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Hot property. The amplifying effects of housing costs on regional inequality ☆

Gabriele Cardullo [a,b,*], Agnese Sechi [a]

[a] *Department of Economics, University of Genoa, Italy*
[b] *IZA, Germany*

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ABSTRACT

We develop a spatial equilibrium model with labor market frictions and non-homothetic preferences to investigate how local changes influence regional inequality. We show that these effects are significantly amplified when the housing expenditure share is large and falls with income (meaning that housing is a necessity good) and wage bargaining is decentralized. Under non-homothetic preferences, local changes induce stronger variations in real income and, in turn, on inter-regional labor mobility. Calibrated to German data, the model provides a quantitative explanation for the narrowing East–West divide in Germany over the past fifteen years.

1. Introduction

Over recent decades, the evolution of interregional inequality has posed a persistent and multifaceted challenge for researchers. In the European Union, regional convergence progressed markedly during the early years of the new millennium but has slowed considerably over the past fifteen years.[1] A similar decline in convergence has been documented for the UK and the United States.[2]

Several theories have been proposed to explain the causes of regional dynamics, ranging from the effects of urban scale and agglomeration to the impacts of globalization and the structural shift from manufacturing to a (digital) services economy.[3] This paper argues that such explanations remain incomplete unless non-homothetic preferences are explicitly incorporated. We show that, regardless of the broader structural drivers emphasized in the literature, two key factors can significantly amplify the process, either accelerating the catching-up phase or deepening the divide. These factors are: (i) the high — and progressively declining with income — share of housing expenditures in household budgets (i.e. preferences are non-homothetic and housing is an expensive necessity good) and (ii) the presence of decentralized wage bargaining systems.

Our quantitative analysis is based on Germany, a country where both factors are notably present. We find that a 1% increase in labor productivity in the tradable sectors in the Eastern poorer states reduces the employment gap with the rest of the country by 2% and the differences in aggregate output and cost of living index by about 3%. If, counterfactually, the housing expenditure share were constant across income levels, the same 1% increase in productivity would generate a much smaller effect: the employment gap would shrink by only 0.4%, the output gap by 1.4%, and the cost-of-living gap by 0.9%. Similarly small effects are obtained if we assume more centralized wage bargaining while keeping a decreasing housing expenditure share.

We derive these results by constructing a novel framework that incorporates the non-homothetic constant elasticity of substitution utility function developed by Comin et al. (2021) into the two-region spatial equilibrium model of Rosen (1979), Roback (1988), and Moretti (2011). This type of preferences allows the housing expenditure share to decrease with income.[4] Labor markets present frictions, resulting

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* Corresponding author at: Department of Economics, University of Genoa, Italy.
E-mail addresses: cardullo@economia.unige.it (G. Cardullo), agnese.sechi@edu.unige.it (A. Sechi).

[1] Partitioning NUTS 2 regions in the EU-28 area into three GDP-per-head classes (as a percentage of the EU average) shows that over 40% of regions moved up to a higher GDP-per-head class between 2002 and 2007. The corresponding figure declined to about 30% in the 2007–2012 period (see Monfort (2020)). This trend of slow to nihil convergence persisted in the subsequent years (see the latest Report on Economic, Social and Territorial Cohesion, available at https://ec.europa.eu/regional-policy/information-sources).

[2] Ganong and Shoag (2017) show that from 1990 to 2010, income gaps between US states fell to less than half the historical average, and in the years leading up to the Great Regional Recession, it nearly disappeared altogether. For the UK, see McCann (2020).

[3] A comprehensive survey of the empirical and theoretical literature is provided by Magrini (2004). A more recent overview is offered by Bathelt et al. (2024).

[4] Conversely, under homothetic preferences the expenditure share on any particular good does not change with income.

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in equilibrium unemployment, and wages are bargained to split the surplus arising from a successful match between the worker and the firm.

In this setting, the amplifying effects of housing as a necessity good and decentralized bargaining come via its impact on real wages and, in turn, on labor mobility. So, if we consider again a positive productivity change in the tradable sector of a poor region, real earnings there rise more than they would under homothetic preferences, prompting stronger migration inflows and accelerating regional convergence. The same magnifying effects with opposite sign occur if we focus on a change that raises regional inequality.

Two channels make real pays more sensitive to local variations under non-homothetic preferences. The first channel originates from a positive feed-back loop between bargained wages and housing prices. Any change that raises labor earnings leads to higher housing prices that in turn augment pays even more. Unlike in a homothetic setting — where the link is one-way — in our non-homothetic scenario higher prices feed back into wage increases, as the higher cost of living raises workers' marginal utility of income in the wage bargaining process.[5] This mechanism is effective in decentralized wage bargaining systems, where local housing costs influence wage negotiations; in centralized systems, national unions are less responsive to local price changes.

The second channel comes from the fact that a change that raises wages also tends to reduce the employees' cost of living index, because better-paid workers will spend a smaller fraction of their income on the more expensive housing services.[6]

Together, these channels reveal a novel mechanism through which the housing sector shapes regional inequality. Viewing housing as a necessity is not just empirically grounded.[7] When the housing expenditure share is large and falls with income, the property market amplifies changes that originate elsewhere. This can help to understand why some countries have seen a significant reduction in income disparities among their regions, while others have seen little to no change.

Germany in the last 15 years is a story of strong income convergence between East and West states. In line with the mechanism outlined in our paper, the relative increase in output per worker in the East states of Germany has been accompanied by (i) a surge in rents and property prices, (ii) a rise in the housing expenditure share, strong and uneven across income groups, (iii) a robust growth in nominal and real earnings, (iv) notable changes in East–West internal migration, with states in East Germany recording overall net migration gain since 2017. It is also well known that since the end of the twentieth century the country went through an unprecedented process of decentralization of wage bargaining.[8]

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the model. Section 4 illustrates the properties of the equilibrium and the comparative statics results. Section 5 shows the quantitative results. Section 6 concludes.

[5] Intuitively, a one euro pay rise is more valuable when housing prices are higher. Since housing expenditure share falls with income, that euro makes workers choose a lower proportion of the good that has become relatively more expensive.

[6] The same two channels are present if the initial change has a negative effect on salaries. In this case the decrease in the real value of the compensations is larger when housing is assumed a necessity good.

[7] Several empirical works confirm this feature. See for instance Quigley and Raphael (2004), Albouy et al. (2016), Larrimore and Schuetz (2017), Fleck et al. (2021) and Diamond and Moretti (2021) for the United States, Dustmann et al. (2021) for Germany, and Belfield et al. (2015) for the UK.

[8] See also Dustmann et al. (2014) for a detailed discussion. More details about decentralization of collective bargaining are provided in Appendix C.

2. Related literature

This work relates to the literature on the housing market's role in shaping labor mobility. Zabel (2012) looks at how the relative mobility of homeowners (versus renters) and the cost of housing affect the flow of workers across US cities. Fischer (2023) and Garriga et al. (2021) show that rural-to-urban migration in China has raised housing prices, slowed convergence, and contributed to inequality. Ganong and Shoag (2017) highlight how rising housing costs hinder low-skilled worker migration, exacerbating inequality—a perspective close to ours. While they emphasize skill sorting, we abstract from heterogeneity to focus on wage-setting systems and their spatial effects.

Our paper is also related to the literature on spatial misallocation, which studies how institutional and regulatory frictions that restrict mobility lead to output and welfare losses. Hsieh and Moretti (2019) show that productivity differences across U.S. states manifest primarily in housing prices and nominal wages, while employment levels remain relatively less affected. This outcome is largely attributed to stricter housing supply regulations in the most productive regions. Boeri et al. (2021) argue that centralized wage bargaining, as in Italy, reduces mobility and increases inequality compared to decentralized systems such as Germany's. While our analysis aligns with theirs, the mechanism differs: in their model, national wages are exogenously fixed above the market-clearing level in poorer regions, making centralized bargaining inherently inefficient and a direct source of income disparities. In our framework, substantial differences between centralized and decentralized systems emerge only when housing is modeled as a necessity good. Moreover, we show that centralized bargaining can sometimes mitigate regional inequality — e.g., following a positive productivity shock in richer regions — highlighting a more nuanced role for national institutions.

Our work also connects to research on non-homothetic preferences and inequality. Albouy et al. (2016) estimate a non-homothetic CES utility function and find that rising rents largely explain the increasing housing expenditure shares in the U.S., worsening affordability for low-income households. Similarly, Dustmann et al. (2021) show that higher housing expenditures have amplified real income inequality in Germany.[9] Gong and Leung (2024) demonstrate that, in a homothetic model, rent caps modestly reduce welfare inequality, but under non-homothetic preferences the same policy increases it. Grossmann et al. (2021) analyze zoning deregulation and find heterogeneous welfare gains due to rent differentials across income groups. These papers are primarily empirical, whereas we develop a theoretical framework to study the interaction between labor and housing markets.[10]

The mechanism in our model also echoes the literature treating housing as a commitment good (e.g., Chetty and Szeidl (2007, 2016), Kaplan and Violante (2014)). In that framework, transaction costs make housing adjustment costly and thus less responsive to shocks. Consequently, wealth or productivity shocks are absorbed mainly through changes in the consumption of other goods—this is the key amplification channel. In our setting, although transaction costs are absent, housing as a necessity good produces similar dynamics: with income elasticity below one, shocks disproportionately affect non-housing consumption. In addition, housing prices influence wages directly—a key feature of our model. Finally, while the commitment-good literature emphasizes aggregate fiscal effects, our focus is on regional inequality.

[9] Apart from spending a greater share of their incomes on housing, poorer individuals are also more likely to pay rents, the relative cost of which has increased over time. Shrinking household sizes (especially at the bottom of the income distribution) and the consequent loss in economies of scale are the other important factors.

[10] Handbury (2021) emphasizes that local income levels shape product variety and prices across U.S. cities, underscoring the broader relevance of non-homothetic preferences. Stores in wealthier cities tend to favor the consumption patterns of high-income household.

To our knowledge, this is the first paper that nests non-homothetic preferences into a standard Rosen (1979) – Roback (1988) spatial equilibrium setting with search and matching frictions (Pissarides, 2000). Other papers combine the Rosen–Roback framework with search and matching models (under homothetic preferences) to study spatial differences in job finding, separation, and vacancy rates (Lkhagvasuren, 2012; Kline and Moretti, 2013; Bilal, 2023; Jung et al., 2023; Kuhn et al., 2021).[11] Our focus is not on these labor market variables. We employ the search and matching framework because it presents in a simple and consistent way the factors affecting the surplus of job–worker match, the bargaining process that divides it, and the resultant equilibrium wage. These are crucial ingredients for our nominal wage to housing price positive relationship. Cardullo (2017) also develops a two-region, two-good general equilibrium framework with labor market frictions; however, the aim of that work is to study the welfare and employment effects of centralized public sector bargaining.

Finally, our paper connects with the extensive literature that analyzes the effects of different wage bargaining procedures on inequality and economic performance. While this literature has not reached a consensus on which system delivers superior results,[12] the present work is the first that connects the impact of wage (de)centralization process with the housing market.

3. Model setup

We briefly outline the model and point to the sections that detail its main components.

We develop a spatial equilibrium model in the spirit of Rosen (1979), Roback (1988), and Moretti (2011). Time is continuous, and the model is studied in steady state. The country consists of two regions, a and b, which differ only in labor productivity; all other product and labor market parameters are identical. This simplification isolates the effects of local demand and supply changes in economies with persistent productivity disparities.

Each region produces two goods: a tradable consumption good and non-tradable housing services. The economy hosts a normalized mass L of infinitely lived, risk-neutral workers with non-homothetic preferences. Specifically, the share of expenditure on housing declines as income rises (Section 3.2). It is also assumed that individuals cannot save.

Firms hire local workers to produce the tradable good. The measure of firms is determined by a free-entry, zero-profit condition (Section 3.3). The labor market is subject to matching frictions (Section 3.1). Individuals can be either employed in tradable production or unemployed (Section 3.2). While unemployed, they choose their region of residence (see Section 3.5) by comparing labor market conditions — namely, wages and job-finding rates — and according to idiosyncratic preferences for specific locations. In the baseline framework, wages are bargained individually between each firm and worker (Section 3.4).

In the housing market, supply is provided by absentee landlords (Section 3.6).

In equilibrium, the following conditions hold in each region: (i) the demand functions of employed and unemployed workers; (ii) the free-entry zero-profit condition for firms producing the tradable good; (iii) the wage outcome from the firm–worker bargaining process; and (iv) the equality of demand and supply in the housing sector. In addition, the measure of workers residing in each region must satisfy the migration decision rule (Section 4.2).

To ease the readability of the section, all the derivations and proofs are presented in Appendix A.

[11] Lindley and Machin (2014) also document spatial variation in some relevant labor market outcomes (such as the education employment shares and the college wage premium) for the US.

[12] Garnero (2021) provides a recent survey on the issue.

3.1. The matching technology

Following Pissarides (2000), hires in the tradable sector of region $i \in \{a, b\}$ are governed by a matching function $M_i = m(U_i, V_i)$, where U_i and V_i denote unemployed workers and vacancies, respectively.[13] There is no on-the-job search. Consistent with Petrongolo and Pissarides (2001), $m(\cdot)$ is homogeneous of degree one, increasing, and concave.

Labor market tightness in region i is defined as $\theta_i = V_i/U_i$. The job-filling rate is $q(\theta_i) = M_i/V_i$, with $q'(\theta_i) < 0$, and the job-finding rate is $f(\theta_i) = M_i/U_i = \theta_i q(\theta_i)$, with $f'(\theta_i) > 0$.[14] Define $\eta \equiv -q'(\theta_i)\left(\theta_i/q(\theta_i)\right)$ as the elasticity of the job-filling rate, assumed constant. Jobs are destroyed at an exogenous rate δ.

Let L_i denote the labor force in region i, with $L_a + L_b = L$ and $E_i + U_i = L_i$. Steady-state flow balance implies the standard Beveridge curve:

$$u_i = \frac{\delta}{\delta + f(\theta_i)}, \quad i \in \{a, b\}, \tag{1}$$

where $u_i \equiv U_i/L_i$ is the unemployment rate in region i.

3.2. Workers' preferences

The core assumption of the model is that all individuals have non-homothetic preferences. Under homothetic utility, the share of income spent on any good is constant with respect to income under fixed prices.[15] This is inconsistent with evidence on housing, where poorer households devote a higher income share to housing (see references in footnote 7 and data in Section 5.1).

To model non-homotheticity, we follow the approach of Comin et al. (2021). Their specific formulation (a non-homothetic constant elasticity of substitution, NHCES, function) is analytically tractable. The instantaneous utility function v_i for all workers living in region $i \in \{a, b\}$ is implicitly defined by the following equation:

$$1 = \left(Q_{nt,i} \cdot v_i^{-(1+\epsilon)}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{t,i} \cdot v_i^{-1}\right)^{\frac{\sigma-1}{\sigma}} \quad \text{with } i \in \{a, b\} \tag{2}$$

in which $Q_{t,i}$ and $Q_{nt,i}$ respectively denote the tradable good and the not tradable housing services consumed in region $i \in \{a, b\}$. As we will see precisely in this section, parameter $\epsilon \geq -1$ measures non-homotheticity; with $\epsilon = 0$, v_i reduces to the standard CES case. The elasticity of substitution between goods is $\sigma > 0$.[16]

Let r denote the discount factor. We take $Q_{t,i}$ as the numeraire. So its price is normalized to 1 and it is equal across the regions. Let also $p_{nt,i}$ denote the housing price in region i.

3.2.1. Unemployed workers

The expected discounted utility of an unemployed worker j searching in region $i \in \{a, b\}$ verifies the following problem:

$$\begin{aligned} rW^U_{j,i} &= \max_{Q^U_{t,i}, Q^U_{nt,i}} \psi_{j,i} + v_{U,i} + f(\theta_i)\left[W^E_{j,i} - W^U_{j,i}\right] \\ \text{s.t.} \quad & p_{nt,i} \cdot Q^U_{nt,i} + Q^U_{t,i} = z. \end{aligned} \tag{3}$$

The instantaneous utility function $v_{U,i}$ is implicitly defined in Eq. (2). Here we simply add the subscript U to recall we are considering the case of unemployed workers. The term $\psi_{j,i}$ stands for the idiosyncratic preference for region i and it is the only difference in preferences across workers. A higher $\psi_{j,i}$ means a stronger attachment to region

[13] For simplicity, unemployed workers in region i cannot search for jobs in region $j \neq i$.

[14] We assume standard Inada conditions: $\lim_{\theta_i \to 0} q(\theta_i) = +\infty$, $\lim_{\theta_i \to \infty} q(\theta_i) = 0$, $\lim_{\theta_i \to 0} f(\theta_i) = 0$, and $\lim_{\theta_i \to \infty} f(\theta_i) = +\infty$.

[15] Engel curves are straight lines from the origin; see Deaton and Muellbauer (1980, ch. 5).

[16] With $\sigma > 1$ (resp. $0 < \sigma < 1$), goods are substitutes (resp. complements). At $\sigma = 1$, preferences are Cobb–Douglas.

i for worker j. The term z stands for the exogenous home production of tradables, identical across regions. Being unemployed yields utility $\psi_{j,i} + v_{U,i}$ and a potential gain from job finding at rate $f(\theta_i)$. The term $W^E_{j,i}$ is the discounted present value of being employed in region i.

First-order conditions yield Hicksian demands:

$$\begin{aligned} Q^U_{nt,i} &= \left(\frac{p_{nt,i}}{z}\right)^{-\sigma} \cdot v_{U,i}^{(1+\epsilon)(1-\sigma)} \\ Q^U_{t,i} &= z^{\sigma} \cdot v_{U,i}^{1-\sigma}, \end{aligned} \tag{4}$$

for $i \in \{a, b\}$. Moreover, the expenditure function is equal to:

$$z = \left[v_{U,i}^{(1+\epsilon)(1-\sigma)} \cdot p_{nt,i}^{1-\sigma} + v_{U,i}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{5}$$

Let $s_{U,i} \equiv p_{nt,i} Q^U_{nt,i}/z$ be the housing expenditure share for the unemployed in region $i \in \{a, b\}$. Using the two equations in (4) to eliminate vU, i gives:

$$s_{U,i} = p_{nt,i}^{1-\sigma} \cdot z^{\epsilon(1-\sigma)} \cdot \left(1 - s_{U,i}\right)^{1+\epsilon} \tag{6}$$

for $i \in \{a, b\}$. Differentiating yields:

$$\frac{d\, s_{U,i}}{d\, z} \cdot \frac{z}{s_{U,i}} = \frac{\epsilon(1-\sigma)(1-s_{U,i})}{1+\epsilon s_{U,i}} \tag{7}$$

for $i \in \{a, b\}$, where the sign depends on $\epsilon(1-\sigma)$. For $-1 \le \epsilon < 0$ and $0 < \sigma < 1$ (goods are complements), $s_{U,i}$ declines with z: housing is a necessity good. If $\sigma > 1$, it becomes a luxury. Under homothetic preferences ($\epsilon = 0$), expenditure shares are income-invariant.

