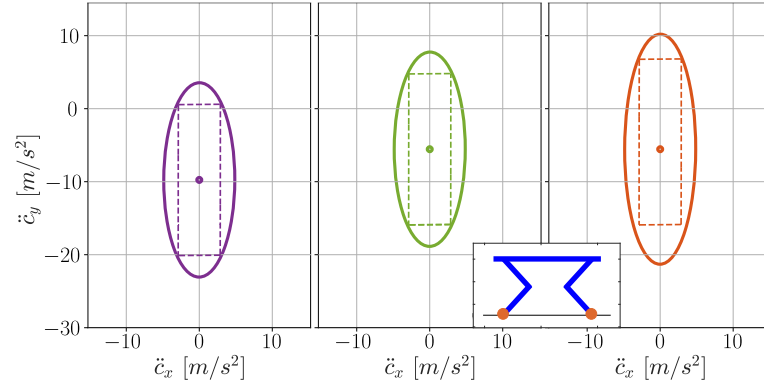


Figure 7.1 DMEs (solid) and FCAPs (dashed lines) are shown for a planar HyQ robot for the following parallel configurations - **without hip-knee biarticulation** (magenta), **with passive hip-knee biarticulation only** (green) and **with active hip-knee biarticulation** (red) for the displayed take-off pose.

7.4 Methods enhancing explosive performance of *eLeg*

One can use (7.10) to comment and understand the benefits of PEAs, FW and multiarticulation towards explosive motions. From (7.7) or (7.11), it can be seen that $\ddot{\mathbf{c}}$ can be suitably enhanced by increasing the torque capacity (first term) as well as by increasing $\dot{\mathbf{c}}'_{vg}$ (second term) through joint velocities. The following two sections presents ways to enhance the aforesaid terms.

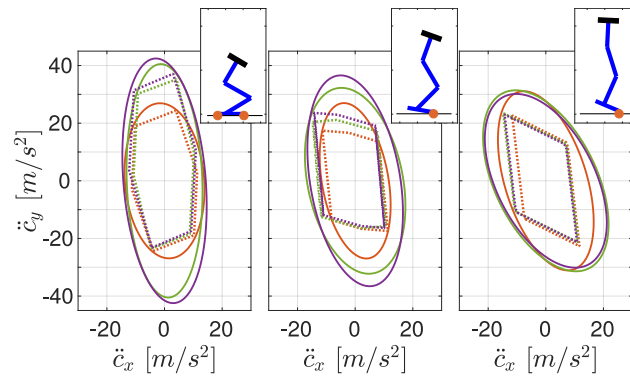


Figure 7.2 DMEs and FCAPs are shown for *noESB* (red), *mono* (green) and *bi* (magenta) actuation configurations in the sagittal plane at the corresponding robot poses displayed in the insets. CoM accelerations due to joint velocities ($\dot{\mathbf{c}}'_{vg}$ in (7.7)) are considered to be zero while plotting the ellipses. While in the first column, heel and toe are in contact (shown by orange dots in the inset) with the ground, in the other two columns, only the toe is in contact.

³Articulation topology transmission ratios are shown for left leg, signs change for right leg, conventions as shown in Fig.7.1.

7.4.1 Explosive motion enhancement due to parallel multi-articulation

DMEs are plotted for three different poses while *eLeg* maintains ground contact(s) for all its actuation configurations in Fig. 7.2. PEAs increase the areas of the ellipsoids as well as negligibly rotate the major axes of the DMEs. Additionally, feasible CoM acceleration polygons (FCAP) as a result of the pretension dependent actuator saturation limits ($\tau_i \geq \underline{\tau}_i, \tau_i \leq \bar{\tau}_i$) are plotted from (7.7) by setting $\ddot{\mathbf{c}}_{vg}$ to zero. The FCAPs are asymmetric about the minor axes of their DMEs as a consequence of the unidirectional PEAs. For the sake of simplicity, the choice of $\bar{\tau}$ for plotting the DMEs does not consider the actuator unidirectionality. It is quite clear that the higher joint torque capacities resulting from PEAs, increase the size of DMEs and FCAPs considerably. For the take-off pose (right column of Fig. 7.2), the increase in areas of DME over the noESB configuration is almost negligible and any change in DME is due to inertial parameter changes. This is solely because the ESBs do not add to the actuation capacity at the particular pose, as the bungees are almost slack by then. The particular observation also indicates that the PEAs are not optimized for jumping motion because *eLeg* can still benefit from ESB torques at that pose.

