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RELIABILITY OF NONLINEAR STATIC ANALYSIS IN CASE OF IRREGULAR URM BUILDINGS WITH FLEXIBLE DIAPHRAGMS

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Abstract: The seismic Performance-Based assessment of existing masonry buildings requires the use of nonlinear models, in order to check the attainment of ultimate limit states. Incremental Dynamic Analysis represents the most accurate method but very few models are available which are able to describe the stiffness and strength degradation, which are typical of masonry buildings, as well as the hysteretic behaviour of piers and spandrels under cyclic actions. At engineering practice level, the Displacement-Based approach is widely adopted, through the use of nonlinear static analysis. However, the application in the case of irregular URM buildings with flexible horizontal diaphragms represents an open issue, due to the various difficulties, for example in the selection of proper load pattern or the definition of performance levels. A wide numerical investigation was made of some case studies, in order to check the applicability of nonlinear static analyses and propose some new procedures. Nonlinear dynamic analyses have been adopted as reference solution, by using the Proper Orthogonal Decomposition technique in order to catch the dominant behaviours to be compared with those obtained by pushover analysis.

Introduction

Due to the high vulnerability of existing unreinforced masonry (URM) buildings observed after all seismic events worldwide, in order to support risk mitigation policies proper methods of analysis and verification procedures are necessary. The Performance-Based Assessment (PBA) requires the evaluation of the seismic response to earthquake of different intensity, till to near collapse conditions. Since masonry has a strongly nonlinear behaviour, also for low horizontal actions, the use of a Displacement-Based Approach (DBA) turns out to be necessary for a reliable assessment, which requires the availability of numerical models for static pushover or dynamic nonlinear analyses. In addition, proper criteria for the definition of Performance Levels (PL) are necessary, which are not straightforward in complex masonry buildings, often irregular in plan and/or in elevation, as well as characterized by the presence of flexible horizontal diaphragms.

Nonlinear static analysis, that is widely adopted in international standards (e.g. ASCE/SEI 41-13 2014, EC8-1 2004, NTC 2008), has been originally developed for RC or steel framed structures, under the hypothesis of rigid horizontal diaphragms and, possibly, in the case of regular configurations. As known, the procedure is based on the following steps: 1) execution of a pushover analysis, with a proper load pattern; 2) derivation of the capacity curve of an equivalent nonlinear single degree or freedom (SDOF) system; 3) identification of displacements related to attainment of different PLs; 4) for each PLs, evaluation of the displacement demand by a proper reduced spectrum; 5) comparison between displacement demand and capacity. Then, several proposals have been formulated in literature to improve the reliability of such procedure in case of structures strongly irregular or for which the contribution of higher modes is not negligible. They follow different approaches based on the execution of multi-modal or adaptive analyses (Aydinoglu and Onem 2010) or the introduction of corrective factors to amplify the displacement demand or corrective eccentricities to reproduce torsional effects induced by irregularities; a state of art of such modified procedures for RC buildings has been recently illustrated in De Stefano and Mariani (2014).

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However critical issues arise in the case of traditional URM buildings and a comprehensive procedure based on DBA for this type of structures has been proposed by Lagomarsino and Cattari (2015a). Furthermore an extensive validation of nonlinear static procedures in the case of irregular URM buildings with flexible diaphragms is still lacking and specific directions on the more appropriate load patterns to be used in these cases for the pushover analysis are needed.
Regarding the availability of nonlinear models, the definition of an equivalent frame (piers and spandrels) represents a feasible approach for modelling 3D complex buildings; Tremuri program (Lagomarsino et al. 2013), which is used in this paper, implements multilinear cyclic hysteretic constitutive laws, with failure criteria for piers and spandrels, and is also able to perform nonlinear dynamic analysis.
The conversion into capacity curve is almost insensitive to the selection of the control node for the pushover analysis in the case of regular buildings with rigid diaphragms; in traditional URM a conventional but effective approach has been identified in the average displacement of nodes at the top level. The definition of PLs is treated in codes by checking corresponding PLs in each single element (ASCE/SEI 41-13 2014) or by considering interstorey drift thresholds and/or heuristic criteria on the stiffness and strength degradation of the pushover curve (EC8-3 2005, NTC2008). However, in case of flexible diaphragms, both methods turn out to be inadequate; therefore a multiscale approach that considers all the above-mentioned three checks seems to be necessary (Lagomarsino and Cattari 2015a).
This paper presents some of the results achieved in a wide parametric analysis on different prototypes of masonry buildings. These models were conceived starting from a regular configuration, then a progressing increase in the in plan/elevation irregularity and a decrease in the stiffness of diaphragms were applied. For each case study, the results obtained by nonlinear Incremental Dynamic Analyses (IDA) are considered as the reference correct behaviour. The large amount of achieved information (time histories of displacements and acceleration in each node of the building) has been processed by the Proper Orthogonal Decomposition (POD) method (Lagomarsino and Cattari 2015b, Cattari et al. 2014), which has been very useful to single out the dominant displacement profiles in each masonry wall, as well as the principal inertial load distributions.
The comparison between nonlinear static and dynamic results has considered deformation modes and load profiles, as well as the estimation of the seismic capacity, in terms of Peak Ground Acceleration (PGA) values that produce the attainment of the different PLs. Finally, some preliminary proposals on the best load patterns to be used in this kind of buildings are presented.