The elasticity with respect to relative price is:

$$\frac{d\, s_{U,i}}{d\, p_{nt,i}} \cdot \frac{p_{nt,i}}{s_{U,i}} = \frac{(1-\sigma)(1-s_{U,i})}{1+\epsilon s_{U,i}} \tag{8}$$

with $i \in \{a, b\}$. Its sign depends only on σ.

3.2.2. Employed workers

The utility maximization problem for an employed worker in region $i \in \{a, b\}$ is:

$$\begin{aligned} rW^E_{j,i} &= \max_{Q^E_{t,i}, Q^E_{nt,i}} \psi_{j,i} + v_{E,i} + \delta \left[W^U_{j,i} - W^E_{j,i} \right] \\ \text{s.t.} \quad & p_{nt,i} \cdot Q^E_{nt,i} + Q^E_{t,i} = w_i. \end{aligned} \tag{9}$$

With w_i we denote the amount of the tradable good that workers in region i receive as wage compensation. Solving as before yields:

$$\begin{aligned} Q^E_{nt,i} &= \left(\frac{p_{nt,i}}{w_i}\right)^{-\sigma} \cdot v_{E,i}^{(1+\epsilon)(1-\sigma)} \\ Q^E_{t,i} &= w_i^{\sigma} \cdot v_{E,i}^{1-\sigma}, \\ w_i &= \left[v_{E,i}^{(1+\epsilon)(1-\sigma)} \cdot p_{nt,i}^{1-\sigma} + v_{E,i}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \end{aligned} \tag{10}$$

for $i \in \{a, b\}$.

Defining $s_{E,i} \equiv p_{nt,i} Q^E_{nt,i}/w_i$, we obtain:

$$s_{E,i} = p_{nt,i}^{1-\sigma} \cdot w_i^{\epsilon(1-\sigma)} \cdot \left(1 - s_{E,i}\right)^{1+\epsilon}. \tag{11}$$

When $\epsilon = 0$, housing shares for unemployed and employed workers (Eqs. (6) and (11)) are identical. The elasticities are:

$$\begin{aligned} \frac{d\, s_{E,i}}{d\, w_i} \cdot \frac{w_i}{s_{E,i}} &= \frac{\epsilon(1-\sigma)(1-s_{E,i})}{1+\epsilon s_{E,i}} \\ \frac{d\, s_{E,i}}{d\, p_{nt,i}} \cdot \frac{p_{nt,i}}{s_{E,i}} &= \frac{(1-\sigma)(1-s_{E,i})}{1+\epsilon s_{E,i}} \end{aligned} \tag{12}$$

for $i \in \{a, b\}$, again depending on ϵ and σ as in the unemployed case.

3.2.3. Price indexes and utility elasticity

Using the Hicksian demands for the tradable good (second equations in (4) and (10)) and the definition of housing expenditure shares, we obtain:

$$\begin{aligned} v_{U,i} &= z\,(1-s_{U,i})^{\frac{1}{1-\sigma}}, \\ v_{E,i} &= w_i\,(1-s_{E,i})^{\frac{1}{1-\sigma}}, \end{aligned} \tag{13}$$

for $i \in \{a, b\}$. Following Deaton and Muellbauer (1980, ch. 7.1) and Comin et al. (2021), we define the average price indexes for unemployed and employed workers as $P_{U,i} \equiv (1 - s_{U,i})^{-\frac{1}{1-\sigma}}$ and $P_{E,i} \equiv (1 - s_{E,i})^{-\frac{1}{1-\sigma}}$. $P_{U,i}$ (resp. $P_{E,i}$) represents the income required by an unemployed (resp. employed) worker to achieve utility $v_{U,i} = 1$ (resp. $v_{E,i} = 1$). Hence, $\frac{z}{P_{U,i}}$ and $\frac{w_i}{P_{E,i}}$ denote real incomes for the unemployed and employed, respectively.[17]

For $0 < \sigma < 1$ and $-1 < \epsilon < 0$, Eqs. (6) and (11) imply $s_{U,i} > s_{E,i}$ whenever $w_i > z$, which holds in equilibrium. Thus, $P_{U,i} > P_{E,i}$: with housing as a necessity, a higher expenditure share raises the cost of living.

From (8) and (12), it follows that $\frac{dP_{U,i}}{dp_{nt,i}} > \frac{dP_{E,i}}{dp_{nt,i}}$,[18] so increases in housing prices disproportionately raise the living costs of unemployed workers.

Next, consider the elasticity of employed workers' instantaneous utility with respect to wages. Using (11) and the second equation in (13):

$$\begin{aligned} \mu_i &\equiv \frac{d\, v_{E,i}}{d\, w_i} \cdot \frac{w_i}{v_{E,i}} = \frac{1}{1+\epsilon \cdot s_{E,i}}, \\ \mu'(p_{nt,i}) &> 0 \quad \text{if and only if} \quad -1 < \epsilon < 0 \ \text{ and } \ 0 < \sigma < 1 \end{aligned} \tag{14}$$

for $i \in \{a, b\}$. When $\epsilon = 0$, $\mu_i = 1$, so utility is linear in income. If $-1 < \epsilon < 0$, $\mu_i > 1$ because $v_{E,i}$ rises not only through w_i but also via the reduction in $s_{E,i}$. With housing as a necessity, higher wages lower the housing share and price index $P_{E,i}$, boosting utility.

Moreover, the fact that $\mu'(p_{nt,i}) > 0$ under $-1 < \epsilon < 0$ and $0 < \sigma < 1$ means that the utility gains from a wage increase are stronger when housing is expensive. The rationale goes as follows: when housing is a necessity, a wage increase reduces the share of income devoted to it. If the relative price of housing services, $p_{nt,i}$, is high, workers shift their consumption toward the relatively cheaper tradable goods. As a result, the utility gains from the wage increase are amplified.

3.3. Firms in the consumption good sector

Following a standard search and matching model (Pissarides, 2000, chapter 1), we impose the one firm — one job assumption. Each firm–worker pair in region i produces an amount y_i of the tradable consumption good, with $i \in \{a, b\}$. We also assume that $y_a > y_b$. This is the only exogenous difference between the two regions.

The expected discounted returns for a firm operating in region $i \in \{a, b\}$, J^E_i verifies the following Bellman equation:

$$rJ^E_i = y_i - w_i + \delta\left(J^V_i - J^E_i\right), \quad \text{with } i \in \{a, b\}. \tag{15}$$

At the RHS of (15) we have the firm's revenues, namely the amount of the units of the consumption good produced y_i net of the wage bill, and the capital loss occurring at rate δ, with J^V_i being the expected value of a vacancy. It is determined as follows:

$$rJ^V_i = -k + q(\theta_i)\left(J^E_i - J^V_i\right), \quad \text{with } i \in \{a, b\}. \tag{16}$$

The expected value of vacancy is given by the vacancy costs k, expressed in terms of the tradable consumption good, and the capital gain that accrues from the match, multiplied by the job filling rate.

As common in search and matching models, a free-entry zero profit condition determines the equilibrium values of tightness θ_i. Free-entry

[17] Although w_i and z are quantities of tradable goods, they only acquire welfare meaning when deflated by the relevant price indexes.

[18] Formally,

$$\frac{dP_{U,i}}{dp_{nt,i}} = \frac{P_{U,i}}{p_{nt,i}} \cdot \frac{s_{U,i}}{1+\epsilon s_{U,i}} > \frac{P_{E,i}}{p_{nt,i}} \cdot \frac{s_{E,i}}{1+\epsilon s_{E,i}},$$

since $P_{U,i} > P_{E,i}$ and $s_{U,i} > s_{E,i}$.

of vacancies and zero profits imply that $J_i^V = 0$. Substituting this into (15) and (16), one gets:

$$\frac{y_i - w_i}{r + \delta} = \frac{k}{q(\theta_i)} \quad \text{with } i \in \{a, b\}. \tag{17}$$

Firms' expected discounted revenues (the LHS of (17)) are equal to the expected cost of posting a vacancy (the RHS of (17)). All firms are domestic and, as in standard in search and matching models (see Pissarides (2000), Chapter 1), are owned by workers. But there are no excess profits in expectation at the aggregate level. All tradable goods produced are either paid to workers as wages w_i or used to recover the vacancy posting costs, k.[19]

3.4. Wage bargaining

In the baseline model, wages are negotiated individually between each firm and worker. In our quantitative exercises, we compare these results with an alternative setting where unions and employers' confederations bargain at the national level.

We assume a Nash bargaining solution that splits the total surplus $W^E j, i - W^U j, i + J^E i - J^V i$ generated by the match. The equilibrium wage w_i solves:

$$w_i = \operatorname{argmax} \left[W_{j,i}^E - W_{j,i}^U \right]^\beta \left[J_i^E - J_i^V \right]^{1-\beta}, \tag{18}$$

with $i \in \{a, b\}$. Parameter β denotes the exogenous bargaining power of a worker ($0 < \beta < 1$). By ordinality, we can consider the $log(.)$ of the function in (18) to find w_i. The F.O.C is equal to:

$$\frac{1-\beta}{J_i^E} = \beta \frac{\frac{d\, v_{E,i}}{d\, w_i}}{W_{j,i}^E - W_{j,i}^U} \quad \text{with } i \in \{a, b\}. \tag{19}$$

The left-hand side represents the marginal cost of higher wages (lower expected firm profits), while the right-hand side gives the marginal gain, i.e. the change in workers' utility relative to their quasi-rents from the match. After some algebra, we obtain:

$$\begin{aligned} \frac{w_i}{P_{E,i}} &= \left[\beta\, \mu_i \frac{y_i + k \cdot \theta_i}{P_{E,i}} + (1-\beta) \frac{z}{P_{U,i}} \right] \cdot \left[1 - \beta \cdot \left(1 - \mu_i \right) \right]^{-1} \\ w_i' &\equiv \left. \frac{d\, w_i}{d\, p_{nt,i}} \right|_{\bar{\theta}_i} > 0 \quad \text{if} \quad -1 < \epsilon < 0 \ \text{ and } \ 0 < \sigma < 1 \end{aligned} \tag{20}$$

with $i \in \{a, b\}$. Under homothetic preferences ($\mu_i = 1$, $P_{U,i} = P_{E,i}$), expression (20) collapses to the standard search-and-matching wage schedule (see Pissarides (2000), ch. 1): w_i is a weighted average of productivity y_i and home production z, with weights determined by β, plus a share of vacancy costs.[20]

With non-homothetic preferences, however, wages also depend on the price of housing services, $p_{nt,i}$, since $P_{U,i}$, $P_{E,i}$, and μ_i vary with it (see Section 3.2.3). The derivative in (20) shows that, for fixed θ_i, wages rise with $p_{nt,i}$ when $-1 < \epsilon < 0$ and $0 < \sigma < 1$. Hence, while higher housing prices only reduce purchasing power under homothetic preferences, they partly translate into higher w_i in the non-homothetic case.

Two competing effects are at work. A higher $p_{nt,i}$ increases $P_{U,i}$ more than $P_{E,i}$, since unemployed workers spend a larger share on housing. This weakens their bargaining position by reducing the real value of $z/P_{U,i}$ relative to $\beta(y_i + k\theta_i)$, lowering w_i.[21] Yet, this effect is dominated by a second, positive one: a higher $p_{nt,i}$ raises μ_i, which in turn increases w_i.[22] As shown in Section 3.2.3, when $-1 < \epsilon < 0$ and $0 < \sigma < 1$, μ_i rises with $p_{nt,i}$. A more expensive housing market raises the marginal value of income for employed workers, strengthening their bargaining position and resulting in higher equilibrium wages.

[19] We are implicitly assuming the existence of functioning capital markets that finance the upfront cost of opening a vacancy.

[20] At $\epsilon = 0$, workers' utility is linear in wages—$u(w) = w$ — as in the canonical matching model.

[21] Formally, $\partial w_i / \partial p_{nt,i} < 0$ for constant μ_i.

[22] The right-hand side of (20) is increasing in μ_i.

3.5. Workers' location decision

To determine the distribution of workers across regions, we adopt the migration condition from Moretti (2011), which assumes that worker j's relative preference for region a over b is given by

$$\psi_{j,a} - \psi_{j,b} \sim g[-\lambda, \lambda], \tag{21}$$

with $g(.)$ being a probability density function. The parameter λ captures the strength of location-specific preferences and thus the degree of labor mobility. A large λ implies strong attachment to one's region and limited mobility, while a small λ indicates high mobility. When $\lambda = 0$, workers are perfectly mobile and indifferent across locations.

Workers choose their region of residence only while unemployed. Hence, each job seeker decides whether to remain unemployed in region a or in b. We assume job search is local — workers can apply only for vacancies within their current region — and exclude job-to-job transitions and voluntary quits. Employed workers, therefore, do not relocate.

Let λ^* denote the preference realization of the marginal worker j^*, who is indifferent between searching in a or b:

$$rW_{j^*,b}^U - rW_{j^*,a}^U = 0.$$

Defining $\lambda^* \equiv \psi_{j \cdot a} - \psi_{j \cdot b}$ and using Eq. (3), we obtain:

$$\lambda^* = v_{U,b} + f(\theta_b) \left[W_{j^*,b}^E - W_{j^*,b}^U \right] - v_{U,a} - f(\theta_a) \left[W_{j^*,a}^E - W_{j^*,a}^U \right]. \tag{22}$$

After some algebra, this simplifies to

$$\lambda^* = \frac{z}{P_{U,b}} - \frac{z}{P_{U,a}} + \frac{\beta \cdot k}{1-\beta} \left[\theta_b \cdot \frac{\mu_b}{P_{E,b}} - \theta_a \cdot \frac{\mu_a}{P_{E,a}} \right]. \tag{23}$$

The labor forces in both regions can be written as:

$$\begin{aligned} L_b &= H(\lambda^*)\, L \\ L_a &= \left(1 - H(\lambda^*)\right) L, \end{aligned} \tag{24}$$

where $H(\cdot)$ is the cumulative distribution function of location preferences. The right-hand side of (23) is increasing (decreasing) in θ_b (θ_a) and decreasing (increasing) in $P_{U,b}$ and $P_{E,b}$ ($P_{U,a}$ and $P_{E,a}$). Thus, a tighter labor market in region b (higher θ_b) raises its job-finding rate, increasing λ^* and the share of workers residing in b. Similarly, a lower cost of living in region b — that is, lower $P_{U,b}$ or $P_{E,b}$— also raises L_b. The opposite holds for region a.

In equilibrium, we will show that a higher $p_{nt,a}$ (or lower $p_{nt,b}$) raises λ^*.[23] When housing becomes less affordable in the more productive region a, unemployed workers find it optimal to relocate to the cheaper region b.[24]

3.6. The housing market

To gain in simplicity, we consider a parsimonious version of the housing market, similar to that adopted in standard spatial equilibrium settings (see for instance Moretti (2011), Beaudry et al. (2014) and Hsieh and Moretti (2019)[25]). As common in models studying local labor markets (see Moretti (2011)), we assume that the housing supply is in

[23] The positive effects of $p_{nt,i}$ on $P_{E,i}$, $P_{U,i}$, and μ_i described in Section 3.2.3 hold for any given w_i. The same signs remain once the wage schedule (20) is incorporated.

[24] In steady state, the measure δE_i of employed workers who become unemployed remains in region i since all variables influencing the location decision — w_i, $f(\theta_i)$, $p_{nt,i}$, and $\psi_{j,a} - \psi_{j,b}$ — are constant for each j and $i \in a, b$.

[25] More complex models feature a construction sector that employs local labor, equilibrium random matching (Genesove and Han, 2012), and endogenous search intensity (Williams, 2018).

the hands of landowners that live abroad.[26] While this assumption is clearly not realistic, separating workers from landowners in the model allows to distinguish the welfare effects of different changes across different types of agents. We also follow Hsieh and Moretti (2019) and consider the following housing supply schedule:

$$Q_{nt,i} = \alpha_i \, p_{nt,i}^{\frac{1}{\gamma}} \tag{25}$$

with $\alpha_i > 0$ a region-specific parameter and $i \in \{a, b\}$. Parameter γ stands for the inverse elasticity of the housing supply. At the equilibrium, the demand must be equal to the supply. This implies:

$$\begin{aligned} \alpha_i \, p_{nt,i}^{\frac{1}{\gamma}} &= U_i \cdot Q_{nt,i}^{U} + E_i \cdot Q_{nt,i}^{E} \qquad \Longleftrightarrow \\ \alpha_i \, p_{nt,i}^{\frac{1}{\gamma}} &= L_i \, \frac{u_i \cdot z \cdot s_{U,i} + (1-u_i)\cdot w_i \cdot s_{E,i}}{p_{nt,i}} \end{aligned} \tag{26}$$

with $i \in \{a, b\}$. The second equation is obtained using the definitions for the housing shares out of total expenditures.

4. Equilibrium

We divide our equilibrium analysis into two steps: a partial equilibrium setting and then a general equilibrium.

4.1. Equilibrium with constant prices and no migration

We begin by examining the variation in labor market variables and workers' expenditure patterns under a partial equilibrium setting, where housing prices $p_{nt,i}$ and the migration parameter λ^* are taken as given. This corresponds to a scenario in which the supply of housing labor is perfectly elastic ($\gamma = 0$) and no migration occurs across regions ($\lambda \to +\infty$ for any worker j). These are, of course, extreme assumptions. We adopt them here to better isolate and highlight certain features of the model that continue to hold under general equilibrium.

Once $p_{nt,i}$ and λ^* are fixed, the only endogenous variables are the housing expenditure share for employed workers $s_{E,i}$, the wage w_i, and labor market tightness θ_i, in each region $i \in \{a, b\}$. The housing share for unemployed workers $s_{U,i}$ is uniquely determined for any given $p_{nt,i}$ and z via Eq. (6).

