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Editorial

Preface to the Special Issue on "Combinatorial Optimization and Applications"

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1. Introduction

Combinatorial optimization is an area of mathematics, particularly operations research, that has attracted many researchers. There are essentially two reasons for this attention. The first lies in the very nature of combinatorial optimization problems, which are typically NP-hard and, especially since the late 1990s, have therefore continuously stimulated a considerable amount of research interest. This interest led to the development of exact models, followed by heuristic methods and subsequently hybrid approaches combining optimization and simulation, along with data-driven methods, designed to find optimal or near-optimal solutions within acceptable amounts of computational time. The second reason is the wide range of application areas, in which problems can be classified as combinatorial optimization problems and formulated accordingly. These areas range from transportation and logistics to finance, manufacturing, healthcare, and many others.

Although this Special Issue does not provide an exhaustive overview of all methods and applications in this field, it presents a collection of papers that cover a broad spectrum of applications and solution approaches, thus offering readers an insight into the significant potential of this research area. All the papers published in this Special Issue have already been cited; our aim is to encourage further research, both in terms of applying the proposed methods to other combinatorial optimization problems and exploring the challenges presented in these studies, in order to facilitate the development of efficient new solution approaches.

The following section briefly describes the 11 original articles selected for this Special Issue after a thorough peer-review process.

2. Description of the Published Papers

The authors of Contribution 1 initially address the management of missing data and the high volatility of historical data relating to financial assets, examining various proposed approaches and assessing their effects on optimal portfolios. For the portfolio optimization problem, three different variants of the Particle Swarm Optimization algorithm are proposed to improve the effectiveness of identifying the optimal solution within the search space. The proposed asset selection strategies can effectively manage a large asset space to create manageable portfolios while maintaining the benefits of diversification and performance characteristics. The overall results obtained from experiments conducted on historical data for the 2015–2023 period show that limiting asset selection based on a single criterion reduces portfolio quality, while increasing portfolio diversification through careful inclusion of high-return/high-risk and low-return/low-risk assets maximizes the profitability of the portfolios constructed.

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The authors of Contribution 2 present a hybrid genetic algorithm that incorporates tabu search to improve the performance of genetic algorithms in solving the max-cut problem. The algorithm novelly integrates solution-level tabu constraints directly into the population dynamics of the genetic algorithm, allowing redundancy prevention without additional memory overhead. An experimental test, conducted with g 31 benchmark graphs of different structural types commonly referenced in the literature, showed that the proposed algorithm outperforms standard genetic algorithms with respect to the max-cut problem, providing a significant contribution to solving classical combinatorial optimization problems.

Contribution 3 addresses the problem of layout reconstruction in oil-gathering systems in the intermediate and final stages of development. To reduce operating costs, the authors propose a model for optimizing layout reconfiguration, with the aim of minimizing the overall cost of investment. The model considers conditions with multiple constraints, such as the current oil-gathering system, processing capacity, and the possibility of pipeline failures or obstructions. Taking a real oil field in the intermediate and final stages of development in China as a case study, the authors present a genetic optimization model for the reconstruction of the oil-gathering system's layout, solved via a hybrid arithmetic-fireworks optimization algorithm. In addition, to increase population diversity, an improved mutation operator is proposed.

Contribution 4 introduces an improved binary crayfish optimisation algorithm designed to address the problem of feature selection, which is a crucial step in data mining and machine learning activities. The aim is to improve handling of high-dimensional data, achieve optimal feature subsets, and maintain classification accuracy by removing unrelated and redundant attributes. The proposed algorithm integrates a local search strategy and a periodic boundary management technique. The performance of the proposed algorithm was evaluated using support vector machine and k-nearest neighbor classifiers on eighteen multi-scale benchmark datasets. The results show that the algorithm offers better performance than nine binary optimisers recently proposed in the literature, achieving 100% accuracy and reducing the feature set size to 0.8.

In Contribution 5, a new discrete differential evolution algorithm is proposed to solve the sequence of operations in a computer-aided process-planning problem. The relationships between operations are represented as a directed graph, in which the nodes are the operations and the edges indicate a relationship between the operations, so that the proposed algorithm operates within a discrete space of feasible solutions. Discrete mutation and crossover operators update solutions to reduce processing and configuration costs using a local search strategy. In addition, a quantile-based statistical method is used to measure the quality of a new solution. The efficiency and robustness of the proposed approach are demonstrated using five prismatic parts with different quantities of machining operations as benchmarks.