Procedure adopted in order to assess the reliability of nonlinear static analyses

As already introduced, the validation of nonlinear static procedures for the PBA of URM buildings, with some proposals for the definition of the load pattern to be used for the pushover analysis, was made with reference to a set of case studies with different sources of irregularity and assuming the results of nonlinear dynamic analyses as the reference correct solution.
The verification according to PBA requires to evaluate the earthquake intensity that produces the attainment of any PL (IM_{PL}) and to check that it is lower than the corresponding target earthquake hazard level, defined in terms of mean return period T_R. The hazard curve of the site gives, as a function of T_R, the seismic input in terms of an Intensity Measure IM (usually the PGA or Spectral Acceleration values). The Acceleration-Displacement Response Spectrum (ADRS), which allows, properly reduced, to evaluate the displacement demand on the capacity curve in the case of nonlinear static procedures, can be scaled by the IM.
In the case of nonlinear dynamic analysis the seismic input is the acceleration time-history at the base of the structure, to which an irregular ADRS corresponds, if compared with those assumed by codes for the design/assessment, which represents an average spectrum of different possible ones, due to the record-to-record variability. For this reason ten records have been used, from the main shock of the five stations in L'Aquila, Italy (2009), conditioned

to the spectral acceleration S_a for the period T=0.35 s, assumed as representative of the main modes of vibration of the considered buildings. IDA have been performed, obtaining for each record the value of IM corresponding to the attainment of each PL; then the median value and 16% and 84% percentiles of IM are evaluated, under the hypothesis IM is log-normally distributed.
Figure 1a shows the acceleration response spectra of the 10 records, scaled in order to have the same IM=S_a(0.35). Figure 1b shows the corresponding median response spectrum, which is almost coincident with the code spectrum, as well as the 16% and 84% percentiles. Therefore, in the case of nonlinear static analysis, the IM correspondent to each PL has been evaluated with these three spectra. The three values of IM are compared with the corresponding ones obtained by IDA.

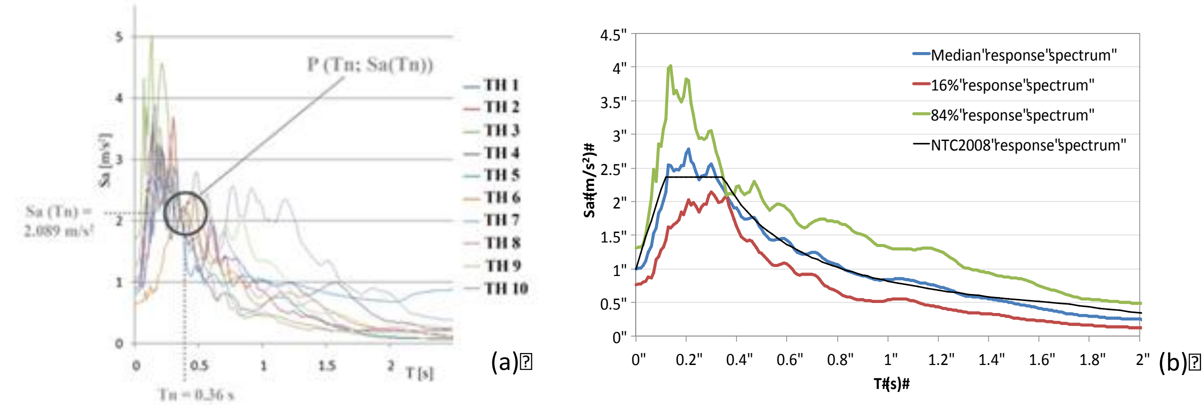


Figure 1. Acceleration response spectra of the 10 records (a) and 16%, 84% e code response spectrum (b)