To determine these three unknowns, we consider the system comprising the demand equation (11), the zero-profit free-entry condition (17), and the wage-setting equation (20).

Lemma 1. *A steady-state partial (i.e., for any given $p_{nt,i}$ and λ^*) equilibrium of the model exists and is unique. Moreover, if $0 < \sigma < 1$ and $-1 < \epsilon < 0$, we have:*

$$\frac{dw_i}{dp_{nt,i}} > 0, \quad \frac{d\theta_i}{dp_{nt,i}} < 0, \quad \frac{ds_{U,i}}{dp_{nt,i}} > 0, \quad \text{and} \quad \frac{ds_{E,i}}{dp_{nt,i}} > 0.$$

The proof follows directly from an application of the implicit function theorem.

Interpretation of the derivatives.

We have already discussed the positive sign of $\frac{dw_i}{dp_{nt,i}}$ in Section 3.4 (albeit under constant θ_i). From the second inequality in Lemma 1, higher housing prices also reduce labor market tightness, via its effect on wages. According to the free-entry zero-profit condition, an increase in labor costs discourages firms from posting vacancies, thereby lowering θ_i—explaining why $\frac{d\theta_i}{dp_{nt,i}} < 0$.

Finally, consider Eqs. (6) and (11). An increase in housing prices raises the share of total expenditures allocated to housing for both unemployed and employed workers. As discussed via elasticities (8) and (12), when housing and the consumption good are gross complements ($0 < \sigma < 1$), both $s_{U,i}$ and $s_{E,i}$ increase with $p_{nt,i}$.

There is, however, an indirect effect on $s_{E,i}$ working in the opposite direction: since $p_{nt,i}$ raises wages, and under non-homothetic preferences higher income reduces the housing share, this would tend to lower $s_{E,i}$. At equilibrium, however, this income effect is dominated by the direct price effect, so that $s_{E,i}$ still increases with $p_{nt,i}$ when $0 < \sigma < 1$.

4.2. General equilibrium

Definition. A steady-state general equilibrium is a vector $[, s_{U,i}, s_{E,i}, w_i, \theta_i, p_{nt,i},]$ for $i \in a, b$, and a value λ^* satisfying: (i) the demand equations for housing shares of unemployed and employed workers, (6) and (11); (ii) the zero-profit conditions for tradable-good firms, (17); (iii) the wage equation, (20); (iv) housing-market clearing, (26); and (v) the migration condition, (23).

Relative to the partial equilibrium in Lemma 1, the general equilibrium introduces two additional endogenous variables — the housing price $p_{nt,i}$ and the migration threshold λ^* — and two corresponding equilibrium conditions, (26) and (23). Solving this extended system determines all other endogenous variables, such as utilities, unemployment rates, and regional labor forces, through their defining relations.

Proposition 1. *In the case $0 < \sigma < 1$ and $-1 < \epsilon \le 0$, a steady-state general equilibrium exists and it is unique if $\frac{1}{\gamma} > (1+\epsilon)(1-\sigma)$. At the equilibrium, we have the following properties: $w_a > w_b$, $p_{nt,a} > p_{nt,b}$, and $P_{U,a} > P_{U,b}$.*

The existence condition is mild: equilibrium requires only that the housing-supply elasticity $1/\gamma$ be sufficiently large.[27]

All inequalities in Proposition 1 hold whether preferences are homothetic ($\epsilon = 0$) or non-homothetic ($-1 < \epsilon < 0$).

Wages are higher in region a, as higher productivity ($y_a > y_b$) increases the surplus captured by workers — consistent with the positive link between wages and productivity in Eq. (20).

The second inequality, $p_{nt,a} > p_{nt,b}$, indicates that the more productive region has higher non-tradable prices. This reflects a Harrod–Balassa–Samuelson effect[28]: while in standard models it arises from intersectoral labor mobility, here it results from higher productivity raising wages and, consequently, housing demand.

This mechanism reinforces the first inequality: wages in region a are higher not only because productivity is greater but also because higher housing prices push wages up through the bargaining channel (Section 3.4).

Given $p_{nt,a} > p_{nt,b}$, unemployed workers in region a spend a larger share of income on housing ($s_{U,a} > s_{U,b}$). By the elasticity in (8) and the definition of $P_{U,i}$ (Section 3.2.3), this implies $P_{U,a} > P_{U,b}$ — a higher cost of living for the unemployed in the more productive region.

The effects on $s_{E,i}$ and $P_{E,i}$ are ambiguous. Higher housing prices tend to increase both, while higher wages — especially under non-homothetic preferences — work in the opposite direction.

Finally, we cannot analytically rank unemployment rates across regions because θ_a and θ_b are affected by offsetting forces. Higher productivity in region a stimulates vacancy creation via the zero-profit condition (17), raising θ_a. Yet it also increases wages (through both y_a

[26] In Appendix B.5, we perform a numerical analysis in which we drop this assumption. The main amplification results of the model hold even in this scenario.

[27] In our framework, $(1+\epsilon)(1-\sigma)$ is the product of two positive terms below one. Thus, equilibrium in the housing market exists even when demand is upward sloping—that is, when the right-hand side of (26) increases in $p_{nt,i}$. This occurs more easily when housing supply is elastic, i.e., when the supply curve is flatter in $(Q_{nt,i}, p_{nt,i})$ space.

[28] See Obstfeld and Rogoff (1996), Ch. 4.

and $p_{nt,a}$), raising labor costs and dampening vacancy posting. Under homothetic preferences ($\epsilon = 0$), the second effect disappears, implying tighter labor markets and lower unemployment in region a. With $\epsilon < 0$, however, the net effect is indeterminate.

In the next section, we study the aggregate implications of two region-specific changes: one to housing supply and another to tradable-sector productivity.

4.3. A housing supply change in one region

The following Proposition summarizes the result:

Proposition 2. *Consider the model with $0 < \sigma < 1$ and $-1 < \epsilon \leq 0$, and a negative housing-supply change in region $i \in a, b$, represented by a decline in α_i. At the new steady state:*

1. *In region i, the housing price rises and the labor force declines. If $\epsilon = 0$, wages and the unemployment rate are unaffected. If $\epsilon < 0$, both wages and unemployment increase.*
2. *In the other region $j \neq i$, both the housing price and labor force increase. Again, if $\epsilon = 0$, wages and unemployment remain unchanged, while for $\epsilon < 0$, both are higher.*

A negative housing-supply change — modeled as a reduction in α_i — can be interpreted as an increase in construction costs or a tightening of regulatory constraints. At the new steady state, the higher local housing price $p_{nt,i}$ induces some migration toward region j, where the inflow of workers raises housing demand and, consequently, $p_{nt,j}$.

When $\epsilon = 0$, wages do not respond to housing prices (Section 3.4). In this case, the zero-profit condition (17) implies that labor-market tightness and unemployment remain unchanged in both regions. When $\epsilon < 0$, however, non-homothetic preferences allow workers to bargain for higher wages as housing costs rise. This, in turn, discourages vacancy creation, leading to an increase in unemployment across regions. Nevertheless, as will be shown in the quantitative analysis, the magnitude of this increase is relatively modest.

The negative housing-supply change also affects welfare. Unemployed workers are worse off nationwide: the cost of living rises as both $p_{nt,i}$ and $p_{nt,j}$ increase, and labor-market conditions deteriorate. For employed workers, the welfare effect is ambiguous—higher wages (w_a, w_b) partly offset the increase in housing costs, leaving the impact on their real income indexes, $P_{E,i}$ and $P_{E,j}$, theoretically indeterminate.

4.4. A productivity change in one region

In contrast to a housing-supply change, a change in tradable-sector productivity in a single region does not yield clear comparative statics for most endogenous variables. We therefore defer a detailed quantitative assessment to Section 5.2.

An increase in productivity y_i raises the local wage w_i, reflecting larger match surpluses. The higher wage boosts housing demand, increasing the equilibrium housing price $p_{nt,i}$ and the cost-of-living index for the unemployed, $P_{U,i}$. The effects on employed workers' housing expenditure and price indexes ($s_{E,i}$, $P_{E,i}$) are ambiguous. On one hand, employees face higher housing costs; on the other, their increased income changes their consumption preferences, potentially reducing their demand for the non-tradable good.

Labor-market tightness and unemployment also respond ambiguously. Greater productivity promotes vacancy creation through the zero-profit condition (17), yet rising wages raise labor costs and curb job posting. As discussed in Proposition 1, under homothetic preferences ($\epsilon = 0$), the second effect vanishes, but with $\epsilon < 0$ the net result cannot be signed analytically.

Migration responses are similarly indeterminate. According to Eq. (23), migration decisions depend on both labor-market conditions and living costs. After a productivity increase in region i, the unemployed face a higher cost of living ($P_{U,i}$), while the directions of change in $P_{E,i}$ and θ_i remain uncertain. Consequently, the responses of the local labor force L_i—and thus of L_j—cannot be determined analytically.

Because this model transmits local changes through adjustments in the labor force, this ambiguity in L_j implies that we cannot analytically determine the resulting impacts on housing and labor market variables in region j.

5. Quantitative results

5.1. Calibration

The model is calibrated using German data for the period 2013–2018. The source or the procedure to derive the numerical values for the exogenous parameters of the theoretical model are shown in Table 1, while the procedure for the endogenous variables is explained in the text.

We define region b as the six re-established states of the former German Democratic Republic (GDR), and region a as the ten "old" states of the Federal Republic of Germany. Time is measured in months. The discount rate r is fixed at 1% on an annual basis. The elasticity of housing supply in Germany during the period under consideration is approximately 0.5, according to Cavalleri et al. (2019).[29] This implies a value of $\gamma = 2$. As for the elasticity of substitution parameter σ, we follow Finlay and Williams (2022) and set it equal to 0.56. A sensitivity analysis on this parameter is provided in Section 5.3. According to Hartung et al. (2018), in the years immediately preceding our calibration window, approximately 0.5% of workers in Germany transitioned each month from employment to unemployment.[30] We thus set the separation rate $\delta = 0.005$. We assume a standard value for workers' bargaining power: $\beta = 0.5$. Regarding the matching function, we adopt the standard Cobb–Douglas form: $M_i = m_i V_i^{1-\eta} U_i^{\eta}$ for $i \in \{a, b\}$. We normalize m_a to 1 and, for simplicity, impose the efficiency condition from Hosios (1990), $\beta = \eta$.[31]

Data on the labor force (L_a, L_b) and unemployment rates (u_a, u_b) are sourced from the German Federal Statistical Office, which also provided figures on housing expenditure shares by income group and by region.[32] More specifically, the Central Information Service of the Federal Statistical Office of Germany (Destatis) provided us with data from the 2013 and 2018 surveys on the consumption expenditure of private households combined by territorial status and household monthly net income.[33]

We must then make assumptions to map these data onto the employed and unemployed workers in our model. For employed workers, we assign the income class corresponding to the median net wage in Germany during those years (about €2500 per month), which falls into the €[2000, 2600] bracket. This yields average housing expenditure shares of 30% in region a and 28% in region b for the years 2013–2018. For unemployed workers, lacking direct data on their income, we assign

[29] Beze (2023) estimates a lower elasticity of around 0.25, while Lerbs (2012) reports a value of 0.4 for the period 2004–2010. We conduct a sensitivity analysis on the parameter γ in Section 5.3.

[30] Similar results are found in Carrillo-Tudela et al. (2021).

[31] A sensitivity analysis on β is also presented in Section 5.3.

[32] In measuring housing expenditure, we refer to the item "Wohnungsmieten", which includes both effective and figurative rents. For a discussion on the pros and cons of including figurative rents, see Dustmann et al. (2021).

[33] We have expenditure share data for both East and West Germany for the years 2013 and 2018, for households with monthly net incomes in the following ranges: under €900, €[900, 1300], €[1300, 1500], €[1500, 2000], €[2000, 2600], €[2600, 3600], and €[3600, 5000].

Table 1
Calibration: Exogenous parameters.

Parameters			
Variables	Values	Interpretation	Source
r	0.00083	Discount rate	1% on annual basis
β	0.5	Workers' bargaining power	Arbitrary choice
δ	0.005	Separation rate	Hartung et al. (2018)
γ	2	Inverse elasticity of the housing supply	Cavalleri et al. (2019)
σ	0.56	Elasticity of substitution between the two goods	Finlay and Williams (2022)
η	0.5	The opposite of the elasticity of the job-filling rate	Hosios condition (Hosios, 1990)
α_b	1	Housing supply parameter in region b	Normalization
m_a	1	Matching function parameter in region a	Normalization
ϵ	−0.8	Non-homotheticity parameter	Explained in the text
α_a	3.6	Housing supply parameter in region a	Housing market equilibrium equation
m_b	0.9	Matching function parameter in region b	Zero profit condition

them to the second-lowest income group, €[900, 1300].[34] We thus obtain housing expenditure shares of 40% in region a and 35% in region b for unemployed workers.

We now turn to the calibration of the variables determined via equilibrium conditions. Starting with the four housing expenditure shares equations, it is straightforward to use Eqs. (6) and (11) (for $i \in \{a, b\}$) to eliminate both $p_{nt,i}$ and z, and to express the real wage gap $\left[w_a\left(1-s_{E,a}\right)^{\frac{1}{1-\sigma}}\right] / \left[w_b\left(1-s_{E,b}\right)^{\frac{1}{1-\sigma}}\right]$ as a function of the housing expenditure shares and the utility parameters ϵ and σ. Since the expenditure shares $s_{E,i}$ and $s_{U,i}$ are directly taken from the data and we have already fixed the elasticity of substitution σ, we exploit empirical estimates of the real wage gap across regions to pin down ϵ. Reported values range from 1.17 (Boeri et al., 2021) to 1.35 (Heise and Porzio, 2019), with Dickey and Widmaier (2021) estimating a gap of around 1.2 even after having accounted for different human capital endowments, location effects, and human capital depreciation. We select a value of 1.22, which implies $\epsilon = -0.8$.

Once ϵ is known, we can use Eqs. (6) to express $p_{nt,i}$ as a function of z for both $i = a$ and $i = b$. We substitute the expression for $p_{nt,b}$ into the housing market equilibrium condition (26) in region b. Assuming the housing supply parameter $\alpha_b = 1$, this equation allows us to solve for the home production value z. With z in hand, we can recover the equilibrium values of $p_{nt,a}$, $p_{nt,b}$, w_a, and w_b using again the housing expenditure shares equations. We then turn to region a's housing market equilibrium condition to compute α_a. We proceed by using the unemployment rate in region a and Eq. (1) to compute θ_a, given $m_a = 1$. The zero-profit conditions (17) are used to write y_a and y_b as functions of k and m_b, respectively. Substituting y_a into the wage Eq. (20) for region a, we solve for k. We then insert y_b into the wage equation for region b and solve for m_b. At this point, only two parameters remain: λ, which determines mobility in Eq. (21), and the threshold λ^*, which splits the total labor force L between regions a and b. We compute λ^* directly from Eq. (23), since all other right-hand-side variables have been determined.

Finally, we assume that the function $g(\cdot)$ in the migration equation is normally distributed with zero mean and standard deviation λ. Then λ is computed knowing that the integral of the normal distribution in the interval $[\lambda^*, +\infty)$ must be equal to L_a/L.

We check the empirical validity of our calibration using three external data moments not used in the calibration process. First, we consider the wage gap w_a/w_b (not price-adjusted), which is reported to be 1.28 in Boeri et al. (2021). Our model produces a value of 1.286. Since we used the real wage gap in our calibration, this result suggests that our assumptions on regional cost-of-living differences are empirically plausible. Second, we examine the productivity ratio y_a/y_b. According to OECD data (OECD, 2023), the gap in gross real value added per employee between West and East Germany was about 1.25 in 2013, falling to 1.2 in 2018. These values align with the estimates reported in Boeri et al. (2021) and Mertens and Mueller (2022). Our model yields a ratio of 1.28, consistent with these findings. Lastly, we validate the job-finding rates implied by our calibration. In our model, $1/f(\theta_i)$ represents the expected duration of unemployment. According to Hartung et al. (2018), during 2004–2014, less than 6% of unemployed workers found a job each month, implying an average unemployment duration of roughly 16 months. Carrillo-Tudela et al. (2021) report slightly lower transition rates (around 4%), corresponding to about 24 months. Our model, based on a later period, yields expected durations of 11 months in region a and 16 months in region b, which appear consistent with available evidence.

5.2. Simulations

As common in search and marching models, our simulations consist of a comparison between two distinct steady-state equilibria: an "initial" state and the one that occurs if the value of an exogenous variable is different.[35]

In our baseline simulations, we examine two alternative changes expected to reduce regional inequality: (i) an increase in productivity in the tradable sector of the Eastern states, and (ii) an increase in the marginal cost of housing in the West. Specifically, we consider changes in y_b and α_a, respectively, of magnitudes sufficient to achieve the same, plausible, reduction of 3% in the West–East output gap.[36]

For all the scenarios, we contrast the results of our standard model (decentralized wage negotiation with non-homothetic preferences) with those of three alternative counterfactual exercises: (1) centralized wage negotiation with non-homothetic preferences ($\epsilon < 0$); (2) decentralized wage negotiation with homothetic preferences ($\epsilon = 0$); (3) centralized

[34] In Germany, unemployment benefits generally cover 60% of prior earnings (or 67% if the recipient has children). Thus, 60% of the median wage places recipients at the upper end of this income bracket. Naturally, this is an approximation. It ignores additional sources of income captured by the variable z in the model. It is also worth noting that not all unemployed individuals receive benefits, as entitlement duration ranges from 6 to 24 months depending on age and prior employment. Furthermore, as highlighted by Carrillo-Tudela et al. (2021), following the Hartz reforms (2005), many long-term unemployed ceased registering as such, and are officially recorded as non-participants with no insurance coverage.

[35] This means that we ignore transitional dynamics. However, Shimer (2005) shows that the elasticities of model variables, evaluated at the steady state, provide a good approximation of their volatility in a fully dynamic stochastic environment.

[36] With the sole exception of the first outbreak of the COVID pandemic (2020–2021), every year between 2011 and 2023 has seen a narrowing of the output gap between the Western and Eastern German states. This reduction has been particularly pronounced over the past decade. Excluding the temporary distortions associated with the COVID period, the West–East output gap has declined by approximately 3% every two years during this time (OECD data: https://data-explorer.oecd.org/).

Table 2
Productivity change in region b: $\Delta y_b/y_b = 1\%$ (percentage changes; for the unemployment rates variation in percentage points).