Two papers focus on sustainable supply chains and transport infrastructure. In Contribution 6, the authors focus on life cycle design optimization for reinforced concrete precast modular frames and expands on previous work, considering optimisation processes in all stages of their development, with a view to sustainability. Costs, environmental impacts, social factors, and structural integrity are considered in the proposed multi-objective problem. The problem is addressed by integrating three evolutionary algorithms for solving multi-objective optimisation problems with known environments for multi-criteria decision-making problems. The algorithms' performance is evaluated against life cycle sustainability metrics across every non-dominated solution. The proposed integration improves the ability to address complex challenges common to various modern engineering projects, enhancing their applicability and robustness. The authors of Contribution 7 present

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a bi-objective optimization mathematical programming model designed to minimize CO_2 emissions in a supply chain while reducing production and transport costs. The model deals with budgeting and location/allocation decisions in an optimization-unregulated scenario in which trading CO_2 is allowed among facilities but no formal limit on the cap of CO_2 emissions is given. The proposed model includes an initial budget for technological investments in the facilities of the supply chain, allowing them to reduce unit production emissions at a different cost. The authors determine the theoretical conditions ensuring that emissions will not increase over time without imposing a formal cap-and-trade mechanism in such a way that the model may act as an unregulated cap-and-trade model.

In Contribution 8, the authors address a real case of the Multi-Depot Periodic Petrol Station Replenishment problem, which pertains to an important logistics activity for all the major oil companies. In the presented case, the company must replenish a set of petrol stations from a set of depots in a weekly planning horizon. The refuelling must satisfy available visiting patterns, which differ from customer to customer, predefining for each of them the number of visits and, in each visit, the amount of fuel to be delivered. To fulfil the weekly demand of each petrol station, it is necessary to select one replenishment plan among the given set of visiting patterns. The aim of the problem is to minimize the total distance travelled by the fleet of tank trucks in the entire planning horizon. A matheuristic approach based on the cluster-first, route-second paradigm is proposed and tested with both random instances and a large real one.

Contribution 9 introduces the Budgeted Labeled Minimum Spanning Tree problem, consisting of minimizing the total cost of a spanning tree while ensuring that, for each type of label, the number of edges using that label does not exceed the specified budget. This problem combines the challenges of cost efficiency and label diversity within a fixed budgetary framework, providing a more realistic and practical approach to network design. The application area is the design of multi-media communication networks. Three distinct mathematical programming formulations are presented and tested under several scenarios: a single-commodity flow formulation, a packing formulation, and a penalization-based formulation. Furthermore, a Lagrangian approach based on a Subgradient algorithm is introduced to derive tighter lower bounds for the optimal solution to the problem.

Two contributions address a location problem within networks. Contribution 10 focuses on the problem of determining the optimal placement of sensors for detecting contaminants in water distribution networks. This multi-objective problem is NP-Hard. The authors propose a MILP is proposed that incorporates the dynamics of contamination events from simulation models. The derived time series are used to estimate the detection time at different sensors and the volume of contaminated water. The problem is solved using an innovative multi-objective evolutionary algorithm based on Tchebycheff's scalarisation. The key contribution of the paper is that the weight vectors in scalarisation are probability measures. This allows us to use Wasserstein distance to guide their selection instead of Euclidean distance. This approach leads not only to a new algorithm that yields better computational results than the standard method but also to a new design approach that can be generalised to other evolutionary algorithms and used in other applications, such as epidemic monitoring. Contribution 11 addresses a variant of the multi-facility location-routing problem involving a B2C company operating in an urban area. The objective is to minimise the costs of the pick-up and delivery processes while minimising pollutant emissions. To achieve this objective, customers are assigned to a pick-up-delivery station only if they can reach it via a zero-carbon route no longer than a given maximum distance. A new exact approach based on partitioning the feasible region of an MILP model is presented, where a branching constraint generates the route that serves the selected stations and the route that serves customers at their site. The binary variables selecting the location of the pick-up delivery stations are set while a classic travelling salesman problem is solved. Computational tests show that there is a threshold value for both the distance and the cost of the stations, beyond which home delivery is more cost-effective for the company than activating additional pick-up delivery stations.