Pushover analyses were carried out by different load patterns, kept invariant during the analysis. It is worth noting that codes usually allows the use of nonlinear static procedure if the participant mass on the first mode is greater than a given percentage (around 75%), a condition that is met for regular buildings with rigid floors. For this reason one possible load pattern is the one correspondent to the first modal shape. In addition, in order to consider the possible formation of a soft story mechanism, the use of a load pattern proportional to masses is suggested. An alternative to first modal load pattern is the “triangular” one, which is obtained by assuming a triangular displacement shape. In the case of irregular URM buildings with flexible floors these load patterns are not always reliable. On one hand, the first mode is not representative because the participant mass can be very low, in particular when diaphragms are flexible; on the other hand, the “triangular” load pattern is not correct because walls have not the same stiffness and in the absence of rigid floors they are not forced to deform in the same way. In order to overcome these critical issues, two additional load patterns have been investigated, obtained by a combination of load patterns derived from all mode shapes which do not present the inversion of sign in displacement (modes of 1st type): 1) SRSS, using the Square Root of Sum of Squares of the 1st type modes; 2) CQC, using a Complete Quadratic Combination of the same modes. All the above-mentioned load patterns have been applied and the results compared.
For each static nonlinear analysis, it is necessary to define the displacement on the pushover curve in which the PL is attained. In this paper the multiscale approach developed within the PERPETUATE research project (Lagomarsino and Cattari 2015a) has been adopted, which takes into account the behavior of single elements (E), macroelement (M) and of the global building (G). For each scale, proper variables are introduced and their evolution in nonlinear phase is monitored: the cumulative rate of panels (piers and spandrels as identified in the equivalent frame idealization of URM walls) that reach a certain damage level (E); drift in masonry walls and horizontal diaphragms (M); normalized total base shear, from global pushover curve (G). The reaching of assigned thresholds for such variables allows to define the displacements on the pushover curve corresponding to the attainment of PL at these

different scales ($u_{E,PLk}$, $u_{M,PLk}$ and $u_{G,PLk}$), being thus the minimum value that establishes the final position of PL. The adoption of this multiscale approach turns out very useful in particular when a damage concentration is expected on single walls that however could not correspond to a significant decay of the overall shear base. In fact, this latter represents the common heuristic and conventional approach adopted in codes, for example equal to 20% for the definition of the Life Safety PL in NTC 2008 or EC8-3 2005, but may reveal not adequate in case of URM existing buildings as those examined. The multiscale approach adopted may be applied with analogous principles in case of both static and dynamic nonlinear analyses guaranteeing a consistent comparison between results provided by two methods (as discussed in Lagomarsino and Cattari 2015b and summarized in Figure 2).

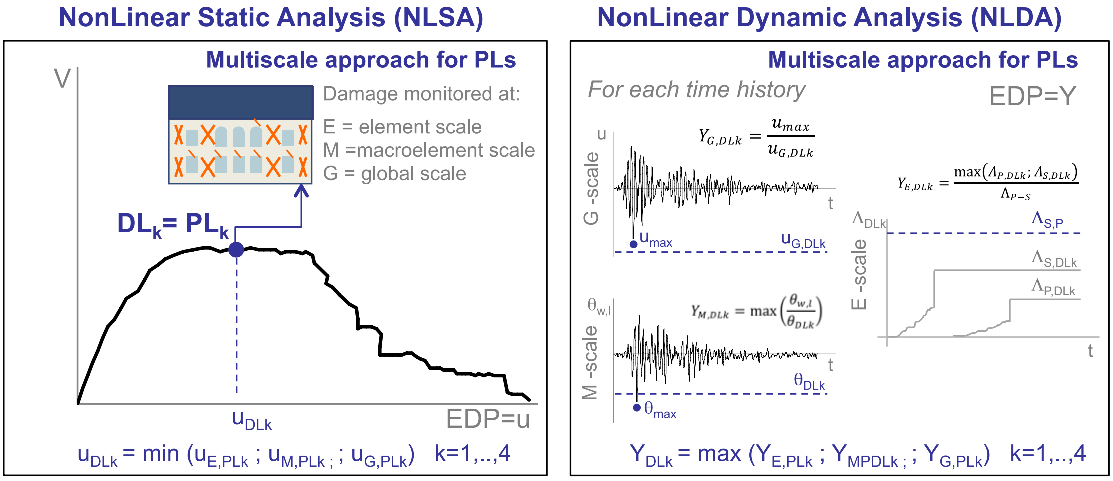


Figure 2. Multiscale approach adopted for the definition of PL in case of static and dynamic analyses (adapted from Lagomarsino and Cattari 2015b)

Then, as aforementioned, the computation of IM_{PL} in case of nonlinear static analyses requires the conversion of the pushover curve (representative of the original Multi Degree of System) into the equivalent SDOF. In the paper, such conversion is performed by adopting the concept of participation factor Γ, as adopted in NTC (2008) and EC8-3 (2005) and originally proposed by Fajfar (2000), which is computed as:

$$\Gamma = \frac{\sum_i m_i \Phi_i}{\sum_i m_i \Phi_i^2} = \frac{m^*}{\sum_i m_i \Phi_i^2} \qquad (1)$$

where m* is the mass of the equivalent SDOF system, m_i is the mass of the i-th node of the EF model (or storey if a stick model is assumed) and Φ_i represents the normalized displacement of the i-th node. In MIT (2009), it is suggested to refer to the displacement pattern of the fundamental modal shape independently from the load pattern chosen. However this factor is very sensitive to the applied load pattern and the related deformed shape, which furthermore varies in the nonlinear response. In the paper, the displacement profile produced in the elastic phase by the application of each load pattern assigned has been adopted as reference for the conversion.