Variable	$\epsilon < 0$		$\epsilon = 0$	
	$w_a \neq w_b$	$w_a = w_b$	$w_a \neq w_b$	$w_a = w_b$
L_a	−0.4	−0.0	−0.1	0.0
L_b	1.7	0.0	0.2	−0.0
u_a (pp)	−0.0	0.0	−0.0	0.0
u_b (pp)	−0.1	−0.1	−0.1	−0.1
$p_{nt,a}$	−0.4	0.1	−0.1	0.2
$p_{nt,b}$	2.2	0.2	0.9	0.2
$P_{E,a}$	−0.1	−0.0	−0.1	0.1
$P_{E,b}$	0.5	0.0	0.6	0.1
$P_{U,a}$	−0.2	0.0	−0.1	0.1
$P_{U,b}$	1.1	0.1	0.6	0.1
w_a (real)	−0.0 (0.1)	0.2 (0.2)	−0.0 (0.1)	0.2 (0.1)
w_b (real)	1.0 (0.5)	0.2 (0.2)	1.0 (0.4)	0.2 (0.1)
Aggregate output	2.4	0.3	0.3	0.4
Aggregate employment	0.0	0.0	0.0	0.0
Real wage gap	−0.3	0.0	−0.4	0.0
Employment gap	−2.1	−0.2	−0.4	−0.1
Output gap	−3.1	−1.2	−1.4	−1.1
Average CPI gap	−2.7	−0.0	−0.9	0.0
Variance of log income (real)	−1.1 (0.0)	−2.8 (−1.7)	−1.7 (−0.5)	−2.3 (−0.3)
W_a^E	0.1	0.2	0.1	0.1
W_a^U	0.1	0.2	0.1	0.1
W_b^E	0.5	0.2	0.4	0.2
W_b^U	0.5	0.2	0.4	0.2

wage negotiation with homothetic preferences ($\epsilon = 0$). This comparison allows us to single out more clearly the amplifying effects of non-homothetic preferences when wages are bargained at local level.

As concerns the counterfactual exercises (1) and (3), we assume that under centralized wage bargaining, the wage w is negotiated by unions within the confederation of employers at the national level and is the same in both regions. The details of the maximization problem are in Appendix B.1.

Under national bargaining, the wage equation becomes:

$$w = \frac{\beta \bar{y}\left(\phi \frac{\mu_a}{P_{E,a}} + (1-\phi)\frac{\mu_b}{P_{E,b}}\right) + (1-\beta) b \left(\frac{\phi}{P_{U,a}} + \frac{1-\phi}{P_{U,b}}\right)}{\frac{\phi}{P_{E,a}}\left(1-\beta(1-\mu_a)\right) + \frac{1-\phi}{P_{E,b}}\left(1-\beta(1-\mu_b)\right)} \tag{27}$$

in which $\phi \equiv E_a/(E_a+E_b)$ and $\bar{y} \equiv \phi y_a + (1-\phi) y_b$. The wage w decided at central level is a weighted average of the ones bargained at regional level in the baseline setting (see Eq. (20)). The weights are the share of total employment in region a and in region b, ϕ and $1-\phi$, respectively. The only difference with respect to Eq. (20) is the absence of vacancy costs, k, which we have not included in the confederation of employers' utility function.

Notice that regional housing prices $p_{nt,a}$ and $p_{nt,b}$ appear in the wage equation (58), since they affect the cost of living indexes $P_{E,i}$ and $P_{U,i}$ for $i \in \{a, b\}$ and the elasticities μ_a and μ_b. Their effect is however weakened by the coefficients ϕ and $1-\phi$.

Productivity changes

In the first simulation we assume a positive 1% increase in labor productivity y_b (East Germany). Of course, this is a regional change that tends to compress spatial disparities. In a second simulation, shown in Appendix B.3, we assume a 1% increase in y_a, a change that is expected to raise regional inequality. Results are respectively summarized in Tables 2 and 4.[37] Since the mechanism at work and the intuitions for the main outcomes of the model are the same, we will present in details only the results of the first simulation. It is easy to see that the figures presented in Table 4 in Appendix B.3 are quantitatively similar to that of Table 2, but with opposite sign.

As concerns the results presented in Table 2, notice first that the housing price and the wage in region b respond significantly to the productivity increase. In the states directly impacted by this change, w_b goes up by the same percentage amount of y_b, while the increase in $p_{nt,b}$ is twice as large in percentage terms. This first effect is mainly the result of a decentralized wage negotiation. We understand the crucial difference introduced by the non-homotheticity assumption by looking at the cost of living index $P_{E,b}$, whose increase is lower when $\epsilon < 0$.[38] When housing is considered a necessity good, an increase in wages reduces the share of income that employed workers in region b allocate to housing (in Eq. (11), $s_{E,i}$ decreases with w_i). As housing becomes relatively more expensive, this shift in household budget composition tends to lower their overall price index.[39] This, in turn, has clear implications for real wages in region b. Attracted by the resulting increase in real compensation, many unemployed workers choose to relocate to region b. As a result, the labor force L_b expands by nearly 2%, a rise at least nine times greater than in the other counterfactual scenarios. This migration reduces housing demand in region a, causing the price $p_{nt,a}$ to decline by 0.4%. More affordable housing in region a then contributes to a lower price index for its residents. This explains the nearly 3% reduction in the average consumer price index (CPI) gap—compared to just 1% under the model with homothetic preferences and decentralized bargaining.[40] In contrast, no variation in the CPI gap is observed under centralized wage. In those scenarios, a positive productivity change in the Eastern regions leads to an increase in wages across the entire country. This, in turn, drives up housing prices — and consequently the cost of living indexes — in both regions.

The results concerning other regional disparities naturally follow from the findings discussed so far. The model with non-homothetic preferences and decentralized wage bargaining implies a larger reallocation of labor, which in turn leads to a more pronounced change in the employment gap. Specifically, the ratio E_a/E_b declines by over 2%

[37] When we report coefficients as +0.00 or −0.00, this indicates that the effects are very small and close to zero. For readability, we report only the zeros in the table.

[38] From Eqs. (6) and (11), we have that if the tradable and the non-tradable goods are gross complements (i.e $0 < \sigma < 1$), a higher $p_{nt,i}$ raises both $s_{U,i}$ and $s_{E,i}$, thereby increasing $P_{E,i}$ and $P_{U,i}$ (see their definition in Section 3.2.3).

[39] Notice that this mechanism is not at work for the unemployed workers, whose income z does not increase in response to the productivity change. The change in $P_{U,b}$ is 0.6 percentage points larger than the change in $P_{E,b}$.

[40] The average CPI gap is computed as $(e_a \cdot P_{E,a} + u_a \cdot P_{U,a})/(e_b \cdot P_{E,b} + u_b \cdot P_{U,b})$.

compared to just 0.2% in the scenario with uniform wages. In the other two counterfactual exercises (fourth and fifth column), the effects are more muted.

It is important to note that the effects of the productivity change on employment gaps are entirely driven by variations in the labor force, not in the unemployment rates. The ratio E_a/E_b shifts because more individuals relocate between regions, not because fewer people are unemployed. As shown in Table 2, increased productivity has no discernible effect on u_a and u_b.[41] There are two key reasons for this. First, the simulations reflect an economy with a low unemployment rate. In such tight labor markets, the rise in vacancy creation following a productivity boost has little impact on unemployment. Second, it is well established that standard search and matching models fail to replicate the large swings in unemployment observed in response to exogenous productivity shocks (see Shimer (2005)). Our results indicate that a 1% increases in y_b raises labor market tightness θ_b by 2%. However, the corresponding decline in u_b is only about half as large.[42] This low elasticity results in a negligible change in unemployment in absolute terms. Naturally, the minimal variation in u_a and u_b also explains why the total employment level in the country remains largely unchanged across all four scenarios.

Greater labor mobility also has significant implications for tradable output.[43] The ratio of output produced in region a to those in region b falls by 3%, while total production of the good increases by 2.4%. Once again, these changes are larger than those observed in the three counterfactual scenarios.

Welfare outcomes are summarized in the last four rows of the table. Two points are worth highlighting. First, the expected lifetime utility of all workers increases. Even unemployed individuals in region b, who face a higher cost of living, experience a net welfare gain. The positive effects of the productivity increase on both the job-finding rate $f(\theta_b)$ and the value of being employed $W^E_{j,b}$ outweigh their temporary loss in real income. This result is intuitive in the context of Germany during the second decade of the 21st century — a period characterized by low separation rates and short unemployment durations.[44] From a lifetime perspective, the time spent job searching is minimal, so the associated income losses are relatively insignificant. Of course, we would expect different welfare outcomes in an economy undergoing a prolonged recession. Second, the magnitude of welfare improvements is greater under our baseline model. The reallocation process not only reduces regional inequality but also delivers welfare gains to all workers.

Housing supply change

We simulate a supply-side shock that increases the marginal cost of housing in West Germany (region a). Specifically, we decrease the parameter α_a in the housing supply function (25) by 3.6%. As previously noted, this adjustment yields the same reduction in the West–East output gap as the 1% productivity change in y_b, thereby allowing a more direct comparison between the two simulations.

This adjustment can be interpreted as reflecting higher construction costs or stricter regulatory constraints on new housing services. The results are summarized in Table 3. As expected, housing prices rise in region a. Similar to the productivity change simulations, an increase in $p_{nt,a}$ raises the cost of living for all workers in the region. Notably, in models where $\epsilon < 0$, $P_{U,a}$ increases by 0.5 percentage points more than $P_{E,a}$. This is because less affordable housing disproportionately affects individuals who allocate a larger share of their income to housing — a distinction absent in models with homothetic preferences, where housing expenditure shares are uniform across households. Consequently, a greater proportion of unemployed workers from region a migrate to region b in models with $\epsilon < 0$. The percentage increase in the labor force L_b is more than twice larger in the baseline model compared to models with $\epsilon = 0$. As in the productivity change simulation, stronger labor reallocation between regions leads to more pronounced reductions in regional disparities. The changes in employment, output, and CPI gaps are larger in magnitude in the second column of Table 3 than in the fourth and fifth columns.

The primary distinction between this exercise and the previous ones lies in the case of non-homothetic preferences with national wage bargaining (third column of Table 3). In scenarios where $\epsilon < 0$, models with decentralized bargaining still exhibit stronger amplification effects, but the difference compared to centralized wage setting is smaller. Specifically, changes in the employment and output gaps are about 30% smaller when $w_a = w_b$, while the disparity in the CPI gap is roughly 40%.

Unlike a labor productivity change, an exogenous change in housing supply does not generate the positive feedback loop between wages and housing prices that is central to our model with non-homothetic preferences and decentralized bargaining. In this scenario, w remains nearly constant across all four cases. This is because housing supply changes do not directly affect workers' earnings. The wage-housing price feedback mechanism in our model is quantitatively significant only when triggered by an initial change in w_i, such as a productivity change that increases the surplus of a job match. In the absence of this mechanism, the non-homothetic nature of housing demand still provides amplification effects, but the type of wage-setting system becomes less critical.

5.3. Sensitivity analysis

We examine the sensitivity of our main simulation results with respect to three key parameters: the inverse of the elasticity of housing supply, γ; workers' bargaining power, β; and the elasticity of substitution, σ.

We begin with γ. As emphasized by Hsieh and Moretti (2019), one of the primary drivers of labor misallocation across U.S. regions is the rigidity of the housing supply. A low elasticity implies that even modest increases in housing demand lead to substantial price hikes, thereby making migration towards more productive areas prohibitively expensive. This, in turn, limits labor reallocation and hinders efficient employment distribution. In our model, changes in regional productivity lead to sizable migration responses. These migration flows alter L_a and L_b, which in turn influence the employment ratio E_a/E_b. Given empirical evidence suggesting that Germany, like the U.S., features a relatively inelastic housing supply (see Cavalleri et al. (2019), Beze (2023), and Lerbs (2012), who estimate supply elasticities in the range of 0.25 to 0.5), it is important to assess how sensitive our results are with respect to changes in γ. In our baseline calibration, we set $\gamma = 2$, corresponding to a housing supply elasticity of 0.5. Table 5 in Appendix B.4 reports the simulation results under an alternative calibration where $\gamma = 0.5$, implying a more elastic housing supply with an elasticity of 2. This scenario amplifies the effect of productivity changes on the employment gap compared to the baseline case.[45] Conversely, the effect of a housing supply change under this more elastic scenario is attenuated compared to the baseline. When the housing supply is already relatively flexible, the model's endogenous amplification mechanisms become less potent.

[41] In the sensitivity analysis presented in Section 5.3, we find that a rigid housing market does not seem to be the culprit. A much larger elasticity $1/\gamma$ produces similar effects on the unemployment rates.

[42] From Eq. (1), it is easy to see that the elasticity of the unemployment rate u_i with respect to y_i, μ_{u_i,y_i} is equal to $(1-u_i)(1-\eta)\mu_{\theta_i,y_i}$ with μ_{θ_i,y_i} being the elasticity with respect to tightness. Recall that η is imposed equal to 0.5.

[43] Recall that housing services are produced by landowners that live abroad.

[44] Recall that the monthly rate δ is equal to 0.005 and the expected duration of unemployment does not exceed 5/6 quarters.

[45] Interestingly, although labor migration intensifies under a more elastic supply, the resulting changes in housing prices are more muted, precisely because the supply is more elastic.

Table 3
Housing supply change in region a: $\Delta\alpha_a/\alpha_a = -3.6\%$ (percentage changes; for the unemployment rates variation in percentage points).

Variable	$\epsilon < 0$		$\epsilon = 0$	
	$w_a \neq w_b$	$w_a = w_b$	$w_a \neq w_b$	$w_a = w_b$
L_a	−0.6	−0.7	−0.6	−0.6
L_b	2.6	1.7	1.2	1.0
u_a (pp)	0.0	−0.0	0.0	−0.0
u_b (pp)	0.0	−0.0	0.0	−0.0
$p_{nt,a}$	2.8	2.7	2.3	2.3
$p_{nt,b}$	2.4	1.5	0.9	0.7
$P_{E,a}$	1.1	1.0	1.6	1.5
$P_{E,b}$	0.9	0.7	0.6	0.5
$P_{U,a}$	1.6	1.3	1.6	1.5
$P_{U,b}$	1.2	0.9	0.6	0.5
w_a (real)	0.0 (−1.1)	−0.1 (−1.1)	0.0 (−1.5)	−0.3 (−1.5)
w_b (real)	0.0 (−0.9)	−0.1 (−0.8)	0.0 (−0.6)	−0.3 (−0.6)
Aggregate output	−0.1	−0.1	−0.1	−0.1
Aggregate employment	−0.0	−0.0	−0.0	−0.0
Real wage gap	−0.2	−0.3	−0.9	−1.0
Employment gap	−3.2	−2.4	−1.8	−1.6
Output gap	−3.2	−2.4	−1.8	−1.6
Average CPI gap	−2.9	−2.1	−0.9	−0.6
Variance of log income (real)	0.3 (0.6)	0.2 (−0.9)	0.1 (−1.9)	−0.0 (−3.9)
W_a^E	−1.1	−1.1	−1.5	−1.5
W_a^U	−1.1	−1.1	−1.5	−1.5
W_b^E	−0.9	−0.8	−0.6	−0.6
W_b^U	−0.9	−0.8	−0.6	−0.6

In a second set of simulations, we vary the workers' bargaining power parameter, β, from its baseline value of 0.5 to 0.4 and 0.6. As shown in Table 6 in Appendix B.4, the results across all scenarios remain strikingly similar to those obtained under the baseline calibration. This suggests that, within this range, workers' bargaining power has a limited influence on the model's key outcomes.

Finally, we investigate the sensitivity of our results to changes in the elasticity of substitution between goods, σ. As reported in Table 7, deviations from the baseline calibration produce only minor changes in the simulation outcomes. One notable exception occurs when σ is reduced to 0.3, reflecting a lower willingness of consumers to substitute between goods. In this case, the effects of a housing supply change on housing prices become more pronounced, leading to a larger increase in the cost-of-living index for all workers.

6. Discussion and concluding remarks

The findings presented in the previous section align closely with Germany's recent progress in regional economic convergence. Over the past fifteen years, the gap between the Western states and the new Eastern states — dissolved by the German Democratic Republic in 1952 and re-established in 1990 — has notably narrowed. In 2008, the value-added per worker ratio between West and East Germany stood at approximately 1.3. By 2018, it had declined to 1.2, and by 2020 to around 1.17 (OECD, 2023). Similarly, the employment rate gap between East Germany and the national average fell from 8 percentage points in 2010 to just 2 points in 2019.[46]

This convergence has coincided with a significant shift in wage-setting institutions. Since the late 20th century, Germany has undergone a marked decentralization of wage bargaining.[47] Between 2015 and 2022, the country also experienced a sharp increase in housing costs, with residential property prices rising by over 50% and rents following a similar trend.[48] Despite this, average nominal wages grew robustly (2%–3% annually), and real earnings increased by an average of 1.2% per year.[49]

This period also saw a profound shift in internal migration patterns. In the mid-2000s, net migration from East to West Germany averaged around 50,000 people per year. A decade later, that number had decreased tenfold. Since 2017, the Eastern states have recorded a net migration gain. While economic disparities between West and East Germany persist — particularly in wealth distribution — the degree of convergence has been significant.[50]

Of course, regional inequality is a complex phenomenon, and no single country perfectly fits all features of our model. Nonetheless, we believe our framework provides useful insights. When housing costs constitute a large portion of household expenditures, individuals have a stronger incentive to migrate in search of better job opportunities—but only if wages are responsive to local economic conditions, including housing market dynamics.

The positive relationship between wages and housing prices, implied by Nash bargaining and matching frictions under non-homothetic preferences, warrants further discussion. In our framework, housing prices enter the wage determination equation because they affect workers' quasi-rents from employment, the difference between the expected value of being employed and the value of being unemployed.[51] These quasi-rents are positive only when labor markets exhibit frictions so

[46] Unless otherwise noted, these figures come from the Federal Statistical Office.

[47] See Appendix C and Dustmann et al. (2014) for a detailed discussion.

[48] In major cities—Berlin, Hamburg, Munich, Cologne, Frankfurt am Main, Stuttgart, and Düsseldorf—property prices increased by about 80%. In 2010, housing costs (including rents, energy, and maintenance) made up 46% of expenditures for households earning less than €1300 per month, compared to 27% for those earning over €8000. By 2020, those shares had risen to 50% and 32%, respectively.

