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FEATURE ARTICLE

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# Optimal consumption, portfolio, and long-term-care health insurance in a dynamic framework

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**Abstract**

We study the optimal dynamic strategy of representative agents who can invest in the financial market and sign an insurance contract to optimise the utility of intertemporal consumption and face the risk of long-term-care (LTC) expenses. The time horizon of the agent coincides with the stochastic death time, and the health expenditure risk takes the form of a jump Poisson process. The agent may hedge against this health risk by signing an insurance contract, on which we assume there exists a mark-up. We find a closed-form solution for the optimal consumption, the optimal portfolio, and the optimal insurance hedge. We show that the decision to purchase LTC insurance is more complex than what emerges from most insurance models. The proportion of LTC expenditure insured decreases with age. Our model predicts substitution between private coverage and savings as a means to finance LTC expenditure. In response to a health shock requiring an increase in LTC expenditure, the individuals sell their assets to keep up the level of consumption (the so-called "consumption smoothing" effect). Richer individuals dissave more than poorer ones. An increase in the interest rate has the same qualitative impact. The reduction in the mark-up, either due to increasing competitiveness or through public subsidies, is likely to increase the welfare of well-off/fit individuals, while an increase in the interest rate may reduce coverage in a very substantial way, an aspect that has been overlooked by the literature so far.

## 1 | INTRODUCTION

Ageing and the related public expenditure on health and long-term care (LTC) are sources of concern in both EU countries and the United States. When the baby boomer generation reaches retirement age, the health care systems of most Western economies will be confronted with one of the largest actuarial risks in history.

In Europe, life expectancy is increasing, but the same is not true for healthy (i.e., disability-free) life years (Eurostat, 2023), and we may expect that the overwhelming majority of the elderly will require LTC at some point in their lives (Atella et al., 2021).

In 2022, US citizens spent $415 billion on LTC (about 2% of the GDP), half of which was financed by Medicaid (Chidambaram & Burns, 2024). Since this expenditure is expected to grow to 1.25% of the GDP by 2050 (He & Gokhale, 2022), rising costs could place federal and state budgets under serious financial strain and cause them to reduce their LTC coverage.

In Europe, in 2019, about half of the population living in private households aged 65+ reported living with disabilities; 26% reported severe impairment, of which 46% reported receiving an inadequate level of assistance (Eurostat, 2022; Hashiguchii & Llena-Nozali, 2020). Family networks are an important provider of LTC services and partially compensate for the lack of public provision. However, family structures are changing rapidly: in 2023, almost one-third of EU citizens aged 65 years or older were single (Eurostat, 2024). Consequently, in the coming decades, in Europe, reliance on informal care will no longer be feasible.

Kotschy and Bloom (2022) estimated an increase in elderly care demand of about 47% in the next 20 years; this, together with the rising costs of these services,[1] raises the question of who will bear the responsibility of financing this increased expenditure between the public and private sectors. Without public support, the total cost of LTC for an individual can represent between one and up to seven times an older person's median income in OECD countries. The out-of-pocket costs left to pay after public benefits or services are received still represent more than 40% of the median income of people with moderate needs and more than 70% of the median income of those with severe needs, making LTC unaffordable for most people (OECD, 2024).

[1]Chidambaram and Burns (2024) estimate that in the United States, in 2021, the median annual cost of care was $108,405 for a private room in a nursing home, $54,000 for an assisted living facility, and $61,776 for a home health aide. Hashiguchii and Llena-Nozali (2020) estimate that, across OECD countries, the expense of institutional LTC for severe needs is, on average, more than twice the median income of older people.

From a life-cycle perspective, individuals who plan to finance their LTC expenses with their household savings need to significantly reduce their current consumption (e.g., Bueren, 2023), and taking out LTC insurance is potentially an option to improve overall well-being. However, as of 2020, only 7% of US adults over the age of 50 (roughly 7.5 million people) have private LTC insurance (American Association for Long-Term Care Insurance, 2021; NAIC, 2023).

The literature on the motives to buy (or not buy) LTC insurance has identified several reasons for the limited diffusion of LTC insurance policies (for a review, see Eling & Ghavibazoo, 2018; Eling et al., 2021 and Einav & Finkelstein, 2023), but most of these models fail to consider that the decision to buy LTC insurance is made in conjunction with decisions about how much to consume/save and how to allocate savings to risky investments. This holistic perspective is the focus of the model we are proposing.[2] Contrary to most of the current literature, our approach makes it possible to study the relationship between LTC insurance and expenses, the prospects of the financial market, and consumption simultaneously.

We show that the proportion of LTC expenditures financed by household wealth increases with age. The LTC insurance market is not perfectly competitive; an increase of the mark-up on the fair price of the insurance reduces the take-out rate of the policy by a magnitude that depends on risk aversion. In response to a health shock requiring an increase in LTC expenditure, individuals sell their assets to keep up the level of consumption (the so-called "consumption smoothing" effect).

From a policy point of view, it is important to assess the magnitude of the effects of changes in the mark-up and interest rates, two variables potentially under the control of the policymaker. We calibrate our model using data derived from the Health and Retirement Study (HRS) and simulate the effects of variation in the interest rate and the mark-up. An increase of 1% in the mark-up reduces the share of wealth invested in the risky asset, as well as the insurance coverage. We show that the size of the reaction depends on income and age. People aged 75+ at the top end of the income distribution reduce LTC coverage by more than 30 percentage points, whereas the reduction for individuals aged 60 in the first income quartile is lower than 10 percentage points. An increase in the interest rate has the same qualitative impact on coverage and asset allocation.

These results have interesting policy implications that have been overlooked by the traditional literature on the determinants of LTC: the reduction in the mark-up achieved through costly public subsidies is likely to increase the welfare of well-off individuals, while an increase in the interest rate may reduce coverage in a substantial way.

The remainder of the paper is organised as follows: in the following Section we present a summary of the literature. in Sections 3 and 4, we present the theoretical framework and the main results of our analysis; in Section 5, we calibrate the model using real-world data. The results and policy implications are then discussed in Section 6 Finally Section 7 concludes.

## 2 | RELATED LITERATURE

Although the epidemiological and social picture suggests a growing need for LTC, to date public coverage remains insufficient, and private coverage through voluntary insurance is very limited. A vast body of literature has tried to understand why the LTC insurance market

[2]An interesting exception is the paper by Dong et al. (2018), who empirically study the effects of being insured.

remains notably small despite the potentially high and highly uncertain costs associated with LTC (Brown & Finkelstein, 2009; Rieger-Fels, 2024). Socio-demographic factors, family networks, and health status all play a role in the decision to insure (e.g., Bernet, 2004; Brown & Finkelstein, 2009), as well as bequest motives (Lockwood, 2018; Sloan & Norton, 1997; Zweifel & Strowe, 1996). Moreover, the low level of coverage can be the outcome of decisions made in the presence of the misperception of the risk (Boyer et al., 2019); low financial literacy and a lack of information (Brown, 2023; Lambregts & Schut, 2020) can make the choice suboptimal ex post.