Then, the computation of IM_{PL} has been finalized by adopting the Capacity Spectrum Method based on the use of overdamped spectra. For the reduction of elastic spectra the law proposed in EC8- 1 (2004) and NTC (2008) has been used that is based on the computation of the factor η equal to $\sqrt{10/(5+\xi_{equ})}$, where the equivalent damping ξ_{equ} (sum of the elastic and hysteretic contribution) has been calibrated through cyclic pushover performed on each prototype building examined by applying as maximum displacement the one corresponding to the attainment of each PL (u_{PL}). For each PL two full cycles of loading have been performed and then the hysteretic damping (ξ_{hist}) has been calculated as:

$$\xi_{hist} = \frac{E_d}{2\pi(E_{S0+} + E_{S0-})} \tag{2}$$

where E_d is the energy dissipated during the cycle considered and E_{S0} is the elastic energy produced (+ for positive direction of loading, - for the negative).
Together with the computation of earthquake intensity associated to the specific PLs, the procedure may be applied for any current point of the pushover curve thus leading to the ISA (Incremental Static Analysis) curve.
By comparing ISA and ISA curves it is possible to provide a more comprehensive comparison of results between static and dynamic nonlinear analyses (Fig.3). In particular, it is useful to highlight if the possible differences in values of IM_{PL} are mainly related to discrepancies in the attainment of PL, for example due to a different spread of damage that may anticipate the attainment of checks at scales monitored (E,M,G), or to those related to intrinsic limits of the static method (e.g. on the conversion into equivalent SDOF, the approximate evaluation of damping, etc.). In fact, in the first case the IDA and ISA curves are expected to be very similar, while not in the second one.

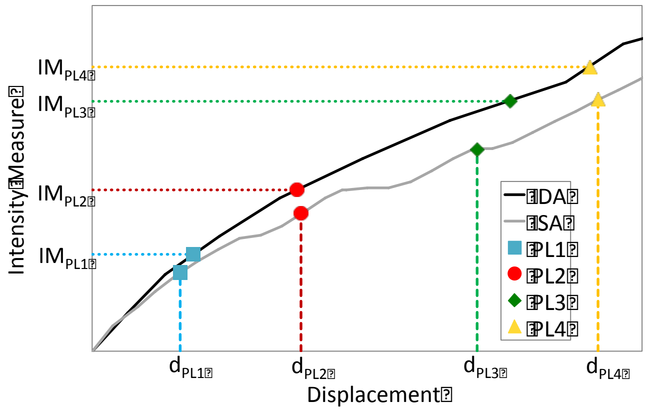


Figure 3. Comparison of IDA and ISA curves to check the reliability of static approach

It is evident dynamic analyses produce a considerable amount of data that go far beyond the strict computation of IM_{PL} and could be very useful to make further comparisons with the results obtained from the static analyses (i.e. the displacement and the acceleration time histories of all nodes of the model). To this aim, the use of the POD technique to process the output of dynamic analyses is adopted in the paper. The method basically consists in the eigenvalue decomposition of the covariance matrix estimated from the acceleration or displacement time histories resulting from the nonlinear dynamic analysis (Lagomarsino and Cattari 2015b). In some recent applications (Cattari et al. 2014), this technique revealed particularly effective to interpret the seismic response in terms of dominant behaviours than referring, as usual, to single and instantaneous peaks of the response (e.g. the maximum displacement occurred in a point of the structure, like as the top level). In particular, in the paper its use it proposed to capture the deformed shapes associated to the attainment of each PL and the distribution of the inertial forces to be compared with those adopted for the static analyses.

Case study and parametric analyses
The research started with the intention to consider a rather simple construction but representative of typical existing buildings of Italy and, more in general, European countries. In particular, it is a three storey full clay masonry building with lime mortar. In Figure 4 a plan view of the basic configuration adopted and a 3D view of the equivalent frame model is depicted. As aforementioned the numerical analyses have been carried on with the software TREMURI (Lagomarsino et al. 2013).