7.4.2 Explosive motion enhancement due to field-weakening

DMEs are plotted for *eLeg* at an arbitrary take-off pose for all actuation configurations in Fig. 7.3. Joint velocities obtained in experimental results in Sec. 6.4 are used for their computation. It can be seen that FW vertically offsets the centers of corresponding DMEs ($\ddot{\mathbf{c}}_{vg}$ in (7.7)) by suitably enhancing the joint velocities at take-off. The lateral variations in DME centers arise from the changes in parallel elasticity and inertia parameters, however they are insignificant.

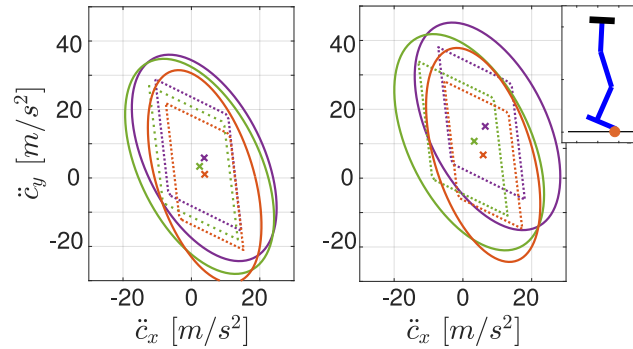


Figure 7.3 DMEs and FCAPs are shown for *noESB* (red), *mono* (green) and *bi* (magenta) actuation configurations of *eLeg* without FW (left column) and with FW (right column) for the displayed take-off pose. FW enhances the vertical CoM acceleration by improving $\ddot{\mathbf{c}}_{vg}$ component (marked by 'x') significantly by improving joint velocities.

7.5 Application to biarticulation design

It is often possible that several scenarios demand highly dynamic motions, more than what an existing humanoid, quadruped or a manipulator is capable of. At such times, one can consider embedding additional PEAs around the robot to quench the need for such additional capabilities, instead of a complete redesign from scratch. Worth noting is that installation of PEAs can be made without significant changes in inertia parameters. Accordingly, DCM measures can be utilized for biarticulation design optimization from (7.11), specifically for maximum tendon pretensions $\mathbf{p}$, tendon stiffnesses

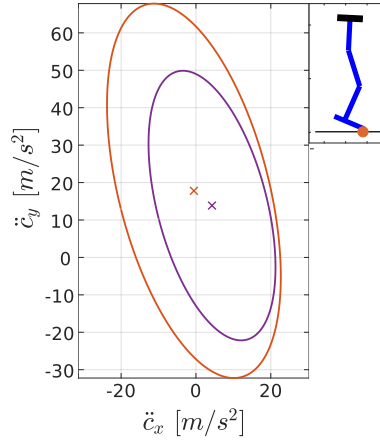


Figure 7.4 DMEs for the eLeg with two biarticulation branches with initial set of parameters (magenta) and optimized parameters (red) are plotted. The centers of DMEs are marked by 'x'. Post optimization, a trade-off is reached between the DME center deviation from y-axis and maximum vertical centroidal acceleration. The tradeoff is determined by the weights set in (7.13).

Table 7.2 Biarticulation design parameter optimization

Specification	Symbol	Unit	Lower limit	Upper limit	Optimized value
Biarticulation stiffness ⁴	K_p	N/m	4e3	20e3	20e3
Ankle-knee biarticulation	t_{11}	m/rad	-0.1	0.1	0.1
topology limits	t_{12}	m/rad	-0.08	0.08	0.010
Knee-hip biarticulation	t_{21}	m/rad	-0.08	0.08	0.072
topology limits	t_{22}	m/rad	-0.08	0.08	0.010
Maximum pretension limits	$\mathbf{p}$	m	0	25e-3	25e-3

(diagonal elements of $\mathbf{K}_p$), tendon attachment point/transmission ratio (elements of $\mathbf{T}$). A minimal example of such a procedure is presented for the constrained optimization of vertical explosive ability of *eLeg* at a particular take-off pose,

$$\begin{aligned}
 \min_{\mathbf{T}, \mathbf{K}_p, \mathbf{p}} \quad & J(\mathbf{T}, \mathbf{K}_p, \mathbf{p}, \mathbf{q}, \dot{\mathbf{q}}) = -\mathcal{W}_y + 0.5 \ddot{c}_{vg, x}^2, \\
 \text{subject to} \quad & \underline{\mathbf{K}}_p \leq \mathbf{K}_p \leq \overline{\mathbf{K}}_p, \quad \underline{\mathbf{K}}_p \geq 0, \\
 & \mathbf{0} \leq \mathbf{p} \leq \overline{\mathbf{p}}, \\
 & \underline{\mathbf{t}}_i \leq \mathbf{t}_i \leq \overline{\mathbf{t}}_i
 \end{aligned} \tag{7.13}$$