Rational decisions coexist with low coverage because of the poor value for money of LTC contracts (Brown & Finkelstein, 2009; Rieger-Fels, 2024), adverse selection (Braun et al., 2019; Colombo et al., 2011; De Donder et al., 2023; Sloan & Norton, 1997), and other market frictions (Doerpinghaus & Gustavson, 2002; Pauly, 1990).

Unlike private insurance for acute health care, the LTC insurance market is individual rather than group-based,[3] and this characteristic may amplify supply-side problems. The number of insurers willing to sell policies to hedge catastrophic or 'tail' risk has been declining significantly, from about one-third in 1995 to less than 10% in 2018 (Cohen et al., 2018). Insurers have also increased their premiums substantially: by 145% between 1995 and 2010 for new policies sold to individuals aged 55–64 and by 134% for new policies sold to people over age 65.

Recent literature has studied the interactions between health care expenditure (including LTC), asset allocation, and consumption but has seldom considered the three components together (see Ameriks et al., 2020; Bueren, 2023; Crainich et al., 2017; Davidoff, 2010). Unlike most of the mentioned literature, we study the evolution of LTC risk coverage across individuals' life cycles by modelling the decision to purchase LTC insurance jointly with individuals' portfolio choices and consumption.[4]

## 3 | THE MODEL

Our model considers the optimal lifetime decision of a representative individual in terms of their consumption choices, asset allocation, and LTC insurance purchase decision. We assume that income is certain since we want to model the optimal choice of an agent close to retirement age (see American Association for Long-Term Care Insurance, 2023).

We assume that health may suddenly deteriorate (through a jump Poisson process), and if this is the case, health care costs require a constant and permanent flow of resources to be devoted to LTC expenditure; each shock adds the same amount to the past level of expenditure.

The agent can invest in a complete and arbitrage-free financial market and may hedge against this LTC expenditure risk by signing an insurance contract (on which we assume there exists a mark-up), which reimburses a given fraction of the amount to be paid for LTC in every period. An agent may face more than one jump in his/her lifetime and he/she can adjust only the coverage of the risk of an increase in future LTC expenditure.

[3] In our model, LTC insurance is underwritten on an individual basis, unlike standard health insurance. In countries with private financed health care, this coverage is often part of the employment package; the same is true for supplementary health insurance in publicly financed health care systems.

[4] Savings patterns and savings puzzles have been extensively studied by the empirical literature (see, e.g., Braun et al., 2019).

The objective of the agent is to maximise the expected discounted hyperbolic absolute risk aversion (HARA) utility of his/her intertemporal consumption over a stochastic lifetime horizon.

Similar to Pauly (1990), we interpret LTC insurance as a contract reimbursing the policyholder in the form of an annuity that covers a fraction of the LTC expenditure. We find a closed-form solution for the optimal consumption, the optimal portfolio, and the optimal insurance hedge.

## 3.1 | The financial market

We assume that on a frictionless and continuously open financial market, two assets are listed:

- a riskless asset whose price $G_t$ solves the ordinary (scalar) differential equation

$$\frac{dG_t}{G_t} = rdt, \tag{1}$$

  where $r$ is the (instantaneously) constant riskless interest rate;
- a risky asset whose price $S_t$ solves the stochastic differential equation

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t, \tag{2}$$

  in which $dW_t$ is a Wiener process whose mean is zero and whose variance is $dt$. The parameters $\mu$ and $\sigma$ measure the average and standard deviation of the asset return, respectively.

This financial market is complete and arbitrage-free or, in other words, there exists a unique market price of risk $\xi$ that solves the linear equation

$$\sigma\xi = \mu - r. \tag{3}$$

The uniqueness of $\xi$ also implies the uniqueness of the neutral probability $\mathbb{Q}$ under which the fundamental theorem of asset pricing holds: the price of any asset is given by the expected value, under $\mathbb{Q}$, of its future cash flows discounted at the riskless interest rate. Finally, Girsanov's theorem allows us to switch from Wiener processes under the real-world probability to Wiener processes under the risk-neutral probability:

$$dW_t^{\mathbb{Q}} = \xi dt + dW_t. \tag{4}$$

The market considered in this study is deliberately simple and does not account for certain empirical phenomena such as interest rate volatility and the heteroskedasticity of financial returns.

While it is entirely feasible to incorporate such features within the optimisation framework we adopt here, doing so introduces a number of limitations. In particular, these phenomena are highly relevant for investors engaged in short-term speculative activities. Our focus, however, is on modelling the behaviour of an agent over a sufficiently long time horizon, one that encompasses the entirety of a life cycle.

Moreover, many stochastic models used to describe interest rate dynamics exhibit, in accordance with empirical evidence, the property of mean reversion. In most cases, the long-run mean to which the rate reverts is assumed to be constant. This implies that more sophisticated models may be useful for capturing short-run fluctuations around equilibrium but do not significantly affect the long-run equilibrium strategy, which is the primary focus of our analysis.

Finally, introducing additional sources of risk in the theoretical model would, to maintain the assumption of market completeness (needed for obtaining a closed-form solution), require the inclusion of additional asset classes appropriately correlated with all risk factors. This would result in a highly complex portfolio structure that lacks empirical support. Indeed, the data at our disposal only report investments in risky financial assets, without any further detail.

## 3.2 | The agent's lifetime

To obtain an optimal closed-form algebraic solution to the agent's problem, we assume that the agent's force of mortality is constant over time ($\kappa$). If we call $\tau \in [t_0, \infty[$ the death time, the probability to be alive at time $t$, given that the agent is alive at time $t_0$, is

$$({}_t p_{t_0}) = \mathbb{E}_{t_0}[\mathbb{I}_{\tau>t}] = e^{-\kappa(t-t_0)},$$

where $\mathbb{I}_\varepsilon$ is the indicator function of the event $\varepsilon$, with a value of 1 if $\varepsilon$ happens and 0 otherwise, and $\mathbb{E}_{t_0}[\bullet]$ is the expected value operator conditional on the information set available at time $t_0$.

Thus, the present value of any amount of money $\Xi_T$ available at a future date $T > t$, conditional on the survival of the agent, can be written as

$$\Xi_t = \mathbb{E}_t^{\mathbb{Q}}[\Xi_T e^{-r(T-t)} \mathbb{I}_{\tau>T}] = \mathbb{E}_t^{\mathbb{Q}}[\Xi_T] e^{-(r+\kappa)(T-t)}, \tag{5}$$

in which we see that the mortality risk increases the rate at which future cash flows are discounted.