The response of masonry panels is simulated by adopting nonlinear beams with piecewise-linear behaviour that have been recently implemented in the software (Cattari and Lagomarsino 2013). A proper constitutive law allows to describe the non linear response until very severe damage levels (from 1 to 5) through progressing strength decay in correspondence of assigned values of drift. In this case, the strength has been computed according to criteria proposed in NTC (2008) for interpreting the shear and flexural behaviour of URM existing buildings. Moreover, also a hysteretic response is formulated through a phenomenological approach, to capture the differences in the various failure modes that may occur (if *flexural* or *shear* prevailing and *mixed* as well) and in the different response of piers and spandrels. This feature is essential to perform in a reliable way nonlinear dynamic analyses and cyclic pushover as well.
Diaphragms are modelled as 3- or 4-nodes orthotropic membrane finite (plane stress) elements. They are identified by a principal direction (floor spanning direction), with two values of Young modulus along the two orthogonal directions (parallel, E_1, and perpendicular, E_2, to the spanning direction), Poisson ratio (ν) and in-plane the shear modulus (G_{eq}). This latter represents the shear stiffness of the floor and influences the horizontal force transferred among the walls, both in linear and nonlinear phases.

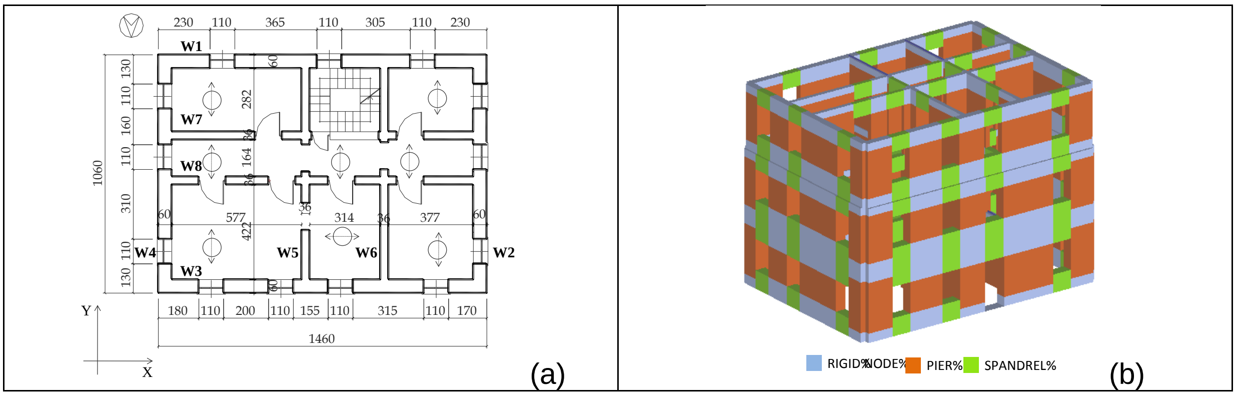


Figure 4. Basic configuration of the prototype building: (a) in plan view and (b) equivalent frame idealisation

Starting from the basic configuration of the prototype building, that has a regular distribution of the openings and is characterized by *rigid* diaphragms (RC slabs), different variants have been defined in order to examine the effects related to modifications in the stiffness of floors and in the plan regularity.
As far as the floor stiffness is concerned, two additional configurations have been defined representative of an *intermediate* and *flexible* condition, respectively. In fact, in the case of URM ancient and existing constructions, the diaphragms are often constituted by timber floors or vaults that are far from the idealisation of *rigid* diaphragms. In case of timber floors the shear stiffness mainly depends on the sheathing (if single or double straight), the presence of steel dowels embedded inside the masonry, the quality of connection provided by joists. Reference values for the shear stiffness are in Brignola et al. (2012). Vaults also represent a wide class and in this case the stiffness contribution strongly depends, beside thickness and material properties, on shape and geometrical proportion (e.g. rise-to-span ratio, as discussed in Cattari et al. 2008). Within this context, for the examined case studies, it may said that: *flexible* condition is representative of a single straight sheathing, for timber floors, and barrel and cross vaults with an high rise-to-span ratio; while *intermediate* condition of double straight sheathing with good connection provided by joists, for timber floors, and cloister vaults or barrel and cross vaults with a low rise-to-span ratio.
In order to introduce plan irregularity, the stiffness of two outer walls has been changed, as shown in Figure 5. Wall 2 (see Fig. 5a), in the base model has three rows with three opening each: the irregularity has been introduced closing six of these nine opening and enlarging the

correspondent six windows in Wall 4. In Figure 5b the red dots represent the centres of masses for the two plan configuration, whereas the blue ones the centre of stiffness. As it is shown, in the irregular configuration the distance between the two points is bigger than the base configuration, as consequence the seismic action induces torsional effects to the building.