3. Conclusions

Looking at the contributions described above, the first thing that emerges is their high quality and originality. Furthermore, it is evident how various applications and real-world problems have been effectively addressed using traditional and innovative combinatorial optimization models and methods. Most contributions use a network as the underlying prototype model for the proposed combinatorial optimization problem, representing, for instance, supply chain and transport infrastructure or pipes. Regarding the objective functions of the proposed optimization problems, sustainability certainly plays an important role. This is in line with the recent scientific literature, in which sustainability issues, especially environmental sustainability, are widely considered. From a methodological perspective, in line with the evolution of optimisation approaches in operational research since the beginning of this century, exact binary programming or MILP models are combined with both heuristic and hybrid methods. The latter, combining, among others, genetic algorithms and tabu search, seem to be valid approaches to solving combinatorial optimisation problems in a computationally effective way.

The 11 published papers have fulfilled the twofold objective of this Special Issue on "Combinatorial Optimization and Application". The first was to suggest new proposals for application problems in order to develop research, showing that there are many contexts that can be traced back to the fertile field of combinatorial optimisation. Secondly, we aimed to present methodological advances in the field of combinatorial optimisation, stimulating and providing useful ideas for further improvements.

As the Guest Editor of this Special Issue, I would first like to thank all the authors who submitted their papers presenting case studies of great interest and the reviewers for their extremely valuable work and the suggestions they provided to the authors to further improve their papers. My warmest thanks go to the editorial staff of Mathematics for their constant professional support. Special thanks go to Nemo Guan, who constantly supported me throughout all stages of the realization of this Special Issue.

Conflicts of Interest: The author declares no conflicts of interest.

List of Contributions

1. Bulani, V.; Bezbradica, M.; Crane, M. Improving Portfolio Management Using Clustering and Particle Swarm Optimisation. *Mathematics* **2025**, *13*, 1623. https://doi.org/10.3390/math13101623.
2. Kim, Y.-H.; Geem, Z.W.; Yoon, Y. Population-Based Redundancy Control in Genetic Algorithms: Enhancing Max-Cut Optimization. *Mathematics* **2025**, *13*, 1409. https://doi.org/10.3390/math13091409.
3. Chen, S.; Wang, S.; Jiang, M.; Li, Y.; Meng, L.; Guan, B.; Yu, Z. Layout Reconstruction Optimization Method of Oil-Gathering Systems for Oilfields in the Mid to Late Stage of Development Based on the Arithmetic–Fireworks Optimization Algorithm. *Mathematics* **2024**, *12*, 2819. https://doi.org/10.3390/math12182819.
4. Sorour, S.E.; Hassan, L.; Abohany, A.A.; Hussien, R.M. An Improved Binary Crayfish Optimization Algorithm for Handling Feature Selection Task in Supervised Classification. *Mathematics* **2024**, *12*, 2364. https://doi.org/10.3390/math12152364.
5. Alvarez-Flores, O.A.; Rivera-Blas, R.; Flores-Herrera, L.A.; Rivera-Blas, E.Z.; Funes-Lora, M.A.; Niño-Suárez, P.A. A Novel Modified Discrete Differential Evolution Algorithm to Solve the Operations Sequencing Problem in CAPP Systems. *Mathematics* **2024**, *12*, 1846. https://doi.org/10.3390/math12121846.

6. Ruiz-Vélez, A.; García, J.; Alcalá, J.; Yepes, V. Enhancing Robustness in Precast Modular Frame Optimization: Integrating NSGA-II, NSGA-III, and RVEA for Sustainable Infrastructure. *Mathematics* **2024**, *12*, 1478. https://doi.org/10.3390/math12101478.
7. Caramia, M.; Stecca, G. Unregulated Cap-and-Trade Model for Sustainable Supply Chain Management. *Mathematics* **2024**, *12*, 477. https://doi.org/10.3390/math12030477.
8. Carotenuto, P.; Giordani, S.; Salvatore, A. A Matheuristic Approach for the Multi-Depot Periodic Petrol Station Replenishment Problem. *Mathematics* **2024**, *12*, 416. https://doi.org/10.3390/math12030416.
9. Cerulli, R.; D'Ambrosio, C.; Serra, D.; Sorgente, C. The Budgeted Labeled Minimum Spanning Tree Problem. *Mathematics* **2024**, *12*, 230. https://doi.org/10.3390/math12020230.
10. Ponti, A.; Candelieri, A.; Giordani, I.; Archetti, F. Intrusion Detection in Networks by Wasserstein Enabled Many-Objective Evolutionary Algorithms. *Mathematics* **2023**, *11*, 2342. https://doi.org/10.3390/math11102342.
11. Sciomachen, A.; Truvolo, M. An Exact Approach for Selecting Pickup-Delivery Stations in Urban Areas to Reduce Distribution Emission Costs. *Mathematics* **2023**, *11*, 1876. https://doi.org/10.3390/math11081876.