## 3.3 | The long-term-care insurance

As mentioned above, we model the total LTC expenditure ($m_t$) as a stochastic variable, which increases from time to time due to a jump that occurs by following a Poisson distribution. Thus, the amount $m_t$ remains unchanged from one jump to another. In particular, we assume that $m_t$ follows

$$dm_t = \phi d\Pi_t, \tag{6}$$

in which $d\Pi_t$ is a Poisson process whose intensity $\lambda$ measures the frequency of the jumps, and $\phi$ is the width of the jump. This means that, after any jump, the total LTC expenditure increases by the amount $\phi$.

The solution to (6) is

$$m_t = m_{t_0} + \phi(\Pi_t - \Pi_{t_0}), \tag{7}$$

and we assume that $m_{t_0} = 0$, that is, the agent does not need LTC services at $t_0$. As it is common in the literature, we also assume $\Pi_{t_0} = 0$ and, accordingly, the difference $(\Pi_t - \Pi_{t_0})$ measures the number of jumps in the time interval $[t_0, t]$. We further assume that $d\Pi_t$ is independent of $dW_t$. The first two moments of $dm_t$ are

$$\mathbb{E}_t[dm_t] = \phi\lambda dt,$$

$$\mathbb{V}_t[dm_t] = \phi^2\lambda dt.$$

We assume that the agent can buy insurance to reduce the amount of money to be spent on LTC services. The scheme of the contract is as follows:

- the agent pays a continuous and constant premium $p$ until his/her death at time $\tau$;
- the insurance company continuously pays the amount of money $m_t$ to cover the agent's LTC expenses for life-saving medications.

This contract is fair if the expected value of the discounted cash flows paid by the agent is the same as the expected value of the discounted cash flows paid by the insurance. This condition can be algebraically written as follows:

$$\mathbb{E}_{t_0}^{\mathbb{Q}}\left[\int_{t_0}^{\infty} \mathbb{I}_{s<\tau} p e^{-r(s-t_0)} ds\right] = \mathbb{E}_{t_0}^{\mathbb{Q}}\left[\int_{t_0}^{\infty} \mathbb{I}_{s<\tau} m_s e^{-r(s-t_0)} ds\right],$$

or, by using (5),

$$p\int_{t_0}^{\infty} e^{-(r+\kappa)(s-t_0)} ds = \int_{t_0}^{\infty} \mathbb{E}_{t_0}^{\mathbb{Q}}[m_s] e^{-(r+\kappa)(s-t_0)} ds,$$

and since $\mathbb{E}_{t_0}^{\mathbb{Q}}[m_s] = \phi\lambda(s - t_0)$, we get

$$p = \phi\lambda \frac{\int_{t_0}^{\infty} (s - t_0) e^{-(r+\kappa)(s-t_0)} ds}{\int_{t_0}^{\infty} e^{-(r+\kappa)(s-t_0)} ds} = \frac{\phi\lambda}{r + \kappa}. \tag{8}$$

Here, we have assumed that the insurance contract has a zero market price of risk, which means that the expected values of $m_t$ computed under $\mathbb{Q}$ and under the historical probability do coincide.

The premium for fully covering the expenditure for LTC medications ($p$) is equal to the expected average of the future increments in LTC medications, weighted by a discount factor that contains both the riskless interest rate and the force of mortality. This insurance contract is akin to a financial instrument called swap. In particular, this is a fix versus floating swap, where the fix part is the constant premium $p$ and the floating part is the medical expenditure $m_t$. Here, we assume that the insurance market is not perfectly competitive and, accordingly, there exists a mark-up ($\eta \geq 1$) on the fair price $p$, such that

$$p = \eta \frac{\phi\lambda}{r+\kappa}. \tag{9}$$

Of course, the agent may be interested in buying partial coverage. If the agent buys, at time $t$, a percentage $h_t$ of the contract, then he/she pays at any instant $ph_t$ and he/she receives $m_t h_t$ from the insurance. Thus, if the percentage $h_t$ is bought, the evolution of the medical expenditure can be written as

$$dm_t = \phi(1-h_t)d\Pi_t, \tag{10}$$

since the percentage $h_t$ is paid by the insurance and the remaining part is paid by the agent.

## 3.4 | The agent's wealth and preferences

The consumer is endowed with an initial level of wealth, which accrues through a constant flow of income (either wage or pension) $w$. Resources are used for consumption ($c_t$) and to buy LTC insurance ($ph_t$), while savings are invested in the financial market. In particular, the consumer optimally chooses the number of risky assets $\theta_t$ and the riskless asset $\theta_{t,G}$ to be held in the portfolio. If we call $R_t$ the agent's total wealth at time $t$, the static portfolio constraint can be written as

$$R_t = \theta_t S_t + \theta_{t,G} G_t, \tag{11}$$

and its dynamics are

$$dR_t = \underbrace{\theta_t dS_t + \theta_{t,G} dG_t}_{dR_{t,1}} + \underbrace{d\theta_t(S_t + dS_t) + G_t d\theta_{t,G}}_{dR_{t,2}}.$$

The wealth differential is formed by two components:

- $dR_{t,1}$ which is due to changes in the asset prices;
- $dR_{t,2}$ which is due to changes in the portfolio allocation.

The portfolio allocation can be chosen by the agent in such a way that the change $dR_{t,2}$ is financed by the positive cash flows and is able to finance the negative cash flows. In particular,

- the positive cash flows are the agent's (instantaneous) wage/pension $w$ and the wealth weighted by the force of mortality $R_t\kappa$, due to the fact that the agent loses all his/her wealth at death;
- the negative cash flows are the insurance premium $h_t p$, the consumption $c_t$, and the health expenditure $m_t$.

Thus, the dynamic constraint can be written as

$$dR_t = \theta_t dS_t + \theta_{t,G} dG_t + (w - m_t - h_t p - c_t + \kappa R_t)dt.$$

Now, we can substitute $dG_t$ and $dS_t$ from (1) and (2), and also substitute $\theta_{t,G}$ from the static constraint (11), and obtain

$$dR_t = (R_t(r+\kappa) + \theta_t S_t(\mu - r) + w - m_t - h_t p - c_t)dt + \theta_t S_t \sigma dW_t. \tag{12}$$

The agent receives utility from consumption. In particular, if we assume that the agent's preferences belong to the HARA family, then we can write his/her utility of consumption as

$$U(t, c_t) = \frac{(c_t - \chi)^{1-\delta}}{1-\delta} e^{-\rho(t-t_0)},$$

where $\rho$ is the subjective discount factor, $\delta$ is the constant Arrow-Pratt relative risk aversion index, and $\chi$ can be interpreted as a subsistence level of consumption that the agent never wants to fall below.