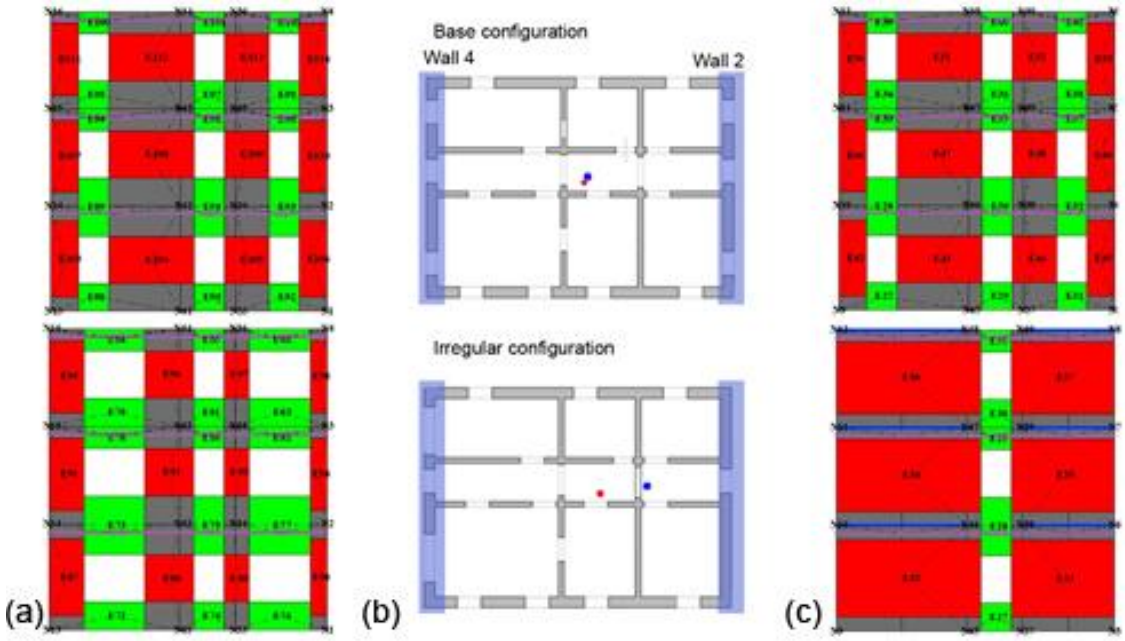


Figure 5. a) Wall 4 base model (top) and irregular model (bottom). b) plane configuration base model (top) and irregular model (bottom) c) Wall 2 base model (top) and irregular model (bottom)

Finally, in order to introduce elevation irregularities further variations have been introduced to the base configuration and other two models have been defined, as illustrated in Figure 6. However, in the paper the results of these latter are not discussed for sake of brevity.

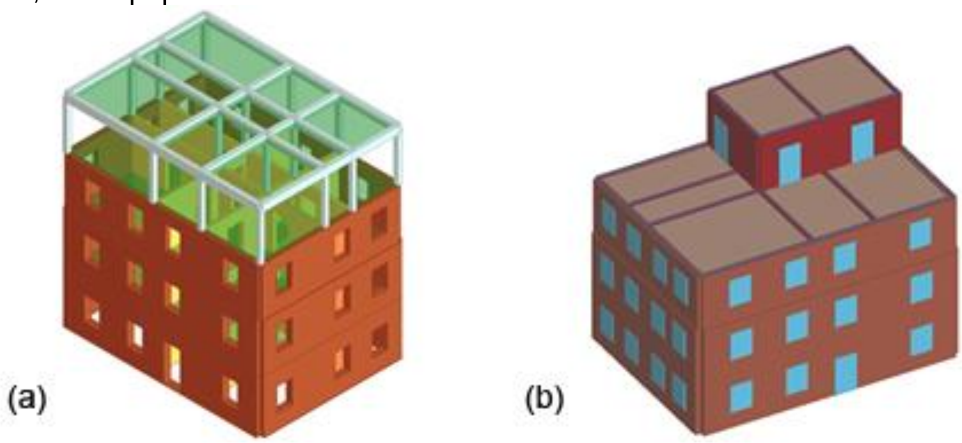


Figure 6. Variations of the base model adding elevation irregularities: a) RC raising-up (b) masonry raising-up

Table 1. Main parameters adopted for the prototype buildings analysed

Masonry properties		**Diaphragms properties**				**Damping [%]**		
			Rigid	**Inter.**	**Flex.**		**Irr./Rigid**	**Irr./Flex**
E_m [MPa]	750	**E_1 [MPa]**	58800	58800	58800	$\xi_{elastic}$	5	5
G_m[MPa]	250	**E_2 [MPa]**	30000	30000	30000	$\xi_{hist,PL1}$	8	10
f_m [MPa]	2.8	**G_{eq}[MPa]**	12500	100	10	$\xi_{hist,PL2}$	12	12
τ_0 [MPa]	0.11	ν	0.2	0.2	0.2	$\xi_{hist,PL3}$	14	13
ρ [kN/m^3]	18	**t [m]**	0.04	0.04	0.04	$\xi_{hist,PL4}$	20	14

Table 1 summarizes the mechanical properties adopted for masonry together with the stiffness properties assigned to diaphragms and the equivalent damping adopted for the computation of IM_{PL}.