where $\mathcal{W}_y$ denotes the point on the DME that lies on the y-axis. $\mathbf{t}_i$ are rows of the topology matrix $\mathbf{T}$. Currently, choices were made to embed biarticulated PEAs for ankle-knee (ankle extension and knee flexion) and knee-hip (knee extension and hip flexion) configurations. Such choices are reflected in the topology matrix limits. However, the optimization process does not necessitate such choices. Fig. 7.4 visualizes the result of biarticulation design parameter optimization on *eLeg*. The optimization criteria penalizes horizontal deviation of ellipse center and maximizes the DME predicted vertical CoM acceleration in a weighted fashion for a particular robot pose. The optimized design parameters are tabulated in Table 7.2.

⁴Uniform stiffness is assumed for both biarticulation branches. In future, this assumption will be dropped.

For the current problem, choices were made as to only biarticulate the ankle-knee joints and knee-hip joints. In future, optimizations will be conducted for the entire topology matrix such that choices of articulation branches and level (mono/bi/tri) will be automated. The essence of this DCM (or DME)-based design optimization approach is understandable while designing a highly articulated robot with many distinct parallel articulation branches. The same procedure can be repeated for multiple poses or joint motions. Additional constraints can enforce greater physical likeness including increasing inertia with pretension parameters (link mass and inertia increases with ballscrew length), tendon stiffness (stiffer bungee cords are thicker and heavier) and topology matrix elements (higher transmission ratio demands a higher pulley radius).

7.6 Summary

While multiple quantitative measures of centroidal capabilities have been developed, centroidal manipulability helps assess the robot's ability to accelerate its CoM. Unlike, serial actuators, parallel actuators play a unique role in that they influence the robot dynamics as well as the joint torque capacities. In this chapter, the definition of centroidal manipulability is generalized to accommodate joints that are powered by parallel actuators. The generalization is further exemplified and a DCM-based optimization process is outlined for parallel biarticulation design. Nevertheless, design co-optimization of series and parallel actuators for robot actuation design can be performed with the same procedure. Efforts are underway to investigate and distinguish the benefits of triarticulation over biarticulation, crossed tendon articulation over normal articulation and topology matrix derivation.

Chapter 8

Conclusion

State-of-the-art actuator technologies that can still profit from improvements are identified in Ch. 1. Literature survey for such identified actuation concepts is presented and possible research gaps are highlighted in Ch. 2. Results of corresponding research activities are then reported in the subsequent chapters. Consequently, the thesis leads to the following major contributions :

Series clutched actuation

- i. An energy-based model of a SCA is derived within the port-Hamiltonian framework. Main difficulties in using the model for control design are highlighted.
- ii. A model-predictive iLQR-based optimal controller is formulated. The controller allows to perform gradual clutching action. Further, the controller is able to dissipate link energy through partially coupling the link with the driving actuator's static friction. Thus, a range of damping coefficients during gravity-driven link motions are obtained.
- iii. The problems in realizing an antagonistic SCA implementation are presented. While the clutch can still regulate torque transmission, it can only not utilize actuator's static friction for link energy dissipation.
- iv. A grid optimization-based component selection process is performed for a serial implementation of an SCA. Furthermore, the ensuing energy savings when compared with a conventional joint are gauged for a range of motion frequencies and amplitudes.

Series-parallel compliant multiarticulation

Evaluation of energy-efficiency

- i. SPEA robots combine the potential of asymmetric series-parallel compliant actuation branches and multi-articulation. A novel QP-based controller is proposed for instantaneous joint torque distribution among the dissimilar actuators.
- ii. This controller can accommodate quadratic cost functions for torque distribution across various parallel articulation topologies. These criteria can also account for actuator hardware specifica-

tions. Accordingly, multiple candidate criteria are composed and their relative energy-efficiencies are compared during periodic squat motions on *eLeg*.

- iii. The energy-based analysis provides valuable insights for control. Allocation of maximum possible torque demand to the higher efficiency PEAs is not the best strategy, at all times. Choice of a joint torque resolution strategy is strongly related to the presence of gravity-driven motion phases and motion frequencies.