Thus, the optimisation problem can be written as

$$\max_{\{c_t,\theta_t,h_t\}_{t\in[t_0,\infty[}} \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \frac{(c_t-\chi)^{1-\delta}}{1-\delta}\mathbb{I}_{t<\tau} e^{-\rho(t-t_0)}dt\right],$$

or, after (5),

$$\max_{\{c_t,\theta_t,h_t\}_{t\in[t_0,\infty[}} \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \frac{(c_t-\chi)^{1-\delta}}{1-\delta} e^{-(\rho+\kappa)(t-t_0)}dt\right]. \tag{13}$$

Here, we assume that the wealth of the agent is fully lost when he/she dies or, in other words, there is no bequest.

## 4 | OPTIMAL SOLUTION

The optimal consumption, portfolio, and insurance that solve problem (13) under the state variable dynamics (10) and (12) are

$$c_t^* = \chi + \alpha \hat{R}_t, \tag{14}$$

$$S_t \theta_t^* = \frac{1}{\delta}\hat{R}_t \frac{\mu - r}{\sigma^2}, \tag{15}$$

$$h_t^* = 1 - \hat{R}_t \frac{r+\kappa}{\phi}\left(1 - \left(\frac{1}{\eta}\right)^{\frac{1}{\delta}}\right), \tag{16}$$

where

$$\alpha := \frac{\rho+\kappa+\lambda}{\delta} + \frac{\delta-1}{\delta}(r+\kappa) + \frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\xi^2 + \lambda\eta\left(\frac{\delta-1}{\delta} - \left(\frac{1}{\eta}\right)^{\frac{1}{\delta}}\right),$$

$$\hat{R}_t := R_t + \frac{w - \chi - m_t}{r + \kappa} - \frac{\phi\lambda\eta}{(r + \kappa)^2}.$$

We see that $\hat{R}_t$ is a measure of life-cycle wealth, which accounts for the current wealth, the discounted expected value of the future net wages of the subsistence level of consumption, and the discounted expected value of the future outflows for insurance premiums and medical expenditures. Since $\hat{R}_t$ is the wealth used for computing the optimal portfolio, we immediately see from (15) that the higher the subsistence level of consumption, the lower the percentage of wealth invested in the risky asset. In fact, the absolute risk aversion of an agent depends on both $\delta$ (the Arrow-Pratt risk aversion index) and $\chi$. In particular, if we compute the absolute risk aversion index, we get

$$-\frac{\frac{\partial^2}{\partial c_t^2}\frac{(c_t-\chi)^{1-\delta}}{1-\delta}}{\frac{\partial}{\partial c_t}\frac{(c_t-\chi)^{1-\delta}}{1-\delta}} = \frac{\delta}{c_t - \chi},$$

in which we see that the higher $\delta$, or $\chi$, the higher the risk aversion. In other words, if the agent's consumption is very close to the minimum threshold $\chi$, the risk aversion of the agent increases because the optimal portfolio cannot contain a too-high percentage of the risky asset, to reduce the risk so that it falls below the threshold $\chi$.

Our results show that the decision to purchase LTC insurance is more articulated than what emerges from most insurance models: the price (in terms of mark-up) is important, but risk aversion and the interest rate also play an important role; furthermore, all the optimal decisions change over the individual's lifetime. In what follows, we explain the role of these variables and their interactions.

Let us start with the decision to purchase insurance, represented by (16). The agent adjusts his/her (marginal) insurance coverage depending on the force of mortality (the derivative with respect to $\kappa$ is negative). More interestingly, if the wealth $\hat{R}_t$ increases, both consumption and portfolio investment will increase, while private coverage for LTC is reduced. In other words, our model predicts substitution between private coverage and wealth as a means to finance LTC expenditure. The mark-up has the negative expected effect on the coverage, which is, however, mediated by risk aversion.

Having considered other forms of investment and financing for LTC, the interest rate becomes an important variable for consumers in their choice of LTC insurance coverage. The effect of the interest rate is ambiguous: there is a direct, negative effect and an indirect effect through $\hat{R}$.

The optimal wealth dynamics are

$$\frac{d\hat{R}_t}{\hat{R}_t} = \left(r + \kappa + \frac{1}{\delta}\left(\frac{\mu - r}{\sigma}\right)^2 + \eta\lambda\left(1 - \left(\frac{1}{\eta}\right)^{\frac{1}{\delta}}\right) - \alpha\right)dt + \frac{1}{\delta}\frac{\mu - r}{\sigma}dW_t,$$

which implies that the modified wealth $\hat{R}_t$ is always strictly positive (if its initial value is positive). Since the wealth $\hat{R}_t$ grows on average over time, (16) implies that the optimal insurance coverage tends to decline over time. This result leads us to conclude that, as the investor ages, the proportion of insurance purchased tends to decrease.

## 4.1 | Comparative statics

From a policy point of view, it is interesting to evaluate how the optimal solution reacts to a change in key parameters. One of the most cited causes for the low level of LTC insurance is the high mark-up ($\eta$) that is charged by the insurance companies. It is then interesting to see how the agent optimally reacts to a change in this variable. Let us then consider the comparative statics of a change in $\eta$:

$$\frac{\partial c_t^*}{\partial \eta} = -\frac{\lambda}{\delta}\left( R_t^*(\delta-1)\left(\left(\frac{1}{\eta}\right)^{\frac{1}{\delta}} - 1\right) + \phi\frac{\left(\frac{1}{\eta}\right)^{\frac{1}{\delta}}\lambda\eta(1-2\delta) + 2\lambda\eta(\delta-1) + \frac{(\delta-1)\xi^2}{2\delta} + \rho + \kappa + \lambda}{(r+\kappa)^2} \right) - \frac{\lambda}{\delta}\left( \frac{(\delta-1)\left((w-\chi-m_t)\left(\left(\frac{1}{\eta}\right)^{\frac{1}{\delta}} - 1\right) + \phi\right)}{(r+\kappa)} \right), \tag{17}$$

$$\frac{\partial S_t\theta_t^*}{\partial \eta} = -\frac{\phi\lambda\eta}{\delta(r+\kappa)^2}\frac{\mu - r}{\sigma^2}, \tag{18}$$

$$\frac{\partial h_t^*}{\partial \eta} = \frac{\lambda}{r+\kappa}\left(1 - \frac{\delta-1}{\delta}\left(\frac{1}{\eta}\right)^{\frac{1}{\delta}}\right) - \frac{R_t^*(r+\kappa) + (w-\chi-m_t)}{\phi\delta\eta}\left(\frac{1}{\eta}\right)^{\frac{1}{\delta}}. \tag{19}$$

An increase in the mark-up undoubtedly reduces the wealth invested in the risky asset, whereas the sign of the other two derivatives cannot be univocally determined. For the LTC coverage, represented by (18), the derivative is the difference between two positive terms. From a policy point of view, it is also interesting to note that in this case, the lower the gap between the income flow ($w$) and medical expenditure ($m_t$), the higher the derivative.