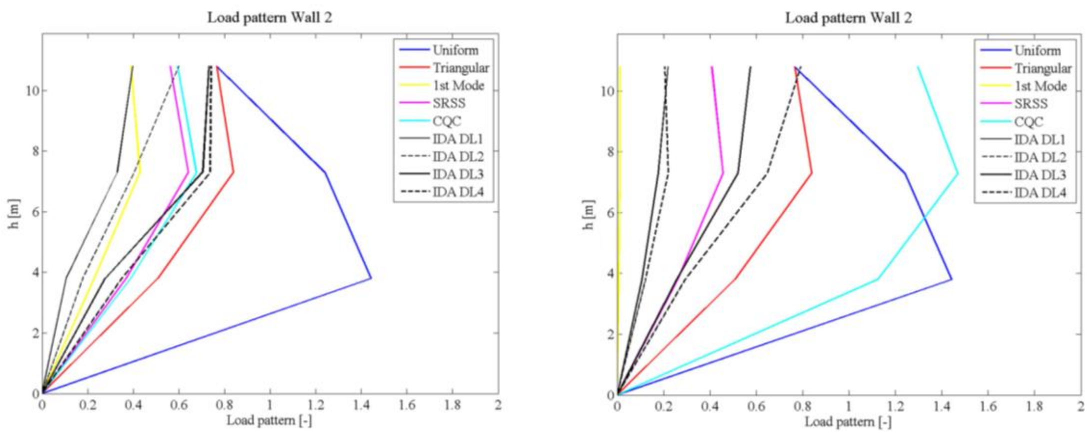
Discussion of the results

Firstly, Table 2 summarizes the results achieved in terms of comparison of IM_{PL} values, by way of example, in case of irregular plan configuration with *rigid* and *flexible* floors. In general it is worth noting that the results of nonlinear static analyses are on the safe side. Through a more detailed comparison in terms of IDA and ISA curves (as those shown in Figure 2), it was possible to establish that such result is mainly due to a general underestimation of the displacement value associated to the attainment of PL through the static approach. In fact, IDA and ISA curves are comparable, being this latter a result that confirm in general the reliability of nonlinear static procedures.

Table 2. IM_{PL} values (in m/s^2) achieved through nonlinear static and dynamic analyses

		PL1		**PL2**		**PL3**		**PL4**	
Load pattern		**Conf. A)**	**Conf. B)**	**Conf. A)**	**Conf. B)**	**Conf. A)**	**Conf. B)**	**Conf. A)**	**Conf. B)**
Uniform	**84%**	0.56	0.62	1.68	1.08	2.15	1.38	2.15	1.74
	50%	**0.68**	**0.69**	**2.2**	**1.4**	**2.84**	**1.87**	**3.32**	**2.29**
	16%	0.84	0.76	2.87	1.83	3.82	2.54	4.91	3.07
Triangular	**84%**	0.68	0.54	1.39	0.80	1.99	1.02	2.22	1.11
	50%	**0.78**	**0.69**	**1.81**	**1.05**	**2.63**	**1.32**	**3.26**	**1.49**
	16%	0.87	0.76	2.37	1.37	3.62	1.73	4.47	2.03
1st mode	**84%**	0.52	0.68	1.39	1.42	1.63	1.42	2.19	1.51
	50%	**0.6**	**0.89**	**1.71**	**1.87**	**2.2**	**1.89**	**2.98**	**2.31**
	16%	0.7	1.16	2.25	2.51	3	2.58	4.11	3.26
SRSS	**84%**	0.56	0.68	1.32	1.42	1.72	1.47	2.25	1.72
	50%	**0.64**	**0.93**	**1.72**	**1.64**	**2.3**	**1.96**	**3.1**	**2.3**
	16%	0.72	1.16	2.25	2.22	3.12	2.63	4.24	3.11
CQC	**84%**	0.54	0.61	1.32	1.78	1.71	2.38	2.25	2.97
	50%	**0.62**	**0.50**	**1.71**	**1.44**	**2.29**	**1.79**	**3.1**	**2.22**
	16%	0.72	0.421	2.25	1.38	3.12	1.38	4.24	1.68
Non Linear Dynamic Analysis	**84%**	0.97	0.70	1.77	1.44	3.13	1.79	3.35	2.22
	50%	**1.26**	**0.92**	**2.32**	**1.73**	**3.96**	**2.63**	**4.41**	**3.06**
	16%	1.63	1.20	3.04	2.19	5.01	3.38	5.81	4.21
Legend: Conf. A) irregular plan - rigid diaphragms; Conf. B) irregular plan- flexible diaphragms									