Enhanced explosive motions

- i. First demonstrations of multi-DoF field-weakening on a legged robot are performed. FW boosts peak velocities resulting in higher mechanical power output. However, the enhanced velocities come at the loss of energy-efficiency when compared to jumping without FW.
- ii. The combined usage of FW, PEAs with biarticulation significantly improves explosive performance of series elastic actuated robots. Since the jump motions are hand-tuned, PEAs only compensate for the loss in efficiency due to FW.
- iii. To simultaneously enhance energy-economy, actuation-aware multi-phase trajectory optimization is adopted. The same helps plan for optimal jump motions with all three *eLeg* configurations. The *actuation awareness* helps obtain physically feasible and continuous jumping motion references.
- iv. With trajectory optimization, FW, PEAs and biarticulation once again incrementally lead to highly dynamic jumping motions. A significant improvement of 39% is obtained over the baseline jump (without these methods). Simultaneously, PEAs lead to more than 15% higher energy-efficiency than the baseline jump. Future SEA-based humanoid and quadruped robots can massively benefit from parallel elasticity and biarticulation. The essence of utilizing power constraints in the optimization process can further be seen from the consensus between optimized and measured joint motions. With FW control, *eLeg* achieves jump heights of more than 93% of the TO predicted values in all three configurations.

Centroidal dynamic manipulability

- i. Definition of centroidal dynamic manipulability is extended to accommodate parallel actuators. Parallel actuators can scale and skew the CoM dynamic manipulability ellipse as well as shift its center. Serial actuators can only scale and skew the ellipse.
- ii. A centroidal dynamic manipulability analysis mathematically explains the incremental benefits of FW, PEAs (in monoarticulation) and parallel biarticulation towards accelerating the CoM of *eLeg* and thereby enhancing its explosive capabilities.
- iii. The concept is applied for a biarticulation design optimization routine through a minimal example. Future robot biarticulation design can greatly benefit from this manipulability measure as it indicates robot capabilities at the centroidal level.

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Appendix A

Passivity of the LuGre Friction Model

For the particular port Hamiltonian description of the LuGre friction model in (3.13), the passivity of the system is hereby shown with respect to the considered Hamiltonian (3.12). The necessary condition for passivity is the positive semi-definiteness of:

$$\begin{bmatrix} \frac{\partial H_z}{\partial x_z} & u_z^T \end{bmatrix} \begin{bmatrix} R_z & P_z \\ P_z^T & S_z \end{bmatrix} \begin{bmatrix} \frac{\partial H_z}{\partial x_z} \\ u_z \end{bmatrix} \geq 0, \quad \forall z \in \left[\frac{-\mu_s}{\sigma_0}, \frac{\mu_s}{\sigma_0} \right], \quad \dot{\phi} \in \mathbb{R}, \quad F_a \in \mathbb{R}_0^+. \quad (\text{A.1})$$

It is sufficient to satisfy this inequality on the domain of the possible values of the F_a , z and $\dot{\phi}$. Note that the LuGre friction parameters are assumed to be monotonically increasing linear functions of F_a . As (A.1) resolves to the following expression:

$$\left(\frac{\sigma_0^2 z^2 |\dot{\phi}|}{g(\dot{\phi})} - \frac{\sigma_0 \sigma_1 z |\dot{\phi}| \dot{\phi}}{g(\dot{\phi})} + (\sigma_1 + \sigma_2) \dot{\phi}^2 \right) \geq 0, \quad (\text{A.2})$$

it can be seen that the equality is satisfied when either or both F_a and $\dot{\phi}$ are equal to zero. Therefore for all $\dot{\phi}$ in $\mathbb{R}/\{0\}$ and F_a in $\mathbb{R}^+/\{0\}$:

$$\left(\frac{\sigma_0^2 z^2 |\dot{\phi}|}{g(\dot{\phi}) \dot{\phi}^2} - \frac{\sigma_0 \sigma_1 z |\dot{\phi}|}{g(\dot{\phi}) \dot{\phi}} + \frac{\sigma_1 + \sigma_2}{\dot{\phi}^2} \right) \geq 0. \quad (\text{A.3})$$

Infinimum of the entire expression (A.3), is evaluated term-wise. For the first term, $\left(\frac{|\dot{\phi}|}{g(\dot{\phi}) \dot{\phi}} \right)$ is non-negative for all $\dot{\phi} \in \mathbb{R}$. Since, $z^2 \geq 0$ and $\lim_{|\dot{\phi}| \rightarrow \infty} \left(\frac{|\dot{\phi}|}{g(\dot{\phi}) \dot{\phi}} \right) = 0$,

$$\inf \left(\frac{\sigma_0^2 z^2 |\dot{\phi}|}{g(\dot{\phi}) \dot{\phi}^2} \right) = 0. \quad (\text{A.4})$$