Assuming that the difference between the two terms is negative, this implies that high-income individuals and/or individuals with limited LTC expenditure are going to increase their LTC coverage more than others when the mark-up is reduced.

Since the derivative for consumption is likely to be negative (for $w - \chi - m_t > 0$), an increase in the mark-up tends to induce a reallocation of wealth.

Additionally, the interest rate has an interesting impact on coverage and portfolio allocation, an aspect that has been overlooked by the traditional literature on the determinants of LTC. The following derivatives show the marginal effects of a change in the interest rate on the optimal insurance and financial investment:

$$\frac{\partial h_t}{\partial r} = -\left(1 - \left(\frac{1}{\eta}\right)^{\frac{1}{\delta}}\right)\left(\frac{R_t}{\phi} + \frac{\lambda\eta}{(r+\kappa)^2}\right), \tag{20}$$

$$\frac{\partial S_t \theta_t^*}{\partial r} = -\frac{1}{\delta\sigma^2}\left(R_t + \frac{w-\chi-m_t}{r+\kappa} - \frac{\phi\lambda\eta}{(r+\kappa)^2}\right) + \frac{\mu - r}{\delta\sigma^2}\left(-\frac{w-\chi-m_t}{(r+\kappa)^2} + \frac{2\phi\lambda\eta}{(r+\kappa)^3}\right). \tag{21}$$

An increase in the interest rate reduces LTC coverage, and the effect increases with wealth. The effect on the risky asset is again less clear, even though it is likely to be negative.

The financial interpretation of this result is straightforward: an increase in $r$ reduces the Sharpe ratio and makes the riskless asset relatively more appealing. The reduction in the risk on the financial market also allows individuals to take on more risk in terms of LTC expenditure by reducing the insurance uptake.

To obtain more insights, the results of the theoretical model will be tested and calibrated by using actual data from the Health and Retirement Study (HRS, Rand data), a biannual longitudinal panel study that surveyed a representative sample of approximately 20,000 people in the United States.

Finally, we can study how the optimal insurance changes with the frequency of adverse events ($\lambda$) and the magnitude of the impact of each jump ($\phi$). These two parameters are always intertwined in the static problems; in fact, the expected medical expenditure is given by the product of the frequency of the jumps ($\lambda$) and the magnitude of each jump ($\phi$). In our dynamic model, we are able to disentangle the effects of the two parameters, which affect the optimal solutions with different strengths. Actually, when we compute the partial derivatives of the optimal insurance $h_t^*$ in (15), we get

$$\frac{\partial h_t^*}{\partial \lambda} = \frac{\eta}{r+\kappa}\left(1 - \left(\frac{1}{\eta}\right)^{\frac{1}{\delta}}\right),$$

$$\frac{\partial h_t^*}{\partial \phi} = \left(R_t \frac{r+\kappa}{\phi^2} + \frac{w-\chi-m_t}{\phi^2}\right)\left(1 - \left(\frac{1}{\eta}\right)^{\frac{1}{\delta}}\right),$$

and from this comparison we obtain

$$\frac{\partial h_t^*}{\partial \phi} > \frac{\partial h_t^*}{\partial \lambda} \Longleftrightarrow \hat{R}_t > (\phi - \lambda)\frac{\phi\eta}{(r+\kappa)^2}.$$

In other words, for a sufficiently high level of modified wealth (i.e., for richer agents), the optimal insurance is affected more by the magnitude of the jumps than by their frequency. Of course, both derivatives are positive, since an increase in both parameters makes the average loss due to illness more important.

## 5 | CALIBRATION

We have shown that the theoretical model allows some policy interventions to impact the optimal insurance coverage. To assess the potential relevance of such impacts, we calibrate the model and then run some policy simulations.

We use available information on financial and insurance markets, life tables, and a medical expenditure database to set some of the variables, and, given these data, we calibrate the key parameters of the consumer preferences to fit the age profile of consumption, risky asset investment, and LTC insurance coverage, which can be observed from the HRS.

The parameters used and the corresponding data sources are fully described in what follows. For the financial market, we consider the daily values of the US S&P 500 index from the beginning of 2013 to the end of 2019. By using the method of moments on the log-returns, we get $\mu = 0.1203387$ and $\sigma = 0.1278036$; the riskless interest rate is assumed to be the average return on the T-bill issued by the US government in the same period (2013–2019). The average daily value of the T-bill return is $r = 0.007690457$.[5] The subjective discount rate is set to the same level as the riskless interest rate.

For the insurance market, the mark-up is set using data from the literature (Brown & Finkelstein, 2007, 2009; Nordman, 2016). The agent 1-year death probability ( ${}_{t+1}q_t$) is derived from the Life Tables for the Social Security area population,[6] as used in the 2016 Trustees Report (TR).[7] Under the hypothesis that the force of mortality is constant, the death probability is

$$({}_{t+1}q_t) = 1 - e^{-\int_t^{t+1} \varkappa du} = 1 - e^{-\varkappa},$$

from which we obtain

$$\varkappa = -\ln(1 - ( {}_{t+1}q_t)).$$

The value of $\varkappa$ is 0.1850327 for females and 0.2026941 for males (on average, females live longer). In the analysis, we will use the average of these two values.

Medical expenditure shock and financial wealth data are derived from the HRS.

Since 1990, the HRS has been collecting longitudinal data from a representative sample of approximately 20,000 US citizens aged at least 50. The HRS covers various aspects of the well-being of older individuals, including income, wealth, health, health care use, employment, and family connections. The core interviews are conducted every 2 years and can be supplemented with additional surveys conducted on a subset of individuals. For our analysis, we merged the HRS data with the Consumption and Activities Mail Survey (CAMS), which has been collecting detailed spending data from around 4000 HRS respondents since 2001. We have exploited data from eight waves spanning the period from 2000 to 2014, focusing on individuals aged between 60 and 80. The resulting data set is an unbalanced panel that provides information from 5153 respondents. Each respondent is observed a varying number of times, with 10% of interviewees observed in all eight waves, while the median number of waves observed is three (the 25th percentile is two, and the 75th percentile is six).

[5]See the series labelled DTB3 in the FRED database (https://fred.stlouisfed.org/series/DTB3).

[6]The Social Security area population is comprised of residents of the 50 states and the District of Columbia; civilian residents of Puerto Rico, the Virgin Islands, Guam, American Samoa, and the Northern Mariana Islands; federal civilian employees and persons in the US Armed Forces abroad and their dependents; non-citizens living abroad who are insured for Social Security benefits; and all other US citizens abroad.

[7]https://www.ssa.gov/oact/STATS/table4c6_2013_TR2016.html.