[49] See Appendix D for evidence on the positive co-movement between housing prices and labor earnings in Germany.

[50] This contrasts with countries like Italy, where regional divides between the North-Center and the South have scarcely changed over the past two decades. Labor earnings and housing costs have grown little, and wage bargaining remains largely centralized. While we do not analyze Italy in detail, its experience underscores that regional convergence is not automatic but depends on institutional and market conditions.

[51] Under non-homothetic preferences, housing costs enter differently in the price indexes of employed and unemployed workers, so they do not cancel out when computing this difference.

that workers and firms must engage in a costly (and time-consuming) search process.

That said, one could imagine alternative settings without search-and-matching frictions but with other labor market distortions in which housing prices might still affect nominal wages. For example, in a fully decentralized labor market without search frictions, firms located in high-cost housing areas might need to offer higher wages to attract workers.

Our model focuses on housing as a key necessity good, but its core insights may also extend to other essential expenditure categories, such as childcare and healthcare. Exploring these extensions is a promising avenue for future research.

In conclusion, we argue that incorporating non-homothetic preferences into analytical models can offer valuable perspectives on regional inequality and the role of the housing market in shaping labor mobility.

CRediT authorship contribution statement

Gabriele Cardullo: Formal analysis, Conceptualization. **Agnese Sechi:** Validation, Software, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Derivations and proofs of the theoretical model

A.1. Derivation of the Bellman equations

We derive the Bellman equation (3). An unemployed individual of type j observes the prevailing price levels and job-finding rates in both regions a and b. Given his relative preference between the two regions, captured by the difference $\psi_{j,a} - \psi_{j,b}$, he decides where to reside and search for employment — this decision is governed by the indifference condition (23). This locational choice is permanent. In the absence of exogenous changes to the economy, the individual will not relocate once the initial decision is made. This implies that, unlike employment or unemployment status, there are no steady-state flows between regions.

Having decided to live in region i, the worker maximizes the following expected utility:

$$U(t) \;=\; \max_{Q_{nt,i},\,Q_{t,i}} E_t \int_t^{+\infty} e^{-r(\tau-t)}\,(\psi_{j,i} + v_i(\tau))d\tau \tag{28}$$

in which r is the time preference rate, $\psi_{j,i}$ stands for the idiosyncratic preference for region i and v_i is a non-homothetic constant elasticity of substitution function implicitly defined in Eq. (2). The budget constraint can be written as follows:

$$p_{nt,i} \cdot Q_{nt,i} + Q_{t,i} \;=\; \omega_i \tag{29}$$

in which $p_{nt,i}$ is the relative price of the not tradable good in terms of the tradable one. A worker's endowment (i.e., the amount of tradable consumption good he possesses) in region $i \in \{a, b\}$ is denoted by ω_i, which includes wage w_i when employed and home production z when unemployed: $\omega_i = w_i, z$. Such an endowment follows a stochastic Poisson differential equation as there is job creation and job destruction. Hence we can write:

$$d\omega_i \;=\; -(w_i - z)\cdot dq_{w_i} + (w_i - z)\cdot dq_b. \tag{30}$$

Job destruction takes place at an exogenous arrival rate δ. The corresponding Poisson process counts how often the individual moves from employment to unemployment, which is q_{w_i}. Job creation takes place at an endogenous rate $f(\theta_i) \equiv m\left(U_i, V_i\right)/U_i = \theta_i\, q(\theta_i)$. The Poisson process related to this matching process is denoted by q_b. It counts how often a job is found. Both arrival rates are state dependent, as no worker can lose a job when he is unemployed and finding one if he is already employed.

We solve this problem by dynamic programming methods (see for instance Malliaris and Brock, 1982, chapter 2.10).

Defining the optimal program $W_{j,i}(\omega_i) \equiv \max_{Q_{t,i}(\tau),Q_{nt,i}(\tau)} U(t)$ subject to constraints (29) and (30), the objective function can be rewritten as follows:

$$W_{j,i}(\omega_i) \;=\; \max_{Q_{t,i}(t),\,Q_{nt,i}(t)} \left\{ \left(\psi_{j,i} + v_i(t)\right)\Delta t + \frac{1}{1+r\Delta t}\,E_t\,W_{j,i}(\omega_i(t+\Delta t)) \right\}. \tag{31}$$

We multiply this expression by $1 + r\Delta t$, then divide by Δt, rearrange and take the limit of $\Delta t \to 0$. As a result, the value function $W_{j,i}(\omega_i) \equiv \max_{Q_{t,i}(\tau),Q_{nt,i}(\tau)} U(t)$ can be written as:

$$rW_{j,i}(\omega_i) \;=\; \max_{Q_{t,i}(t),\,Q_{nt,i}(t)} \left\{ \psi_{j,i} + v_i(t) + \frac{1}{dt}\,E_t\,dW_{j,i}(\omega_i) \right\} \tag{32}$$

with the constraint (29) and the Poisson differential equation (30). We follow Sennewald (2007, Theorem 1) and Sennewald and Wälde (2006, Proposition 1) and apply the change of variable formulas (CVFs) to compute the differentials for the Poisson process. We get:

$$\begin{aligned} dW_{j,i}(\omega_i) \;=\;& \left\{ W_{j,i}(\omega_i - \varsigma) - W_{j,i}(\omega_i) \right\} dq_{w_i} \\ &+ \left\{ W_{j,i}(\omega_i + \varsigma) - W_{j,i}(\omega_i) \right\} dq_b \end{aligned} \tag{33}$$

in which $\varsigma \equiv w_i - z$. Recall that $E_t\,dq_{w_i} = \delta dt$ and $E_t\,dq_b = f(\theta_i)dt$. We divide by dt and obtain:

$$\begin{aligned} & rW_{j,i}(\omega_i) \\ &= \max_{Q_{t,i}(t),\,Q_{nt,i}(t)} \left\{ \psi_{j,i} + v_i + \delta\left[W_{j,i}(\omega_i - \varsigma) - W_{j,i}(\omega_i)\right] \right. \\ &\quad \left. + f(\theta_i)\left[W_{j,i}(\omega_i + \varsigma) - W_{j,i}(\omega_i)\right] \right\} \end{aligned}$$

given the budget constraint (29). Denote $W^U_{j,i} \equiv W_{j,i}(z)$ the expected present value of being unemployed and $W^E_{j,i} \equiv W_{j,i}(w_i)$ the expected present value of being employed.

Consider the problem of the unemployed worker. Knowing that the probability of losing a job for an unemployed worker is equal to 0, the above equation can be written as:

$$\begin{aligned} rW^U_{j,i} \;&=\; \max_{Q^U_{t,i},\,Q^U_{nt,i}} \psi_{j,i} + v_{U,i} + f(\theta_i)\left[W^E_{j,i} - W^U_{j,i}\right] \\ \text{s.t.}\quad p_{nt,i}\cdot Q^U_{nt,i} + Q^U_{t,i} \;&=\; z. \end{aligned}$$

This coincides with Eq. (3). Similarly, knowing that the probability of finding a job for an employed worker is equal to 0, the same steps allow to obtain Eq. (9).

A.2. The wage equation

We derive the wage equation (20). Knowing that $J^V_i = 0$ and taking $W^U_{j,i}$ as given, the F.O.C. of (18) is:

$$\beta\,\frac{\frac{d\,W^E_{j,i}}{d\,w_i}}{W^E_{j,i} - W^U_{j,i}} + (1-\beta)\,\frac{\frac{d\,J^E_i}{d\,w_i}}{J^E_i} \;=\; 0 \quad \text{with } i \in \{a, b\}. \tag{34}$$

Using (9) and (15) yields

$$\frac{d\,W^E_{j,i}}{d\,w_i} \;=\; \frac{\frac{d\,v_{E,i}}{d\,w_i}}{r+\delta}; \quad \text{and} \quad \frac{d\,J^E_i}{d\,w_i} \;=\; \frac{-1}{r+\delta} \quad \text{with } i \in \{a, b\}.$$

So the F.O.C. in (34) becomes:

$$\frac{1-\beta}{J^E_i} \;=\; \beta\,\frac{\frac{d\,v_{E,i}}{d\,w_i}}{W^E_{j,i} - W^U_{j,i}} \quad \text{with } i \in \{a, b\}. \tag{35}$$

From Eq. (15) and the condition $J_i^V = 0$ we easily get that

$$\frac{1-\beta}{J_i^E} = (1-\beta)\frac{r+\delta}{y_i - w_i} \tag{36}$$

for $i \in \{a, b\}$.

As concerns the RHS of (35), using the Eqs. (3) and (9) to derive an expression for $W_{j,i}^E - W_{j,i}^U$ and Eq. (13) to compute the derivative $d\,v_{E,i}/d\,w_i$ we obtain:

$$\frac{\frac{d\,v_{E,i}}{d\,w_i}}{W_{j,i}^E - W_{j,i}^U} = \frac{r+\delta}{\psi_{j,i} + v_{E,i} - rW_{j,i}^U}\cdot\frac{\left(1-s_{E,i}\right)^{\frac{1}{1-\sigma}}}{1+\epsilon s_{E,i}} \tag{37}$$

for $i \in \{a, b\}$. Putting together this expression with Eq. (36), using the second equation in (13), and recalling that $\mu_i \equiv \frac{1}{1+\epsilon\cdot s_{E,i}}$, $P_{U,i} \equiv \left(1-s_{U,i}\right)^{\frac{-1}{1-\sigma}}$, and $P_{E,i} \equiv \left(1-s_{E,i}\right)^{\frac{-1}{1-\sigma}}$, we obtain:

$$\frac{1-\beta}{y_i - w_i} = \frac{\beta\,\frac{\mu_i}{P_{E,i}}}{\frac{w_i}{P_{E,i}} + \psi_{j,i} - rW_{j,i}^U}, \tag{38}$$

with $i \in \{a, b\}$ and for a generic worker j.

Next we find an expression for $rW_{j,i}^U$ to insert into the RHS of (37). First, we rearrange equation (16) with $J_i^V = 0$ to get $J_i^E = k/q(\theta_i)$. We use this expression to write the F.O.C (19) as follows:

$$W_{j,i}^E - W_{j,i}^U = \frac{\beta}{1-\beta}\frac{k}{q(\theta_i)}\frac{\left(1-s_{E,i}\right)^{\frac{1}{1-\sigma}}}{1+\epsilon s_{E,i}}$$

with $i \in \{a, b\}$. Using this equation and the fact that $v_{U,i} = z\left(1-s_{U,i}\right)^{\frac{1}{1-\sigma}}$ (from Eq. (13)), Eq. (3) becomes:

$$rW_{j,i}^U = \psi_{j,i} + \frac{z}{P_{U,i}} + \frac{\beta}{1-\beta}\,\mu_i\,\frac{k}{P_{E,i}}\,\theta_i$$

Inserting this expression into Eq. (38), we get the wage equation (20). To show that w_i is increasing in $p_{nt,i}$, we consider Eq. (38). The term at the LHS is an increasing function of w_i and it does not depend on $p_{nt,i}$. The term at the RHS is a decreasing function of w_i. If we show that it is also increasing in $p_{nt,i}$, we have that a higher $p_{nt,i}$ implies a higher equilibrium w_i. Computing the derivative of the term at the RHS of (38) with respect to $p_{nt,i}$, we get:

$$\beta\left[\frac{\frac{d\,\mu_i}{d\,p_{nt,i}}\cdot\left(1-s_{E,i}\right)^{\frac{1}{1-\sigma}}}{w_i\cdot\left(1-s_{E,i}\right)^{\frac{1}{1-\sigma}} + \psi_{j,i} - rW_{j,i}^U} - \mu_i\frac{\frac{1}{1-\sigma}\left(1-s_{E,i}\right)^{\frac{1}{1-\sigma}-1}\cdot\frac{d\,s_{E,i}}{d\,p_{nt,i}}\left[\psi_{j,i} - rW_{j,i}^U\right]}{\left[w_i\cdot\left(1-s_{E,i}\right)^{\frac{1}{1-\sigma}} + \psi_{j,i} - rW_{j,i}^U\right]^2}\right].$$

From Eqs. (10) and (14), it is easy to see that both $\frac{d\,\mu_i}{d\,p_{nt,i}}$ and $\frac{d\,s_{E,i}}{d\,p_{nt,i}}$ are positive in our scenario with $0 < \sigma < 1$ and $-1 < \epsilon < 0$. Moreover, from Eq. (3) we also get that $\psi_{j,i} - rW_{j,i}^U < 0$. So we conclude that the derivative the RHS of Eq. (38) with respect to $p_{nt,i}$ is unambiguously positive.

A.3. The Workers' location equation

We derive Eq. (23). We first rewrite that the F.O.C. in the Nash bargaining problem (19) using the second equation in (13) and that $\mu_i \equiv \frac{1}{1+\epsilon\cdot s_{E,i}}$

$$W_{j,i}^E - W_{j,i}^U = \frac{\beta}{1-\beta}\cdot\frac{d\,\mu_i}{d\,w_i}\cdot J_i^E = \frac{\beta}{1-\beta}\cdot\frac{\mu_i}{P_{E,i}}\cdot J_i^E = \frac{\beta}{1-\beta}\cdot\frac{\mu_i}{P_{E,i}}\cdot\frac{k}{q(\theta_i)}.$$

The last equality is obtained by imposing $J_i^V = 0$ in Eq. (16). Inserting this and the first equation in (13) into (22), we get Eq. (23).

A.4. Proof of Lemma 1

If $p_{nt,i}$ and λ^* are assumed to be fixed, an equilibrium exists if the system composed by the three Eqs. (11), (17), and (20) admits a solution for the $s_{E,i}$, θ_i, and w_i in the case $0 < \sigma < 1$ and $-1 < \epsilon < 0$, for $i \in \{a, b\}$. If the equilibrium values for these unknowns exist, then all the other endogenous variables of the model (the expected lifetime and the instantaneous utilities) are easily obtained using their corresponding equations.[52] We first apply the implicit function theorem to Eq. (11). It is easy that w_i is a decreasing function of $s_{E,i}$ in the case $0 < \sigma < 1$ and $-1 < \epsilon < 0$. Moreover, $w_i \to +\infty$ as $s_{E,i} \to 0$ and $w_i \to +0$ as $s_{E,i} \to 1$. So for any value of $s_{E,i}$, Eq. (11) allows to identify a corresponding value of w_i. We can write $w_i(s_{E,i})$ with $w_i'(s_{E,i}) < 0$.

We write down the system of Eqs. (17) and (20) in the following way:

$$\begin{cases} \mathbb{ZP}_i \equiv \frac{y_i - w_i(s_{E,i})}{r+\delta} - \frac{k}{q(\theta_i)} = 0 \\ \mathbb{W}_i \equiv w_i(s_{E,i}) - \frac{\beta\left(y_i + k\cdot\theta_i\right) + (1-\beta)\,z\,(1+\epsilon\cdot s_{E,i})\left(\frac{1-s_{U,i}}{1-s_{E,i}}\right)^{\frac{1}{1-\sigma}}}{1+(1-\beta)\epsilon\cdot s_{E,i}} = 0 \end{cases} \tag{39}$$

for $i \in \{a, b\}$. Notice that $\frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}} = \frac{\partial\,\mathbb{ZP}_i}{\partial\,w_i}\cdot w_i'(s_{E,i})$ is positive because the first derivative is negative and we have just seen that w_i is decreasing in $s_{E,i}$. Moreover, since $q(\theta_i)$ is a decreasing function, then $\frac{d\,\mathbb{ZP}_i}{d\,\theta_i} < 0$, for $i \in \{a, b\}$. So the first equation of the system describes an increasing relationship in the $(s_{E,i}, \theta_i)$ space. In addition, with $s_{E,i} \to 0$ we have $w_i \to +\infty$ and, for the conditions in footnote 14, $\theta_i \to 0$. Conversely, if $s_{E,i} \to 1$ we have $w_i \to 0$ and θ_i is a positive finite number. Under a Cobb–Douglas matching function $\theta_i = \left(\frac{y_i}{(r+\delta)k}\right)^{\frac{1}{\eta}} \equiv \bar{\theta}_i$.

As concerns the second equation of the system, we get:

$$\frac{d\,\mathbb{W}_i}{d\,s_{E,i}} = \frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} + \frac{\partial\,\mathbb{W}_i}{\partial\,w_i}\cdot w_i'(s_{E,i}) \tag{40}$$

with

$$\frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} = -\frac{1-\beta}{1+(1-\beta)\epsilon\cdot s_{E,i}}\left[z\left(\frac{1-s_{U,i}}{1-s_{E,i}}\right)^{\frac{1}{1-\sigma}}\left(\frac{1+\epsilon\cdot s_{E,i}}{\left(1-s_{E,i}\right)(1-s)} + \epsilon\right) - \epsilon w_i\right] \tag{41}$$

for $i \in \{a, b\}$. Notice that $\frac{1+\epsilon\cdot s_{E,i}}{(1-s_{E,i})(1-s)} + \epsilon > 0$ as long as $-1 < \epsilon < 0$ and $0 < \sigma < 1$. So we have $\frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} < 0$. Since $\frac{\partial\,\mathbb{W}_i}{\partial\,w_i} = 1$, the second term at the RHS of (40) is negative, and we obtain that $\frac{d\,\mathbb{W}_i}{d\,s_{E,i}} < 0$. It is also easy to see that $\frac{d\,\mathbb{W}_i}{d\,\theta_i} < 0$. Therefore $\mathbb{W}_i = 0$ describes a decreasing relationship in the $(s_{E,i}, \theta_i)$ space. In addition, if $s_{E,i} \to 0$ we have $w_i \to +\infty$ and, $\theta_i \to +\infty$. We can also show (details are available on request) that if $\theta_i = \bar{\theta}_i$, $s_{E,i} \in (0, 1)$.