For all $\dot{\phi} \in \mathbb{R}$ and $z \in \left[\frac{-\mu_s}{\sigma_0}, \frac{\mu_s}{\sigma_0} \right]$, the infimum of the second term is given by:

$$\inf \left(\frac{\sigma_0 \sigma_1 z |\dot{\phi}|}{g(\dot{\phi}) \dot{\phi}} \right) = \lim_{|\dot{\phi}| \rightarrow \infty} \left(\frac{\sigma_0 \sigma_1 z |\dot{\phi}|}{g(\dot{\phi}) \dot{\phi}} \right) \Big|_{z=\frac{\mu_s}{\sigma_0}} = -\frac{\sigma_1 \mu_s}{\mu_c}. \quad (\text{A.5})$$

Writing the third term in (A.3) along with (A.5) and restating the dependencies on F_a , yields the passivity condition:

$$-\frac{\sigma_1(F_a)\mu_s(F_a)}{\mu_c(F_a)} + (\sigma_1(F_a) + \sigma_2(F_a)) \geq 0. \quad (\text{A.6})$$

This condition is similar to the one obtained in [137, 134] and only additionally requires $F_a \in \mathbb{R}_0^+$. When (A.6) is satisfied, for the Hamiltonian (3.12), the system described in (3.13) is input-output passive. Now it is true that at times when free link motion is desired to follow its natural dynamics, the clutch is required to completely decouple the motor from the link leading to zero normal force ($F_a = 0$). To account for the change in dynamics arising from this clutch contact transition, one can consider modelling the series clutched actuator system as a hybrid combination of two independent systems with a variable Dirac structure ([123], Ch.2, Pg.73).

Appendix B

Centroidal dynamic manipulability ellipsoid inequality

By substituting (7.7) in (7.9) one can obtain,

$$\boldsymbol{\tau} = \mathbf{J}_\tau^+ (\ddot{\mathbf{c}} - \ddot{\mathbf{c}}_{vg}) + \mathbf{N}_\tau \boldsymbol{\tau}_0, \quad (\text{B.1})$$

where $\mathbf{N}_\tau$ is the null-space projection of $\mathbf{J}_\tau$ given by,

$$\mathbf{N}_\tau = \mathbf{I} - \mathbf{J}_\tau^+ \mathbf{J}_\tau, \quad \mathbf{J}_\tau^+ = \mathbf{W}_\tau^{-1} \mathbf{J}_\tau^T (\mathbf{J}_\tau \mathbf{W}_\tau^{-1} \mathbf{J}_\tau^T)^{-1}.$$

Substituting (B.1) in (7.9), one can obtain

$$\mathbf{1} = (\ddot{\mathbf{c}} - \ddot{\mathbf{c}}_{vg})^T \mathbf{J}_\tau^{+T} \mathbf{W}_\tau \mathbf{J}_\tau^+ (\ddot{\mathbf{c}} - \ddot{\mathbf{c}}_{vg}) + \boldsymbol{\tau}_0^T \mathbf{N}_\tau^T \mathbf{W}_\tau \mathbf{N}_\tau \boldsymbol{\tau}_0. \quad (\text{B.2})$$

Since $\mathbf{W}_\tau$ is positive definite, (B.2) simplifies to,

$$\mathbf{0} \leq (\ddot{\mathbf{c}} - \ddot{\mathbf{c}}_{vg})^T (\mathbf{J}_\tau \mathbf{W}_\tau^{-1} \mathbf{J}_\tau^T)^{-1} (\ddot{\mathbf{c}} - \ddot{\mathbf{c}}_{vg}) \leq \mathbf{1},$$

which is already described in (7.10). The last equation describes an ellipsoid in the CoM acceleration space. SVD decomposition of the matrix $\mathbf{J}_\tau \mathbf{W}_\tau^{-1} \mathbf{J}_\tau^T$ yields the eigenvalues and eigenvectors that determine the axes lengths and orientation of the ellipsoid, respectively.

Appendix C

Parametrization of motion plans

The second order continuous curve parametrization for joint motion plans can be obtained as follows. Given the initial and terminal points, x_0 and x_t , respectively and their derivatives $\dot{x}_0$ and $\dot{x}_t$ for a second order curve,

$$x(t) = a_0 + a_1 t + a_2 t^2$$

it can be alternatively parametrized with the curve duration T as,

$$a_0 = x_0$$

$$a_1 = \dot{x}_0$$

$$a_2 = (x_0 - x_t) T^{-2} + \dot{x}_t T^{-1}$$

x_0 and $\dot{x}_0$ are known since *eLeg* starts from a squatting pose at rest. Joint motion plans can be heuristically obtained by setting x_t , T and $\dot{x}_t$. x_t is set based on typical take-off robot (near-singularity) poses. $\dot{x}_t$ can be set to a typical peak velocity obtained during FW. T is set to a value between 0.35 and 0.5.