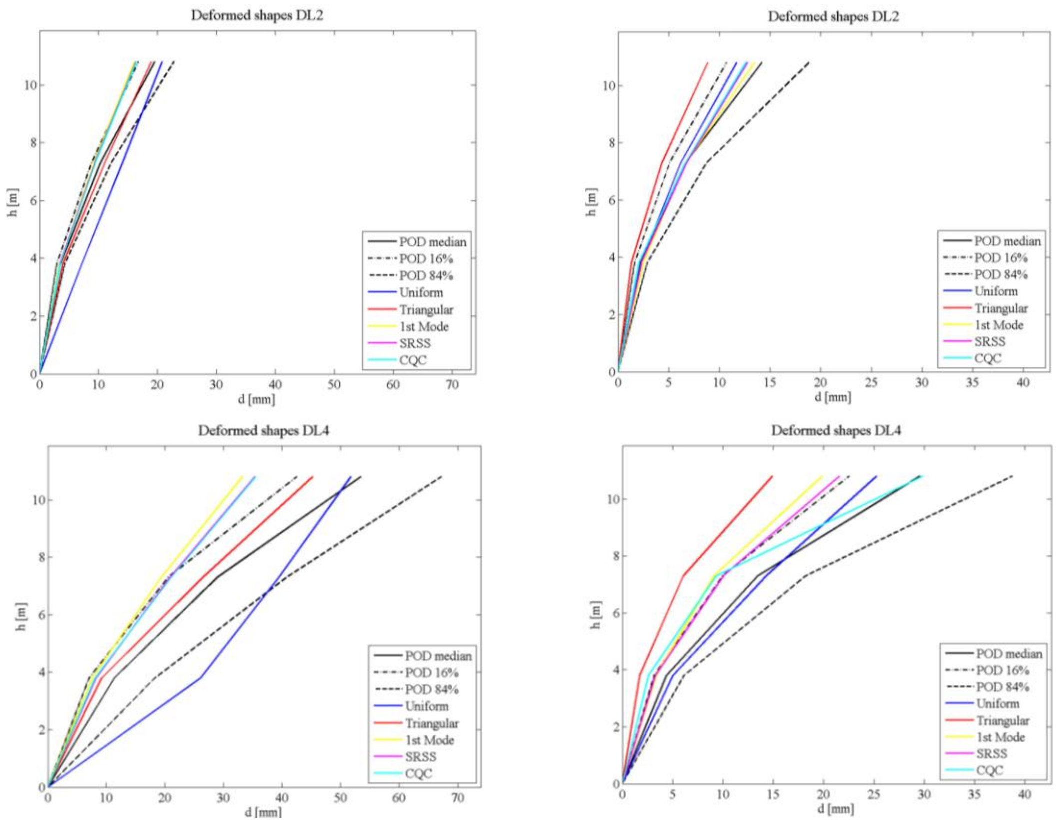
Figure 7 illustrates the comparison in terms of load patterns.



(a) irregular plan - rigid diaphragms (b) irregular plan - flexible diaphragms

Figure 7. Comparison of the load patterns deriving from pushover analyses and dynamic analyses.

The uniform load pattern always results very different from the others, while among those aimed to simulate the “modal shape” the SRSS revealed to be the more reliable, also a function of the various cases examined (of which herein, for sake of brevity, is presented only a selection, being a more comprehensive discussion of results illustrated in Camilletti 2015). In fact, the use of the 1st modal load pattern revealed quite questionable in particular in case of flexible floors since it tends to some walls are almost not loaded (see wall 2 in Figure 7b). Although the response promoted by the adoption of the uniform load pattern in the examined cases has not confirmed by results of nonlinear dynamic analyses, its use combined with other load pattern is necessary, as suggested also by codes. In fact, it is able to highlight different types of failure modes, such as soft storey mechanisms.



(a) irregular plan - rigid diaphragms (b) irregular plan - flexible diaphragms

Figure 8. Comparison of the deformed shape deriving from pushover analyses and dynamic analyses..

Finally, from the comparison of the results in terms of deformed shapes associated to the PL attainment (Figure 8), it is possible to observe that if the floors are stiff the load pattern does not have significant influence on the displacement capacity, whereas for flexible floors the triangular load pattern tends to provide excessive conservative results.

Final remarks

A procedure to assess the reliability of nonlinear static procedures in case of URM buildings based on the comparison with nonlinear dynamic analyses has been presented in the paper. It is comprehensive of proper tools to define performance levels in a coherent way through such two approaches and interpret in an effective way the rich amount of data provided by nonlinear dynamic analyses (through the POD technique). Results presented on the first set of case studies outlined, representative also of irregular configurations and in presence of flexible floors, showed as, hopefully, nonlinear static procedure are on the safe side. Concerning the choice of load pattern, the results achieved suggest the adoption of that coming from the combination through the SRSS rule as that more robust in providing reliable assessments. Thus, its use, in combination with the uniform load pattern, is proposed in the paper as alternative to that proportional to the first mode or the triangular one.

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