This implies that there exists a unique equilibrium in $(s_{E,i}, \theta_i)$ levels that satisfy system (39). The equilibrium value of $s_{E,i}$ allows to uniquely identify w_i. In turn, all the other endogenous variables are determined via their corresponding equations. As concerns the derivatives presented in Lemma 1, they are just obtained applying the implicit function theorem to the system (39). More specifically, we get:

$$\frac{d\,s_{E,i}}{d\,p_{nt,i}} = -\left[\frac{d\,\mathbb{W}_i}{d\,p_{nt,i}}\cdot\frac{d\,\mathbb{ZP}_i}{d\,\theta_i} - \frac{d\,\mathbb{W}_i}{d\,\theta_i}\cdot\frac{d\,\mathbb{ZP}_i}{d\,p_{nt,i}}\right] \times\left[\frac{d\,\mathbb{W}_i}{d\,s_{E,i}}\cdot\frac{d\,\mathbb{ZP}_i}{d\,\theta_i} - \frac{d\,\mathbb{W}_i}{d\,\theta_i}\cdot\frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}}\right]^{-1} > 0$$

[52] With $p_{nt,i}$ fixed, $s_{U,i}$ is obtained via Eq. (6).

since we have:

$$\frac{d\,\mathbb{W}_i}{d\,p_{nt,i}} = -(r+\delta)\cdot\frac{d\,\mathbb{ZP}_i}{d\,p_{nt,i}} > 0; \qquad \frac{d\,\mathbb{W}_i}{d\,\theta_i} < 0 \tag{42}$$

for $i \in \{a, b\}$. Moreover, using Eqs. (40) and (41), we have:

$$\frac{d\,\mathbb{W}_i}{d\,s_{E,i}} = \frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} + w_i'(s_{E,i}) < 0; \qquad \frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}} = -\frac{w_i'(s_{E,i})}{r+\delta} > 0; \qquad \frac{d\,\mathbb{ZP}_i}{d\,\theta_i} < 0 \tag{43}$$

for $i \in \{a, b\}$. We also get:

$$\frac{d\,\theta_i}{d\,p_{nt,i}} = -\left[\frac{d\,\mathbb{W}_i}{d\,s_{E,i}}\cdot\frac{d\,\mathbb{ZP}_i}{d\,p_{nt,i}} - \frac{d\,\mathbb{W}_i}{d\,p_{nt,i}}\cdot\frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}}\right] \times \left[\frac{d\,\mathbb{W}_i}{d\,s_{E,i}}\cdot\frac{d\,\mathbb{ZP}_i}{d\,\theta_i} - \frac{d\,\mathbb{W}_i}{d\,\theta_i}\cdot\frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}}\right]^{-1}$$

for $i \in \{a, b\}$. The second difference at the RHS is positive for the inequalities in Eqs. (42) and (43). From Eqs. (42) and (43) we can also write the first difference as follows:

$$\left[\frac{d\,\mathbb{W}_i}{d\,s_{E,i}}\cdot\frac{d\,\mathbb{ZP}_i}{d\,p_{nt,i}} - \frac{d\,\mathbb{W}_i}{d\,p_{nt,i}}\cdot\frac{d\,\mathbb{ZP}_i}{d\,s_{E,i}}\right] = -\left(\frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} + w_i'(s_{E,i})\right)\frac{1}{r+\delta}\frac{d\,\mathbb{W}_i}{d\,p_{nt,i}} + \frac{w_i'(s_{E,i})}{r+\delta}\frac{d\,\mathbb{W}_i}{d\,p_{nt,i}}$$

for $i \in \{a, b\}$. This term is positive because the we have proved that $\frac{\partial\,\mathbb{W}_i}{\partial\,s_{E,i}} < 0$. In turn, this means that $\frac{d\,\theta_i}{d\,p_{nt,i}} < 0$.

Notice that, for the zero profit condition (17), $\frac{d\,\theta_i}{d\,p_{nt,i}} < 0$ implies that $\frac{d\,w_i}{d\,p_{nt,i}} > 0$. The only reason labor market tightness decreases after a positive variation in $p_{nt,i}$ is because w_i has increased, the other variables of Eq. (17) being exogenously given.

A.5. Proof of Proposition 1

We divide the proof in four steps.

STEP 1.

From Lemma 1, we have seen that the system composed by the housing share equation for employed workers (11), the free entry zero profit condition (17), and the wage Eq. (20) admits a unique equilibrium in $s_{E,i}$, θ_i, and w_i for any given $p_{nt,i}$, and λ^* ($i \in \{a, b\}$). Now, we focus on the market clearing conditions in the housing sector in both regions, (26). Notice first that using the Hicksian demand functions (4) and (10) and Eq. (13), we can write

$$u_i\cdot Q^U_{nt,i} + (1-u_i)\cdot Q^E_{nt,i} = p_{nt,i}^{-\sigma}\left[u_i\, z^{1+\epsilon(1-\sigma)}\left(1-s_{U,i}\right)^{1+\epsilon} + (1-u_i)\, w_i^{1+\epsilon(1-\sigma)}\left(1-s_{E,i}\right)^{1+\epsilon}\right]$$

for $i \in \{a, b\}$. We can insert this expression into the equilibrium condition (26). Using Eq. (24), we get:

$$\left(1-H(\lambda^*)\right)L = \frac{\alpha_a\, p_{nt,a}^{\frac{1}{\gamma}+\sigma}}{u_a\, z^{1+\epsilon(1-\sigma)}\left(1-s_{U,a}\right)^{1+\epsilon} + (1-u_a)\, w_a^{1+\epsilon(1-\sigma)}\left(1-s_{E,a}\right)^{1+\epsilon}} \tag{44}$$

$$H(\lambda^*)L = \frac{\alpha_b\, p_{nt,b}^{\frac{1}{\gamma}+\sigma}}{u_b\, z^{1+\epsilon(1-\sigma)}\left(1-s_{U,b}\right)^{1+\epsilon} + (1-u_b)\, w_b^{1+\epsilon(1-\sigma)}\left(1-s_{E,b}\right)^{1+\epsilon}}. \tag{45}$$

We want to show that the RHS of both equations are increasing in $p_{nt,i}$ for $i \in \{a, b\}$. The numerator is of course increasing in $p_{nt,i}$. As concerns the denominator, applying the implicit function theorem to the system composed by Eqs. (11), (17), and (20) (details are available on request), we obtain that $\frac{d\,w_i}{d\,p_{nt,i}} > 0$, $\frac{d\,s_{E,i}}{d\,p_{nt,i}} > 0$, $\frac{d\,s_{U,i}}{d\,p_{nt,i}} > 0$ and $\frac{d\,\theta_i}{d\,p_{nt,i}} < 0$. Since the unemployment rate u_i is decreasing in θ_i for Eq. (1), we get that the only positive term[53] of the derivative of the denominator of (44) with respect to $p_{nt,i}$ is :

$$(1+\epsilon(1-\sigma))\,(1-u_i)\left(1-s_{E,i}\right)^{1+\epsilon} w_i^{\epsilon(1-\sigma)}\,\frac{d\,w_i}{d\,p_{nt,i}}$$

for $i \in \{a, b\}$. Computing the derivative of (44) with respect to $p_{nt,i}$, we then obtain that sufficient condition for the term at the RHS of (44) to be increasing in $p_{nt,i}$ is that

$$p_{nt,i}^{\frac{1}{\gamma}+\sigma}\,[1+\epsilon(1-\sigma)]\,(1-u_i)\left(1-s_{E,i}\right)^{1+\epsilon} w_i^{\epsilon(1-\sigma)}\,\frac{d\,w_i}{d\,p_{nt,i}} < \left(\frac{1}{\gamma}+\sigma\right)p_{nt,i}^{\frac{1}{\gamma}+\sigma-1}(1-u_i)\left(1-s_{E,i}\right)^{1+\epsilon} w_i^{1+\epsilon(1-\sigma)}.$$

This is equivalent to prove that

$$\frac{d\,w_i}{d\,p_{nt,i}}\,\frac{p_{nt,i}}{w_i} < \frac{\frac{1}{\gamma}+\sigma}{1+\epsilon(1-\sigma)}.$$

In a supplementary note (available on request), we prove that this is always verified if the term at the RHS is greater than 1. This in turn is equivalent to imposing $\frac{1}{\gamma} > (1+\epsilon)(1-\sigma)$.

Therefore, if this condition holds, we have that the term at the RHS of (44) is an increasing function of $p_{nt,i}$

STEP 2

We then use both equations in (44) to get rid of the terms with λ^*:

$$\mathbb{L} \equiv L - \frac{\alpha_a\, p_{nt,a}^{\frac{1}{\gamma}+\sigma}}{u_a\, z^{1+\epsilon(1-\sigma)}\left(1-s_{U,a}\right)^{1+\epsilon} + (1-u_a)\, w_a^{1+\epsilon(1-\sigma)}\left(1-s_{E,a}\right)^{1+\epsilon}} + - \frac{\alpha_b\, p_{nt,b}^{\frac{1}{\gamma}+\sigma}}{u_b\, z^{1+\epsilon(1-\sigma)}\left(1-s_{U,b}\right)^{1+\epsilon} + (1-u_b)\, w_b^{1+\epsilon(1-\sigma)}\left(1-s_{E,b}\right)^{1+\epsilon}} = 0. \tag{46}$$

For the result obtained in STEP 1, the implicit function $\mathbb{L} = 0$ describes a decreasing relationship in the $(p_{nt,a}, p_{nt,b})$ space. Notice also that $\frac{d\,\mathbb{L}}{d\,y_i} > 0$ for $i \in \{a, b\}$, as w_i is increasing in y_i (see Eq. (20)) and $\frac{d\,u_i}{d\,y_i} = \frac{\partial\,u_i}{\partial\,\theta_i}\cdot\frac{\partial\,\theta_i}{\partial\,y_i} < 0$ and $\frac{d\,s_{E,i}}{d\,y_i} = \frac{\partial\,s_{E,i}}{\partial\,w_i}\cdot\frac{\partial\,w_i}{\partial\,y_i} < 0$.

STEP 3

For convenience we re-write here the migration equation (23):

$$\Lambda \equiv \lambda^* - z\left[\left(1-s_{U,b}\right)^{\frac{1}{1-\sigma}} - \left(1-s_{U,a}\right)^{\frac{1}{1-\sigma}}\right] - \frac{\beta\cdot k}{1-\beta}\left[\frac{\theta_b(1-s_{E,b})^{\frac{1}{1-\sigma}}}{1+\epsilon\cdot s_{E,b}} - \frac{\theta_a(1-s_{E,a})^{\frac{1}{1-\sigma}}}{1+\epsilon\cdot s_{E,a}}\right] = 0.$$

Notice that $\frac{d\,\Lambda}{d\,\lambda^*} > 0$. Moreover we get:

$$\frac{d\,\Lambda}{d\,p_{nt,a}} = \frac{\partial\,\Lambda}{\partial\,p_{nt,a}} + \frac{\partial\,\Lambda}{\partial\,p_{nt,b}}\cdot\left.\frac{\partial\,p_{nt,b}}{\partial\,p_{nt,a}}\right|_{\mathbb{L}=0}$$

in which $\left.\frac{\partial\,p_{nt,b}}{\partial\,p_{nt,a}}\right|_{\mathbb{L}=0} < 0$ is obtained by total differentiating the implicit function $\mathbb{L} = 0$ (see STEP 2). Moreover, we have:

$$\frac{\partial\,\Lambda}{\partial\,p_{nt,a}} = -z\left(1-s_{U,a}\right)^{\frac{1}{1-\sigma}-1}\cdot\frac{d\,s_{U,a}}{d\,p_{nt,a}} + \frac{\beta\cdot k}{1-\beta}\,\frac{(1-s_{E,a})^{\frac{1}{1-\sigma}}}{1+\epsilon\cdot s_{E,a}}\cdot\frac{d\,\theta_a}{d\,p_{nt,a}} + - \frac{\beta\cdot k}{1-\beta}\cdot\frac{\theta_a(1-s_{E,a})^{\frac{1}{1-\sigma}}}{1+\epsilon\cdot s_{E,a}}\cdot\frac{(1+\epsilon\cdot s_{E,a})+\epsilon(1-\sigma)(1-s_{E,a})}{(1-\sigma)(1-s_{E,a})(1+\epsilon\cdot s_{E,a})}\cdot\frac{d\,s_{E,a}}{d\,p_{nt,a}}.$$

[53] This is because the denominator at the RHS of (44) is decreasing in $s_{E,i}$ and $s_{U,i}$, that are increasing in $p_{nt,i}$. Moreover, the same denominator is decreasing in u_i (that is in turn positively affected by $p_{nt,i}$, because $z^{1+\epsilon(1-\sigma)}\left(1-s_{U,i}\right)^{1+\epsilon} < w_i^{1+\epsilon(1-\sigma)}\left(1-s_{E,i}\right)^{1+\epsilon}$ as $z < w_i$ and $s_{U,i} > s_{U,i}$ if $-1 < \epsilon < 0$).

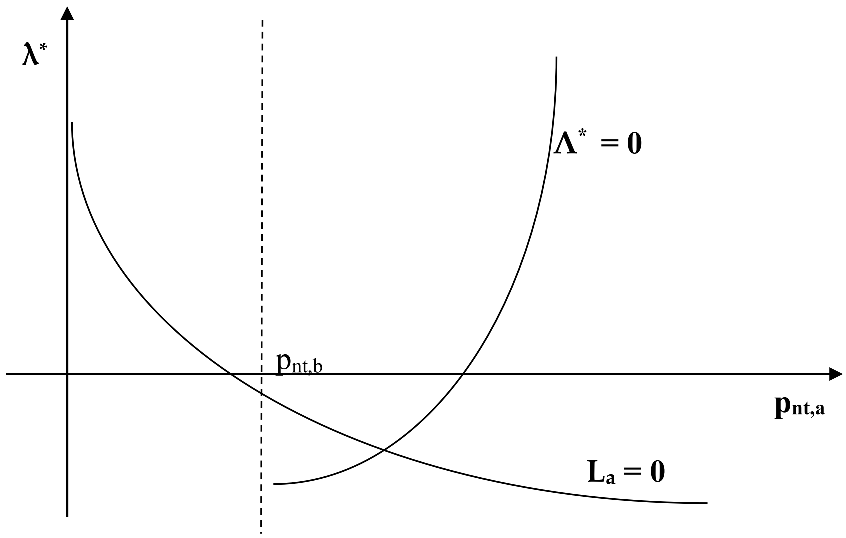

Fig. 1. Equilibrium of system (47).

This derivative is negative, since $\frac{d\, s_{U,a}}{d\, p_{nt,a}} > 0$, $\frac{d\, s_{Ea}}{d\, p_{nt,a}} > 0$, $\frac{d\, \theta_a}{d\, p_{nt,a}} < 0$ and $(1 + \epsilon \cdot s_{E,a}) + \epsilon(1-\sigma)(1 - s_{E,a}) > 0$. Following the same procedure, it is easy to see that $\frac{\partial \Lambda}{\partial\, p_{nt,b}} > 0$. So we have that $\frac{d\, \Lambda}{d\, p_{nt,a}} < 0$. The implicit function $\Lambda = 0$ describes a positive relationship in the $(p_{nt,a}, \lambda^*)$ space.

STEP 4

We consider the system composed by the migration equation (23) and the market clearing condition in the housing market in region a, the first equation in (44):

$$\begin{cases} \Lambda = 0 \\ \mathbb{L}_a \equiv (1 - H(\lambda^*))\, L - \dfrac{\alpha_a\, p_{nt,a}^{\frac{1}{\gamma}+\sigma}}{u_a\, z^{1+\epsilon(1-\sigma)} \left(1-s_{U,a}\right)^{1+\epsilon} + (1-u_a)\, w_a^{1+\epsilon(1-\sigma)} \left(1-s_{E,a}\right)^{1+\epsilon}} = 0. \end{cases} \tag{47}$$

We have proved in STEP 3 that $\Lambda = 0$ a positive relationship in the $(p_{nt,a}, \lambda^*)$ space. From STEP 1 we also know that the term at the LHS of the second equation of system (47), is decreasing in $p_{nt,a}$. Moreover, the same term is also decreasing in λ^*, as $H(.)$ is cumulative density function. Therefore, the second equation of system (47) describes a positive relationship in the $(p_{nt,a}, \lambda^*)$ space. As concerns the limit of the second equation of the system, as $\lambda^* \to \lambda$, there is no labor force in region a and $p_{nt,a} \to 0$. If $\lambda^* \to -\lambda$, $p_{nt,a}$ takes a positive finite number.

We focus now on the limit cases for the implicit function $\Lambda = 0$, that is obtained using the migration equation (23) and he implicit function $\mathbb{L} = 0$, defined in (46). If $\lambda^* < 0$, $L_a > L_b$ and the first negative term at the LHS of Eq. (46) must be greater in absolute value the second one. Since $y_a > y_b$ and $\frac{d\, \mathbb{L}}{d\, y_i} > 0$ for $i \in \{a, b\}$ (see STEP 3), this is possible only if $p_{nt,a} > p_{nt,b}$. This implies that the implicit function $\Lambda = 0$ is negative for values of $p_{nt,a}$ close but greater than $p_{nt,a}$. Since it is an increasing function in the $(p_{nt,a}, \lambda^*)$ space, an equilibrium for the system (47) exists and it is unique. See Fig. 1. Once the equilibrium values of $p_{nt,a}$ and λ^* are determined, $p_{nt,b}$ is uniquely obtained via Eq. (46). From Lemma 1, all the other endogenous variables of the model can be found solving system (39).

Notice that in Fig. 1 we have considered an equilibrium with $\lambda^* < 0$ and $L_a > L_b$. We cannot rule out, however, the possibility that the two curves intersect in the positive hortant.