The information gathered in the HRS-CAMS data set was used to compute two sets of parameters to determine how adverse health shocks affect individuals' financial wealth over their lifetimes. Specifically, we computed, for each respondent, the variation in medical expenses[8] between two consecutive waves. Based on the distribution of this variable, we estimated the top-5% highest annual increase in medical spending to be equal to $5418 and consider this as $\phi$, the increase in medical expenditure due to health shocks. Finally, the representative individual is endowed with an initial financial net wealth[9] and a flow of income,[10] which are both taken to be equal to median values in the HRS data.

The subsistence level of consumption ($\chi$) and the risk aversion parameter ($\delta$) are set in such a way that the simulated age profile of medical expenditure, risky assets,[11] and consumption[12] (all three as a percentage of net financial wealth) mimic the actual age profiles based on HRS data. Figures 1–3 show the simulated (graphs on the left) and HRS-CAMS (graphs on the right) trends over the lifespan of the three variables for our preferred choice of the parameters, that is, $\chi = 0$ and $\delta = 30$.

For the simulated data, the bold lines are the median profiles, whereas the shaded areas of the figures highlight the area between the 20th and 80th percentiles for the simulated data and the 95% confidence interval for the real data.

The simulated age profile for the percentage of wealth used to cover medical expenditure is flatter than that from the HRS data, but the 20–80 percentile range is wide enough to include the estimated confidence interval for the smoothed HRS data age profile for all but ages above 75. The correlation between the two profiles is 0.75.

The simulated and actual age profiles of the share of wealth invested in risky assets are similar after the age of 65 (correlation of 0.5 in the 65–80 range). For HRS data, the share of risky assets becomes almost constant after an initial growth phase, whereas the simulated pattern slightly decreases over the considered age range. The simulated pattern is smoother, also because the simulation does not take into account the sudden changes in the financial market that occurred during the years covered by HRS data that households have actually faced, impacting their portfolios.

Consumption is also a bit more stable in the simulated model than in the real data. This is due to several reasons, which include the fact that the theoretical model assumes that individuals have a risk-free certain income source (a pension), whereas many US citizens over 60 years old are still active in the labour market and therefore also rely on uncertain flows of incomes. The overall correlation between the actual and simulated age profiles is 0.48.

[8]In the HRS-CAMS data set, medical expenditure is defined as the total out-of-pocket medical expenditure in the reference period.

[9]Net financial wealth is defined as the sum of financial assets—including the net value of IRAs, Keogh accounts, stocks, mutual funds, investment trusts, checking, savings, money market accounts, certificates of deposit (CDs), government bonds, and T-bills, as well as other bonds and bond funds—the net of nonmortgage debts.

[10]Income reflects the total income from the previous calendar year and is calculated as the sum of respondent and spouse earnings; pensions and annuities; SSI and Social Security Disability; Social Security retirement benefits; unemployment and workers' compensation; other government transfers; household capital income; and other income sources.

[11]Based on the HRS-CAMS data set, the risky assets include the net value of stocks, mutual funds, investment trusts, the net value of bonds and bond funds, and a portion of the net value of IRA and Keogh accounts.

[12]In the HRS-CAMS data set, the total consumption is calculated by summing up all the expenditures, encompassing expenses related to durables, non-durables, transportation, and housing.

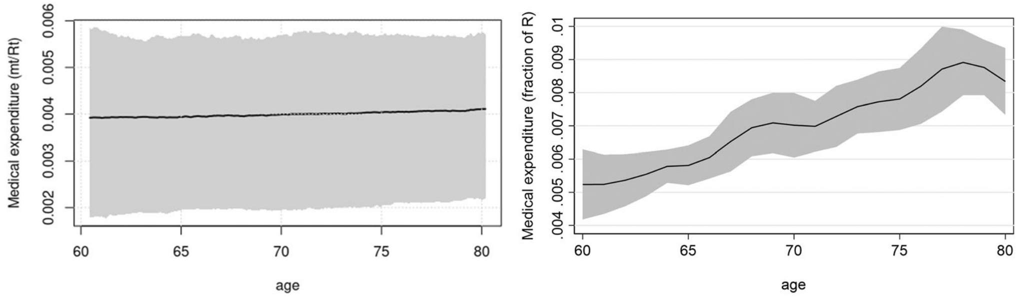


**FIGURE 1** Medical expenditure as percentage of wealth. Simulated and real data.

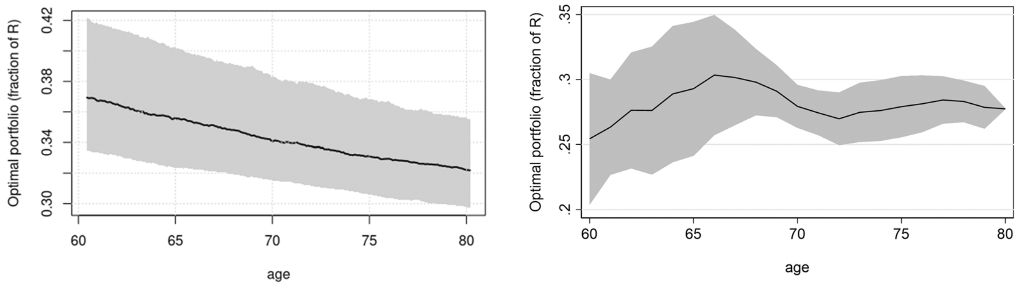


**FIGURE 2** Investment in the risky asset as percentage of wealth. Simulated and real data.

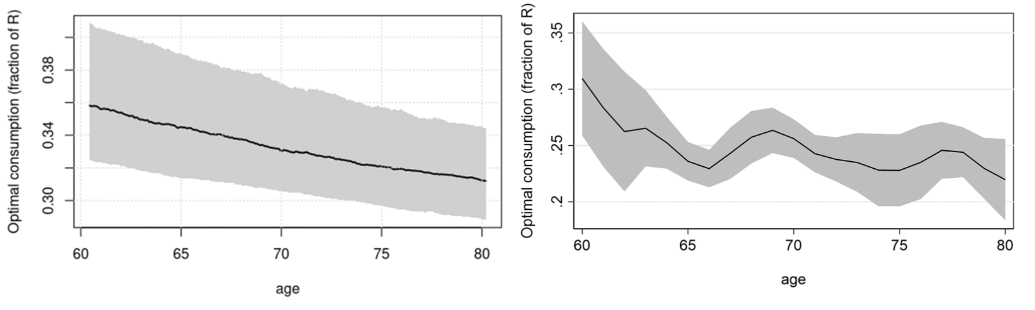


**FIGURE 3** Consumption as percentage of wealth. Simulated and real data.

Unsurprisingly, the simulated portfolio and consumption age profiles are smoother than the actual, micro-funded profiles, but we still consider the correlation satisfactory.