As concerns the inequalities presented in Proposition 1, we use the implicit function theorem to system (47):

$$\frac{d\, p_{nt,a}}{d\, y_a} = -\left[\frac{d\, \mathbb{L}_a}{d\, \lambda^*} \cdot \frac{d\, \Lambda}{d\, y_a} - \frac{d\, \mathbb{L}_a}{d\, y_a} \cdot \frac{d\, \Lambda}{d\, \lambda^*}\right] \cdot \left[\frac{d\, \mathbb{L}_a}{d\, \lambda^*} \cdot \frac{d\, \Lambda}{d\, p_{nt,a}} - \frac{d\, \mathbb{L}_a}{d\, p_{nt,a}} \cdot \frac{d\, \Lambda}{d\, \lambda^*}\right]^{-1} > 0 \tag{48}$$

since we have:

$$\frac{d\, \mathbb{L}_a}{d\, \lambda^*} < 0; \quad \frac{d\, \mathbb{L}_a}{d\, y_a} > 0; \quad \frac{d\, \mathbb{L}_a}{d\, p_{nt,a}} < 0; \quad \frac{d\, \Lambda}{d\, \lambda^*} > 0$$

$$\frac{d\, \Lambda}{d\, y_a} = \underbrace{\frac{\partial\, \Lambda}{\partial\, y_a}}_{+} + \underbrace{\frac{\partial\, \Lambda}{\partial\, p_{nt,b}}}_{+} \cdot \underbrace{\left.\frac{\partial\, p_{nt,b}}{\partial\, y_a}\right|_{\mathbb{L}=0}}_{+} > 0 \quad \text{and}$$

$$\frac{d\, \Lambda}{d\, p_{nt,a}} = \underbrace{\frac{\partial\, \Lambda}{\partial\, p_{nt,a}}}_{-} + \underbrace{\frac{\partial\, \Lambda}{\partial\, p_{nt,b}}}_{+} \cdot \underbrace{\left.\frac{\partial\, p_{nt,b}}{\partial\, \partial_{nt,a}}\right|_{\mathbb{L}=0}}_{-} < 0.$$

All the signs of the derivatives in the first line are easily obtained by differentiating the equations in system (47). The signs of the derivatives in the second line are computed by differentiating the implicit equation $\mathbb{L} = 0$ in (46). The sign of the derivative in Eq. (48) means that $p_{nt,a} > p_{nt,b}$ as long as $y_a > y_b$. In turn, this implies that $w_a > w_b$ because

$$\frac{d\, w_a}{d\, y_a} = \frac{\partial\, w_i}{\partial\, y_i} + \frac{\partial w_i}{\partial p_{nt,i}} \cdot \frac{\partial p_{nt,i}}{\partial y_i} > 0.$$

The first term at the RHS is positive and it is obtained by applying the implicit function theorem to system (39). The second term at the RHS is also positive for the results in Lemma 1 ($\frac{\partial w_i}{\partial p_{nt,i}} > 0$) and for what we have obtained by totally differentiating system (47) ($\frac{\partial p_{nt,i}}{\partial y_i} > 0$). The fact that $p_{nt,a} > p_{nt,b}$ also means that $s_{U,a} > s_{U,b}$ for Eq. (6).

A.6. Proof of Proposition 2

The proof consists on applying the implicit function theorem to system (47). We consider a marginal change in α_a but the same procedure applies in case of a variation in α_b. More specifically, we get that

$$\frac{d\, p_{nt,a}}{d\, \alpha_a} = -\left[\frac{d\, \mathbb{L}_a}{d\, \lambda^*} \cdot \frac{d\, \Lambda}{d\, \alpha_a} - \frac{d\, \mathbb{L}_a}{d\, \alpha_a} \cdot \frac{d\, \Lambda}{d\, \lambda^*}\right] \cdot \left[\frac{d\, \mathbb{L}_a}{d\, \lambda^*} \cdot \frac{d\, \Lambda}{d\, p_{nt,a}} - \frac{d\, \mathbb{L}_a}{d\, p_{nt,a}} \cdot \frac{d\, \Lambda}{d\, \lambda^*}\right]^{-1} < 0 \tag{49}$$

since we have:

$$\frac{d\, \mathbb{L}_a}{d\, \lambda^*} < 0; \quad \frac{d\, \mathbb{L}_a}{d\, \alpha_a} < 0; \quad \frac{d\, \mathbb{L}_a}{d\, p_{nt,a}} < 0; \quad \frac{d\, \Lambda}{d\, \lambda^*} > 0$$

$$\frac{d\,\Lambda}{d\,\alpha_a} = \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,b}}}_{+} \cdot \underbrace{\left.\frac{\partial\,p_{nt,b}}{\partial\,\alpha_a}\right|_{\mathbb{L}=0}}_{-} < 0 \quad \text{and}$$

$$\frac{d\,\Lambda}{d\,p_{nt,a}} = \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,a}}}_{-} + \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,b}}}_{+} \cdot \underbrace{\left.\frac{\partial\,p_{nt,b}}{\partial\,\partial_{nt,a}}\right|_{\mathbb{L}=0}}_{-} < 0.$$

All the signs of the derivatives in the first line are easily obtained by differentiating the equations in system (47). The signs of the derivatives in the second line are computed by differentiating the implicit equation $\mathbb{L} = 0$ in (46).

To evaluate the effects on $p_{nt,b}$ and λ^* of a marginal reduction in α_a we find it easier to focus on an alternative equilibrium system, in which we consider the equilibrium condition in the housing market in region b, the second equation in (44):

$$\begin{cases} \Lambda \;=\; 0 \\ \mathbb{L}_b \;\equiv\; H(\lambda^*)L \;-\; \dfrac{\alpha_b\, p_{nt,b}^{\frac{1}{\gamma}+\sigma}}{u_b\, z^{1+\epsilon(1-\sigma)}\left(1-s_{U,b}\right)^{1+\epsilon} + (1-u_b)\, w_a^{1+\epsilon(1-\sigma)}\left(1-s_{E,b}\right)^{1+\epsilon}} = 0. \end{cases} \tag{50}$$

The same procedure described in STEP 4 in the previous Appendix allows to find a unique equilibrium in the $(p_{nt,b}, \lambda^*)$ space.

Comparative statics is easier if we consider this system as $\frac{d\mathbb{L}_b}{d\,\alpha_a} = 0$. So we get:

$$\frac{d\,p_{nt,b}}{d\,\alpha_a} = -\left[\frac{d\,\mathbb{L}_b}{d\,\lambda^*}\cdot\frac{d\,\Lambda}{d\,\alpha_a}\right]\cdot\left[\frac{d\,\mathbb{L}_b}{d\,\lambda^*}\cdot\frac{d\,\Lambda}{d\,p_{nt,b}} - \frac{d\,\mathbb{L}_b}{d\,p_{nt,b}}\cdot\frac{d\,\Lambda}{d\,\lambda^*}\right]^{-1} < 0 \tag{51}$$

and

$$\frac{d\,\lambda^*}{d\,\alpha_a} = \left[\frac{d\,\mathbb{L}_b}{d\,p_{nt,b}}\cdot\frac{d\,\Lambda}{d\,\alpha_a}\right]\cdot\left[\frac{d\,\mathbb{L}_b}{d\,\lambda^*}\cdot\frac{d\,\Lambda}{d\,p_{nt,b}} - \frac{d\,\mathbb{L}_b}{d\,p_{nt,b}}\cdot\frac{d\,\Lambda}{d\,\lambda^*}\right]^{-1} < 0 \tag{52}$$

since we have:

$$\frac{d\,\mathbb{L}_b}{d\,\lambda^*} > 0\,; \qquad \frac{d\,\mathbb{L}_b}{d\,p_{nt,b}} < 0\,; \qquad \frac{d\,\Lambda}{d\,\lambda^*} > 0$$

$$\frac{d\,\Lambda}{d\,\alpha_a} = \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,a}}}_{-} \cdot \underbrace{\left.\frac{\partial\,p_{nt,a}}{\partial\,\alpha_a}\right|_{\mathbb{L}=0}}_{-} > 0 \quad \text{and}$$

$$\frac{d\,\Lambda}{d\,p_{nt,b}} = \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,b}}}_{+} + \underbrace{\frac{\partial\,\Lambda}{\partial\,p_{nt,a}}}_{-} \cdot \underbrace{\left.\frac{\partial\,p_{nt,a}}{\partial\,\partial_{nt,b}}\right|_{\mathbb{L}=0}}_{-} > 0.$$

Again, all the signs of the derivatives in the first line are easily obtained by differentiating the equations in system (50). The signs of the derivatives in the second line are computed by differentiating the implicit equation $\mathbb{L} = 0$ in (46).

We get that a marginal reduction in α_a increases $p_{nt,a}$, $p_{nt,b}$, and λ^*. Since the labor force L_a is a decreasing function of λ^*, a lower α_a reduces L_a and raises L_b. For the properties described in Lemma 1, for any $i \in \{a, b\}$ a higher $p_{nt,i}$ raises w_i and $P_{U,i}$, while it reduces θ_i (so u_i goes up for Eq. (1)).

Appendix B. Numerical exercises

B.1. Identical wage across regions

We consider the case in which w is identical across regions. The equilibrium value for w is the result of bargaining between a national workers' union and the confederation of all employers. More specifically, we assume that the workers' union utility maximizes the sum of the utilities of all its members:

$$rU^W \;=\; \sum_j E_a\cdot(\psi_{j,a} + v_{E,a}) \,+\, E_b\cdot(\psi_{j,b} + v_{E,b}). \tag{53}$$

Table 4
Productivity change in region a: $\Delta y_a/y_a = 1\%$ (percentage changes; for the unemployment rates variation in percentage points).

Variable	$\epsilon < 0$		$\epsilon = 0$	
	$w_a \neq w_b$	$w_a = w_b$	$w_a \neq w_b$	$w_a = w_b$
L_a	0.5	0.1	0.2	0.0
L_b	−2.1	−0.2	−0.3	−0.1
u_a (pp)	−0.0	−0.0	−0.0	−0.0
u_b (pp)	−0.0	0.1	−0.0	0.0
$p_{nt,a}$	1.1	0.4	0.9	0.4
$p_{nt,b}$	−2.0	0.2	−0.3	0.3
$P_{E,a}$	0.1	−0.0	0.6	0.2
$P_{E,b}$	−0.7	−0.1	−0.2	0.2
$P_{U,a}$	0.6	0.2	0.6	0.3
$P_{U,b}$	−0.9	0.1	−0.2	0.2
w_a (real)	1.0 (0.9)	0.6 (0.6)	1.0 (0.4)	0.2 (0.3)
w_b (real)	−0.0 (0.7)	0.6 (0.7)	−0.0 (0.2)	0.2 (0.3)
Aggregate output	1.0	0.8	0.8	0.7
Aggregate employment	0.0	−0.0	0.0	−0.0
Real wage gap	0.2	−0.1	0.3	−0.0
Employment gap	2.7	0.3	0.6	0.2
Output gap	3.7	1.3	1.6	1.2
Average CPI gap	0.0	0.3	1.3	0.2
Variance of log income (real)	2.3 (1.4)	3.6 (1.5)	3.0 (0.9)	3.1 (0.1)
W_a^E	0.9	0.6	0.4	0.3
W_a^U	0.9	0.6	0.4	0.3
W_b^E	0.7	0.6	0.2	0.3
W_b^U	0.7	0.6	0.2	0.3

Table 5
Sensitivity analysis with $\gamma = 0.5$ (percentage changes — for the unemployment rates variation in percentage points).

Percentage change	$\Delta y_a/y_a = 1\%$	$\Delta y_b/y_b = 1\%$	$\Delta\alpha_a/\alpha_a = -3.6\%$
L_a	0.9	−0.8	−0.4
L_b	−4.1	3.3	1.9
u_a (pp)	−0.0	−0.0	0.0
u_b (pp)	−0.0	−0.1	0.0
$p_{nt,a}$	0.6	−0.3	1.2
$p_{nt,b}$	−1.6	1.5	0.7
$P_{E,a}$	−0.1	−0.1	0.5
$P_{E,b}$	−0.6	0.3	0.3
$P_{U,a}$	0.4	−0.2	0.7
$P_{U,b}$	−0.8	0.7	0.4
w_a (real)	1.0 (1.1)	−0.0 (0.1)	0.0 (−0.5)
w_b (real)	−0.0 (0.6)	1.0 (0.7)	0.0 (−0.3)
Aggregate output	0.2	−0.1	−0.1
Aggregate employment	0.1	−0.0	−0.0
Real wage gap	0.5	−0.6	−0.2
Employment gap	5.3	−4.0	−2.3
Output gap	6.3	−4.0	−2.3
Average CPI gap	5.8	−4.3	−2.0
Variance of log income (real)	2.1 (1.5)	−0.9 (−0.2)	0.2 (0.2)
W_a^E	1.1	0.1	−0.5
W_a^U	1.1	0.1	−0.5
W_b^E	0.6	0.7	−0.3
W_b^U	0.6	0.7	−0.3

Similarly, the utility of the employers' confederation U^F is just the sum of the revenues raised in the country:

$$rU^F \;=\; E_a\cdot(y_a - w_a) + E_b\cdot(y_b - w_b). \tag{54}$$

The fall-back positions for the two players are respectively equal to:

$$r\bar{U}^W \;=\; \sum_j E_a\cdot(\psi_{j,a} + v_{U,a}) \,+\, E_b\cdot(\psi_{j,b} + v_{U,b}) \tag{55}$$

$$r\bar{U}^F \;=\; 0. \tag{56}$$

The Nash bargaining problem takes the following form:

$$w \;=\; \operatorname{argmax}\left[U^W - \bar{U}^W\right]^{\beta}\left[U^F - \bar{U}^F\right]^{1-\beta}. \tag{57}$$

Table 6
Sensitivity analysis with $\beta = 0.4$ and $\beta = 0.6$ (percentage changes — for the unemployment rates variation in percentage points).

	$\beta = 0.4$	$\beta = 0.4$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.6$	$\beta = 0.6$
Percentage change	$\Delta y_a/y_a = 1\%$	$\Delta y_b/y_b = 1\%$	$\Delta\alpha_a/\alpha_a = -3.6\%$	$\Delta y_a/y_a = 1\%$	$\Delta y_b/y_b = 1\%$	$\Delta\alpha_a/\alpha_a = -3.6\%$
L_a	0.5	−0.4	−0.6	0.5	−0.4	−0.6
L_b	−2.1	1.7	2.6	−2.1	1.7	2.6
u_a (pp)	−0.0	−0.0	0.0	−0.0	−0.0	0.0
u_b (pp)	−0.0	−0.1	0.0	−0.0	−0.1	0.0
$p_{nt,a}$	1.1	−0.4	2.8	1.1	−0.4	2.8
$p_{nt,b}$	−2.0	2.2	2.4	−1.9	2.2	2.4
$P_{E,a}$	0.1	−0.1	1.1	0.1	−0.1	1.1
$P_{E,b}$	−0.7	0.5	0.9	−0.7	0.5	0.9
$P_{U,a}$	0.6	−0.2	1.7	0.6	−0.2	1.6
$P_{U,b}$	−0.9	1.1	1.2	−0.9	1.1	1.2
w_a (real)	1.0 (0.9)	−0.0 (0.1)	0.0 (−1.1)	1.0 (0.9)	−0.0 (0.1)	0.0 (−1.1)
w_b (real)	−0.0 (0.7)	1.0 (0.5)	0.0 (−0.9)	−0.0 (0.7)	1.0 (0.5)	0.0 (−0.9)
Aggregate output	0.1	−0.1	−0.1	0.1	−0.1	−0.1
Aggregate employment	0.0	0.0	−0.0	0.0	0.0	−0.0
Real wage gap	0.2	−0.3	−0.2	0.2	−0.3	−0.2
Employment gap	2.7	−2.1	−3.2	2.7	−2.1	−3.2
Output gap	3.8	−2.1	−3.2	3.7	−2.1	−3.2
Average CPI gap	3.6	−2.7	−2.9	3.6	−2.7	−2.9
Variance of log income (real)	2.3 (1.4)	−1.1 (0.0)	0.3 (0.6)	2.3 (1.4)	−1.1 (0.0)	0.3 (0.6)
W_a^E	0.9	0.1	−1.1	0.9	0.1	−1.1
W_a^U	0.9	0.1	−1.1	0.9	0.1	−1.1
W_b^E	0.7	0.5	−0.9	0.7	0.5	−0.9
W_b^U	0.7	0.5	−0.9	0.7	0.5	−0.9

Table 7
Sensitivity analysis with $\sigma = 0.3$ and $\sigma = 0.7$ (percentage changes — for the unemployment rates variation in percentage points).

	$\sigma = 0.3$	$\sigma = 0.3$	$\sigma = 0.3$	$\sigma = 0.7$	$\sigma = 0.7$	$\sigma = 0.7$
Percentage change	$\Delta y_a/y_a = 1\%$	$\Delta y_b/y_b = 1\%$	$\Delta\alpha_a/\alpha_a = -3.6\%$	$\Delta y_a/y_a = 1\%$	$\Delta y_b/y_b = 1\%$	$\Delta\alpha_a/\alpha_a = -3.6\%$
L_a	0.4	−0.3	−0.6	0.5	−0.4	−0.6
L_b	−1.7	1.4	2.7	−2.4	1.9	2.6
u_a (pp)	−0.0	−0.0	0.0	−0.0	−0.0	0.0
u_b (pp)	−0.0	−0.1	0.0	−0.0	−0.1	0.0
$p_{nt,a}$	1.1	−0.4	3.4	1.1	−0.4	2.6
$p_{nt,b}$	−1.9	2.3	3.0	−2.0	2.2	2.2
$P_{E,a}$	0.2	−0.1	1.2	0.0	−0.2	1.1
$P_{E,b}$	−0.7	0.6	1.0	−0.8	0.5	0.8
$P_{U,a}$	0.6	−0.2	1.8	0.7	−0.2	1.7
$P_{U,b}$	−0.9	1.0	1.4	−1.0	1.1	1.1
w_a (real)	1.0 (0.8)	−0.0 (0.1)	0.0 (−1.2)	1.0 (1.0)	−0.0 (0.2)	0.0 (−1.1)
w_b (real)	−0.0 (0.7)	1.0 (0.4)	0.0 (−1.0)	−0.0 (0.8)	1.0 (0.5)	0.0 (−0.8)
Aggregate output	0.1	−0.0	−0.1	0.2	−0.1	−0.1
Aggregate employment	0.0	0.0	−0.0	0.0	0.0	−0.0
Real wage gap	0.1	−0.3	−0.2	0.2	−0.4	−0.2
Employment gap	2.2	−1.8	−3.3	3.0	−2.4	−3.1
Output gap	3.2	−1.8	−3.3	4.0	−2.4	−3.1
Average CPI gap	3.1	−2.4	−3.1	3.9	−2.9	−2.9
Var. log income (real)	2.7 (1.5)	−1.3 (0.1)	0.4 (0.8)	2.0 (1.3)	−1.0 (0.0)	0.3 (0.6)
W_a^E	0.8	0.1	−1.2	1.0	0.2	−1.1
W_a^U	0.8	0.1	−1.2	1.0	0.2	−1.1
W_b^E	0.7	0.4	−1.0	0.8	0.5	−0.8
W_b^U	0.7	0.4	−1.0	0.8	0.5	−0.8

The F.O.C. is equal to:

$$E_a\left[\frac{\beta\frac{d\,v_{E,a}}{d\,w}}{U^W-\bar{U}^W}-\frac{(1-\beta)}{U^F-\bar{U}^F}\right]+E_b\left[\frac{\beta\frac{d\,v_{E,b}}{d\,w}}{U^W-\bar{U}^W}-\frac{(1-\beta)}{U^F-\bar{U}^F}\right]=0 \qquad (58)$$

Using Eq. (14) to compute the derivatives at the numerators and Eqs. (53), (54), (55), and (56), we get:

$$w=\frac{\beta\bar{y}\left(\phi\frac{\mu_a}{P_{E,a}}+(1-\phi)\frac{\mu_b}{P_{E,b}}\right)+(1-\beta)b\left(\phi\frac{1}{P_{U,a}}+(1-\phi)\frac{1}{P_{U,b}}\right)}{\phi\frac{1}{P_{E,a}}\left(\beta\mu_a+1-\beta\right)+(1-\phi)\frac{1}{P_{E,b}}\left(\beta\mu_b+1-\beta\right)} \qquad (59)$$

in which $\phi \equiv E_a/(E_a+E_b)$ and $\bar{y} \equiv \phi y_a + (1-\phi)y_b$.

For the quantitative analysis, we proceed as follows. For the exogenous parameters of the model, we take exactly the same values considered in our calibration.[54] The endogenous variables of the model are obtained by solving the equilibrium system with Eq. (59) instead of (20). The changes on y_a, y_b, and α_a are then computed using these figures as starting values.

B.2. Homothetic scenario

We follow two alternative procedures to calibrate our model under homothetic preferences (i.e. $\epsilon = 0$). The first one consists of following the same steps presented in Section 5.1. We make only one notable departure. Since with $\epsilon = 0$, the expenditure shares are equal for all workers belonging to the same region, we consider the average values

[54] So the values for r, β, δ, γ, σ, η, L z, ϵ, k, y_a, y_b, α_a, α_b, m_a, m_b, and λ are the same of our baseline exercise with $w_a \neq w_b$ and $\epsilon < 0$.

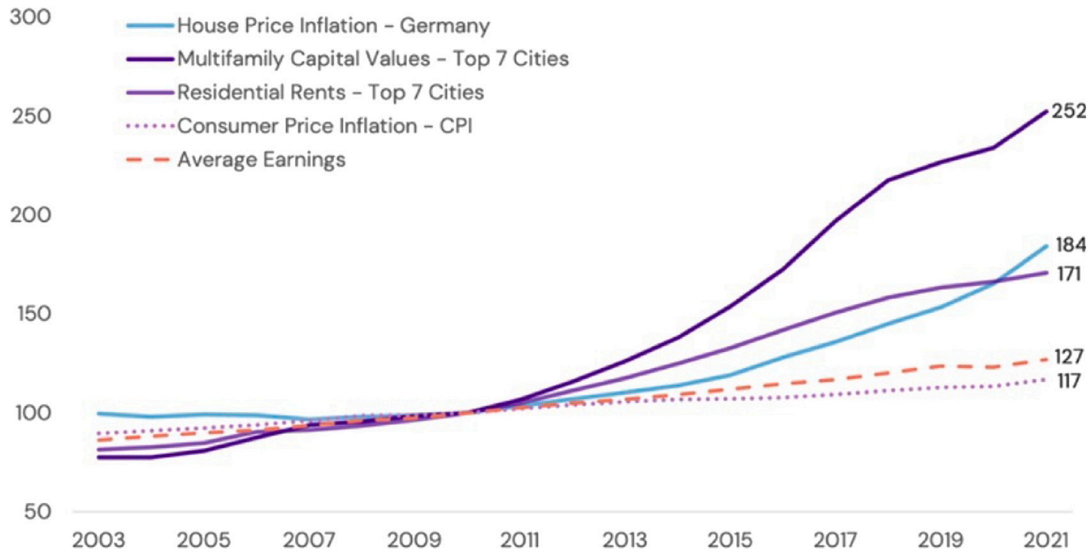

Fig. 2. German residential performance indexes. 2010 = 100.
Source: QSix (QSIX, 2022).

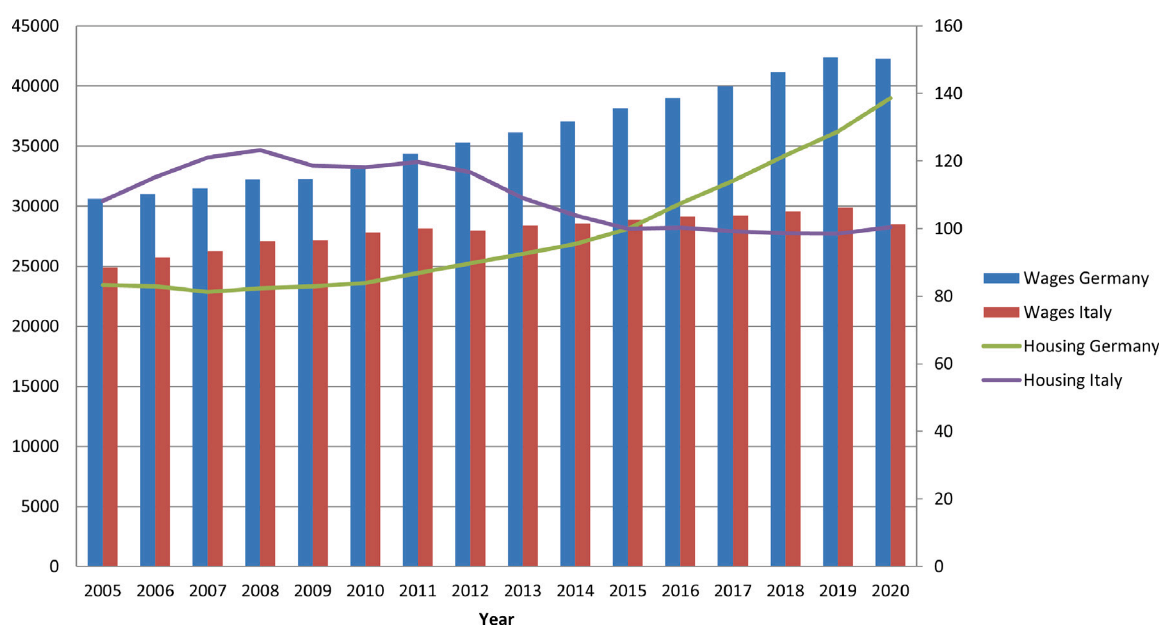

Fig. 3. Average annual wages (Current €) and nominal house price Index.
Data Source: OECD - https://data-explorer.oecd.org/?lc=en.

of the expenditure share for the Western and the Eastern states of Germany, instead of distinguishing between employed workers and poorer, unemployed ones. The resulting calibrated model is then simulated in response to the same changes considered in Section 5.2.

In the second alternative approach, we take the baseline calibrated model and simulate a change on ϵ. Its initial calibrated value is -0.8. We look at the new values for the endogenous variables of the model if $\epsilon = -0.08$, ten times larger. We then simulate the same changes considered in Section 5.2 using as starting point the model at $\epsilon = -0.08$.

They are both counterfactual exercises. The first one considers the real average values for the housing expenditure shares, but all the variables that are calibrated conditional on a specific value of ϵ (most of those presented in Table 2) are obtained imposing homotheticity, an assumption that does not match German data, that show housing expenditure shares decrease with income. On the other hand, the second procedure is equivalent to considering a change on ϵ that changes households' preferences on housing (in the sense it is no longer considered a necessity good) and then look at how the resulting economy would react to the changes. Luckily, results are very similar for either option. In Tables 4, 2, and 3 we present the results of the second procedure. The ones obtained via the first approach are available on request. As concerns the other sensitivity analyses presented in

Table 8
Resident landowners. Productivity change in region a: $\Delta y_a/y_a = 1\%$ (percentage changes; for the unemployment rates variation in percentage points).

Variable	$\epsilon < 0$		$\epsilon = 0$	
	$w_a \neq w_b$	$w_a = w_b$	$w_a \neq w_b$	$w_a = w_b$
L_a	0.4	0.1	0.1	0.0
L_b	−1.5	−0.2	−0.2	−0.1
u_a (pp)	−0.0	−0.0	−0.0	−0.0
u_b (pp)	−0.0	0.0	−0.0	0.0
$p_{nt,a}$	0.9	0.5	0.9	0.6
$p_{nt,b}$	−1.2	0.3	0.5	0.4
$P_{E,a}$	0.1	−0.0	0.6	0.3
$P_{E,b}$	−0.5	−0.1	0.3	0.3
$P_{U,a}$	0.2	0.1	0.6	0.3
$P_{U,b}$	−0.8	−0.0	0.3	0.3
w_a (real)	1.0 (1.0)	0.7 (0.7)	1.0 (0.4)	0.6 (0.2)
w_b (real)	−0.0 (0.5)	0.7 (0.8)	−0.0 (−0.3)	0.6 (0.3)
Aggregate output	1.0	0.8	0.8	0.7
Aggregate employment	0.0	−0.0	0.0	0.0
Real wage gap	0.4	−0.1	0.8	−0.1
Employment gap	1.9	0.3	0.3	0.1
Output gap	3.0	1.3	1.3	1.1
Average CPI gap	1.1	0.4	1.6	2.6
Variance of log income (real)	0.9 (−0.2)	1.6 (0.3)	1.3 (−0.4)	0.7 (−0.3)
W_a^E	1.0	0.7	0.5	0.4
W_a^U	1.0	0.7	0.5	0.4
W_b^E	0.9	0.8	0.5	0.4
W_b^U	0.9	0.8	0.5	0.4

Section 5.3, we simply change the values of the parameter of interest in the calibration and then proceed with the simulation.

B.3. Productivity change in region a

See Table 4.

B.4. Sensitivity analysis

See Tables 5–7.

B.5. Simulations in case of resident landowners

An important assumption of the model concerns the foreign ownership of the housing supply. In this Appendix, we run the same simulations of Section 5.2 under the hypothesis that revenues from the housing sector are instead distributed among all residents of the country. Hence, in this revised setting, individuals have two sources of income: wages (or home production if unemployed), and proceeds from the sale of housing services.

Unfortunately, we lack empirical data on how real estate income or rental earnings are distributed between employed and unemployed individuals in Germany. As a result, we adopt the admittedly unrealistic assumption that this second source of income is equally shared across all individuals in the economy. Therefore, any individual's income from housing is equal to $\left(p_{nt,a} \cdot Q_{nt,a} + p_{nt,b} \cdot Q_{nt,b}\right)/L$ (in terms of tradable consumption good). Given this limitation, this alternative setting is not introduced solely to enhance realism, but rather as a robustness check. Our aim is to assess whether the main findings of our baseline model — particularly the amplification mechanism driven by non-homothetic preferences and decentralized bargaining — are sensitive to the assumption of foreign versus domestic ownership of housing assets.

Tables 8, 9, and 10 confirm that the combination of non-homothetic preferences (i.e., $\epsilon < 0$) and regional wage bargaining (i.e., $w_a \neq w_b$) has a substantially greater effect on regional (di)convergence than the other three scenarios analyzed.

For example, Table 8 examines the impact of a 1% positive productivity change in region a. In the baseline model, this results in employment and output increasing by approximately 2% and 3%, respectively—compared to only approximately 0.3% and 1.3% in the other scenarios. As explained in Section 5.2, the magnifying results of the baseline setting have been triggered by larger migration inflows: the increase (respectively, decrease) in L_a (resp. L_b) is much larger if $\epsilon < 0$ and $w_a \neq w_b$.

A similar disparity in the order of magnitude emerges if we consider a productivity change in region b (y_b) (Table 9) or a housing supply change in region a (Table 10).

Productivity changes

See Tables 8 and 9.

Housing supply change

See Table 10.

Appendix C. Decentralized collective bargaining in Germany

Germany's system of industrial relations is built on two main pillars: collective agreements, and statutory co-determination at both workplace and corporate board levels. Collective agreements are negotiated between trade unions and either employers' associations or individual employers. In general, these agreements establish minimum standards. In contrast, management and works councils — statutory employee representatives elected at the workplace or company level — share the task of implementing collective agreements at company level. They may also sign works agreements that, in principle, may not address pay or employment conditions, but may include supplementary social benefits. While sectoral and company-level bargaining are largely distinct, sectoral agreements frequently include opening clauses that permit limited company-level adjustments (Schulten and Bispinck, 2020).

Starting from the end of the 20th century, decentralization has become a defining feature of the German dual system (Haipeter and Rosenbohm (2022) and Schulten and Bispinck (2020)). Germany's bargaining system has seen a shift in regulatory authority from industry-level actors (unions and employer associations) to workplace-level actors (works councils and local management), along with a move from collective to workplace agreements. The initial phase of such decentralization process gained momentum in the 1990s amid a severe economic crisis following reunification. Employers increasingly criticized sectoral agreements as overly rigid and lacking flexibility at the company level. They asked and often obtained that sectoral agreements contained some opening clauses whereby companies obtained the right to derogate from sectoral standards in exchange for the safeguarding of jobs. In later years, however, the drive toward greater bargaining decentralization was no longer solely a response to the economic challenges of reunification. Opening clauses increasingly served not just to address hardship but also to enhance competitiveness. This trend is reflected in the decline of industry-level bargaining coverage: in 1996, coverage was 70% in West Germany and 56% in the East; by 2020, it had dropped to 45% and 32%, respectively (Ellguth and Kohaut, 2021).

Appendix D. Housing prices and bargained wages: Empirical evidence

In this section we present some empirical evidence of the relationship between housing prices and labor earnings. We first present the German case. Fig. 2 (reported in a in-depth study by the real estate asset management firm QSIX, 2022) illustrates a positive relationship between housing prices and average earnings in Germany in the last fifteen years.

In Fig. 3 we juxtapose the evolution of wages and housing prices in Germany and Italy in the last twenty years. In Italy, wages (expressed

Table 9
Resident landowners. Productivity change in region b: $\Delta y_b/y_b = 1\%$ (percentage changes; for the unemployment rates variation in percentage points).

Variable	$\epsilon < 0$		$\epsilon = 0$	
	$w_a \neq w_b$	$w_a = w_b$	$w_a \neq w_b$	$w_a = w_b$
L_a	−0.3	0.0	−0.0	0.0
L_b	1.1	−0.0	0.0	−0.0
u_a (pp)	−0.0	0.0	−0.0	0.0
u_b (pp)	−0.0	−0.1	−0.1	−0.0
$p_{nt,a}$	−0.2	0.1	0.2	0.2
$p_{nt,b}$	1.4	0.1	0.4	0.2
$P_{E,a}$	−0.1	−0.0	0.1	0.1
$P_{E,b}$	0.3	−0.0	0.3	0.1
$P_{U,a}$	−0.1	0.0	0.1	0.1
$P_{U,b}$	0.7	0.0	0.3	0.1
w_a (real)	−0.0 (0.1)	0.2 (0.2)	−0.0 (−0.1)	0.2 (0.1)
w_b (real)	1.0 (0.7)	0.2 (0.2)	1.0 (0.7)	0.2 (0.1)
Aggregate output	0.1	0.3	0.0	0.3
Aggregate employment	0.0	0.0	0.0	0.0
Real wage gap	−0.6	−0.0	−0.9	−0.0
Employment gap	−1.4	−0.0	−0.1	0.0
Output gap	−2.4	−1.0	−0.1	−1.0
Average CPI gap	−0.9	0.0	−0.3	1.0
Variance of log income (real)	−0.7 (0.2)	−1.5 (−1.0)	−2.0 (−0.4)	−0.7 (−0.1)
W_a^E	0.1	0.2	0.1	0.2
W_a^U	0.1	0.2	0.1	0.2
W_b^E	0.0	0.2	0.2	0.2
W_b^U	0.0	0.3	0.2	0.2

Table 10
Resident landowners. Housing supply change in region a: $\Delta\alpha_a/\alpha_a = -3.6\%$ (percentage changes; for the unemployment rates variation in percentage points).

Variable	$\epsilon < 0$		$\epsilon = 0$	
	$w_a \neq w_b$	$w_a = w_b$	$w_a \neq w_b$	$w_a = w_b$
L_a	0.0	−0.1	−0.4	−0.2
L_b	−0.1	0.1	0.8	0.3
u_a (pp)	0.0	0.0	0.0	0.0
u_b (pp)	0.0	0.0	0.0	0.0
$p_{nt,a}$	−0.7	−0.5	−5.6	−1.7
$p_{nt,b}$	−1.1	−0.4	−5.3	−1.4
$P_{E,a}$	0.1	0.0	−3.8	−1.0
$P_{E,b}$	0.0	0.1	−3.7	−0.9
$P_{U,a}$	1.0	0.5	−3.7	−0.9
$P_{U,b}$	0.6	0.7	−3.6	−0.8
w_a (real)	0.0 (−0.1)	0.3 (0.2)	0.0 (4.0)	2.1 (3.2)
w_b (real)	0.0 (−0.0)	0.3 (0.2)	0.0 (3.9)	2.1 (3.0)
Aggregate output	0.0	−0.0	−0.1	−0.0
Aggregate employment	−0.0	−0.0	−0.0	−0.0
Real wage gap	−0.0	0.1	0.1	0.1
Employment gap	0.2	−0.2	−1.1	−0.4
Consumption gap	0.2	−0.2	−1.1	−0.4
Average CPI gap	0.2	−0.4	−6.2	−7.9
Variance of log income (real)	3.8 (4.3)	3.9 (2.7)	18.7 (7.4)	16.7 (1.0)
W_a^E	−1.3	−0.8	−3.6	−1.2
W_a^U	−1.3	−0.8	−3.6	−1.2
W_b^E	−1.6	−0.9	−4.5	−1.3
W_b^U	−1.6	−0.9	−4.5	−1.3

in current prices) have remained largely stagnant, while nominal house prices have slightly declined.[55]

At first glance, these patterns appear to support the core implications of our model: in countries where housing costs are high and decrease with income — and where wage bargaining is decentralized (as in Germany) — the correlation between housing prices and wages tends to be stronger than in countries with predominantly centralized bargaining systems (such as Italy). However, we must interpret this evidence with caution. A positive relationship between housing prices and wages is not unique to our framework with non-homothetic preferences. Even under homothetic preferences these variables tend to co-move: for instance, a productivity improvement that increases wages will also raise the demand for housing — assuming it is a normal good — thereby pushing up housing prices. A robust empirical validation of our model would require analyses beyond the scope of this paper. Specifically, we should identify an instrumental variable to address the potential endogeneity in the causal relationship from housing prices to wages, as well as a separate instrument to tackle the reverse endogeneity from wages to housing prices.

[55] We use average annual wages at current prices (Euro, 2023) and Nominal house price index by OECD https://data-explorer.oecd.org/?lc=en.

Data availability

Data will be made available on request.

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