On average, apart from the portfolio decision, whose volatility is a bit higher than what our model predicts, our model allows us to represent the actual households' optimal behavior.[13]

[13]The less satisfactory fit with portfolio data may be due to the fact that this choice may also be affected by unforeseen gains and losses in the financial market.

## 6 | POLICY SIMULATIONS

In this section, we show how the optimal solutions (14)–(16) would react to a change in some policy parameters.

As shown in Section 4, an increase in $\eta$, the mark-up on LTC insurance, always causes a reallocation in the portfolio for individuals, which reduces their risky assets. The effect on LTC insurance is less straightforward, as it depends on variables such as income, wealth, and health care expenditure. In what follows, we simulate the effects of a change in $\eta$, stratifying households according to their wealth quartile.

These simulations are based on the HRS data presented in Table 1, which illustrates the median values of the income, net financial wealth, and income/wealth ratio of the households by net financial wealth quartiles.

As expected, the median income tends to increase with growing wealth, moving from a median value of $19,820 for those in the first wealth quartile to $50,289 for those in the fourth quartile. The income-to-wealth ratio decreases when transitioning from the first to the fourth financial wealth quartile (from 27.47 to 0.13). Notably, the impact of a health shock that causes an increase of $5418 is particularly pronounced for households with low financial wealth, specifically those in the first quartile. For these households, the impact of a health shock is nearly five times their total wealth; conversely, the impact of a health shock on financial wealth is nearly negligible for wealthier households.

Let us now examine the simulations of the impact of a change in the mark-up and in the interest rate on the optimal choices. An increase in the price for LTC insurance reduces consumption, as shown in Figure 4.

A marginal decrease of the mark-up (Figure 5) produces a reduction in consumption for all the wealth quartiles ($1200–1300 per year). However, the impact in relative terms is quite heterogeneous: about 6% of the median income for the lowest wealth quartile and 2.4% for the wealthiest. The impact of a change in the LTC insurance premium is nearly constant across ages, ranging from approximately $1290 for a 60-year-old in the first quartile to $1220 for an 80-year-old in the fourth quartile.

Figure 5 shows (on the left-hand side of the diagram) the effect of a marginal change in $\eta$ on the optimal insurance coverage $h_t^*$ for different wealth quantiles. The derivative is always negative, but the effect is definitely higher (in absolute value) for people at the top end of the wealth distribution and for those that are older. An increase in $\eta$ by one point has a very small effect on the LTC coverage of individuals aged 60 in the first three quartiles. This means that policies aimed at reducing the mark-up (a reduction in $\eta$) have a strong pro-rich effect, which increases as the age increases. Figure 5 (right-hand side) shows that the effect of an increase in the interest rate is similar to an increase in the mark-up: the effect is significant for the richest individuals and is (in absolute value) higher for older individuals. This result has interesting policy implications: incentives to reduce the mark-up (which are usually quite costly for the government) have roughly the same effect as a reduction in the interest rate.

Let us then examine how changes in the risk-free rate alter the portfolio allocation. As expected, a higher risk-free rate implies a reduction in risky asset investment, but the heterogeneity of this reduction is rather high, with poorer and younger individuals being the most reactive.

Figure 6

For the other variables, the variance becomes higher as individuals get older, while in this case, the absolute values of the derivatives converge as the age increases. The intuition for

**TABLE 1** Parameters for simulations for wealth quartiles.

| Quartile of net financial wealth | Net financial wealth (median value) | Income (median value) | Income/net financial wealth (median value) | Jump medical expenditure ($5,418)/net financial wealth (median value) |
|---|---|---|---|---|
| I | $823.2 | $19,819.8 | 27.47 | 4.920 |
| II | $16,904.39 | $31,422.06 | 1.92 | 0.210 |
| III | $85,670 | $37,580.26 | 0.44 | 0.041 |
| IV | $339,226.5 | $50,289.36 | 0.13 | 0.010 |
| Total | $40,737 | $32,686.09 | 0.91 | 0.086 |

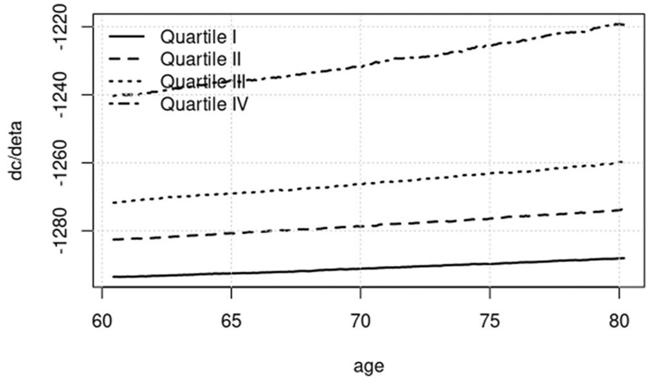


**FIGURE 4** Effect of marginal change in $\eta$ on optimal consumption.

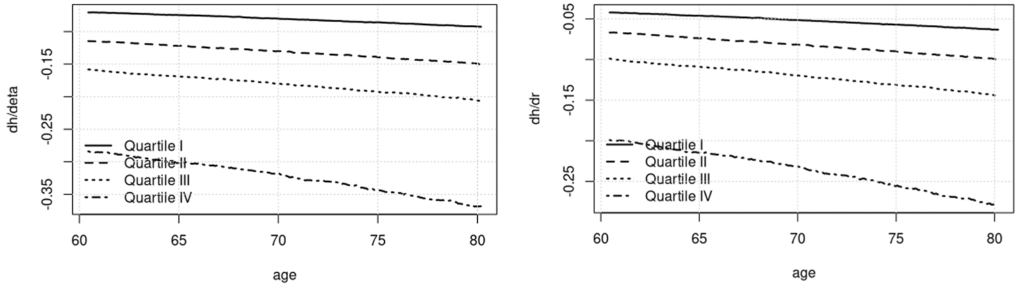


**FIGURE 5** Effect of marginal change on optimal insurance coverage for $\eta$ (left) and r (right).

this result is that at the early stage of the accumulation process, a change in the interest rate causes an important reallocation between financial assets because the individual is at the start of this accumulation process. Later in life, this effect is less pronounced because most of the resources that the individual wants to obtain as financial assets have already been accumulated.

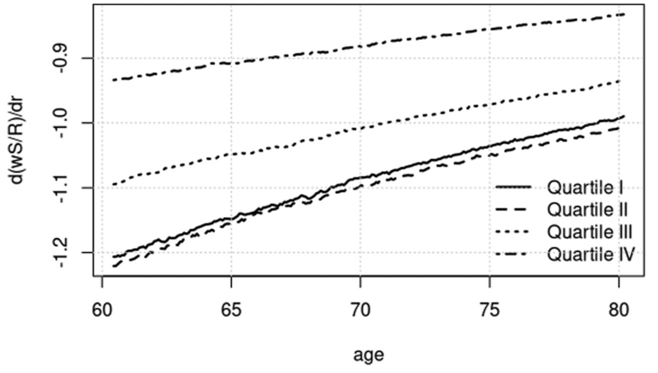


FIGURE 6 Effect of marginal change in $r$ on optimal risky asset allocation.

## 7 | DISCUSSION AND CONCLUSIONS

The increasing demand for health care and LTC, coupled with the financial vulnerability of older persons, creates a significant financial burden for elderly people whose needs are not supported by the social protection system. This raises questions about health care expenditure sustainability (Chidambaram & Burns, 2024). LTC insurance is not widespread, even in countries where care is mostly financed by private health insurance. The literature has tried to explain the reasons for this phenomenon (Brown & Finkelstein, 2007, 2009, 2011) without reaching firm conclusions.

In this paper, we study the decision to sign an LTC insurance contract, along with optimal consumption and investment decisions, within a model where information is complete, albeit stochastic. The life cycle approach we implement allows us to consider the direct and indirect determinants of this decision. Our model shows that while the mark-up on the insurance is an important deterrent in insurance coverage, the degree of risk aversion and the interest rate play an important role, which changes over the individual's lifetime. This result has important policy implications that derive from our more general approach. In a framework where whether to purchase LTC insurance is the only decision, the interest rate affects this choice only through the discount rate; when portfolio and investment decisions are taken into account, the interest rate, by changing the expected wealth, becomes an important determinant in LTC insurance choices.

As one might expect, private coverage and savings are substitute means to finance LTC expenditure, but our model shows that the substitution rate increases with the individual's age. This means that external shocks on both the determinants of LTC expenditure and wealth allocation determine LTC coverage. Finally, in response to a health shock requiring more LTC, an individual prefers to smooth the consumption pattern rather than wealth, in line with the life-cycle model.

From a policy point of view, we show that the mark-up changes the price of LTC coverage and changes the income and savings decision, but a similar effect is produced by a change in the interest rate because it alters the portfolio composition, which in turn changes LTC coverage and consumption. Age and income significantly affect these responses.

An increase in the mark-up leads to a reduction in the LTC insurance coverage; this reduction is higher for high-income individuals and increases with age. As shown in Figure 5, the difference is lower at the early stage of the life path considered. The same increase in the mark-up causes a reduction that is 10 points higher for older individuals in the highest income quartile (−0.35 vs. −0.25), while the effect is less significant for the other quartiles.[14] Income is the main driver for this behavioural change. At 60, an individual in the first quartile reduces their coverage by about −0.05, while in the fourth quartile, this value is about −0.25. An increase in the interest rate creates the same qualitative effect, although it is a bit smaller. Thus, any policy aimed at reducing the mark-up could be offset by an increase in the interest rate and may become a pro-rich policy.

An increase in the mark-up also reduces consumption, but the age effect is again quite interesting: for older and low-income groups, the effect may become negligible as the age increases. Furthermore, a change in the riskless interest rate has a significant effect on the portfolio allocation, which decreases for older agents.

Our model shows that LTC insurance decisions have a broader dimension than simply the insurance market and the price of insurance policies. Given the impact that LTC expenses have on the lifespan, they encompass all the decisions individuals make. For this reason, policies aimed at increasing coverage should be designed accordingly. The first policy implication is that subsidies and incentives are more effective if they are means-tested and age-targeted. Furthermore, while fiscal policies may improve coverage (with some form of financial subsidy or tax relief), the government should also take into account the effect of monetary policies, especially those deemed to change the interest rate, when designing incentives to promote LTC. In fact, we have shown that such incentives may be offset by a change in the interest rate.

Our model could be extended in several directions: for example, it is possible to make the life duration stochastic and also to consider a more complete range of consumer investments, for example, pension schemes, and a wider range of assets.

## AUTHOR CONTRIBUTIONS

**Lucia Leporatti**: Conceptualisation, methodology, data collection, formal analysis, writing. **Rosella Levaggi**: Conceptualisation, methodology, formal analysis, writing. **Francesco Menoncin**: Conceptualisation, methodology, data collection, formal analysis, writing. **Raffaele Miniaci**: Conceptualisation, methodology, data collection, formal analysis, writing.

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## CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

## DATA AVAILABILITY STATEMENT

Data derived from public domain resources.

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[14]For example, for individuals in the third quartile, the change is from −0.20 to −0.17.

## REFERENCES

American Association for Long-Term Care Insurance. (2021). Long-term care insurance facts. Technical report, American Association for Long-Term Care Insurance.

American Association for Long-Term Care Insurance. (2023). Long-term care insurance facts. Technical report, American Association for Long-Term Care Insurance.

Ameriks, J., Briggs, J., Caplin, A., Shapiro, M. D., & Tonetti, C. (2020). Long-term-care utility and late-in-life saving. *Journal of Political Economy*, *128*(6), 2375–2451.

Atella, V., Belotti, F., Kim, D., Goldman, D., Gracner, T., Piano Mortari, A., & Tysinger, B. (2021). The future of the elderly population health status: Filling a knowledge gap. *Health Economics*, *30*(S1), 11–29.

Bernet, P. M. (2004). The influence of prescription drug use on long-term care insurance ownership. *Risk Management and Insurance Review*, *7*(2), 107–120.

Boyer, M., De Donder, P., Fluet, C., Leroux, M.-L., & Michaud, P.-C. (2019). Long-term care risk misperceptions. *The Geneva Papers on Risk and Insurance—Issues and Practice*, *44*(2), 183–215.

Braun, R. A., Kopecky, K. A., & Koreshkova, T. (2019). Old, frail, and uninsured: Accounting for features of the US long?term care insurance market. *Econometrica*, *87*(3), 981–1019.

Brown, J. H. (2023). The impact of a long-term care information campaign on insurance coverage. *Journal of Health Economics*, *92*, 102822.

Brown, J. R., & Finkelstein, A. (2007). Why is the market for long-term care insurance so small? *Journal of Public Economics*, *91*(10), 1967–1991.

Brown, J. R., & Finkelstein, A. (2009). The private market for long-term care insurance in the United States: A review of the evidence. *Journal of Risk and Insurance*, *76*(1), 5–29.

Brown, J. R., & Finkelstein, A. (2011). Insuring long-term care in the United States. *Journal of Economic Perspectives*, *25*(4), 119–142.

Bueren, J. (2023). Long-term care needs and savings in retirement. *Review of Economic Dynamics*, *49*, 201–224.

Chidambaram, P., & Burns, A. (2024). 10 things about long-term services and supports (LTSS). Technical report, Kaiser Fund.

Cohen, M., Feder, J., & Favreault, M. M. (2018). A new public-private partnership: Catastrophic public and front-end private LTC insurance. Technical report, Urban Institute.

Colombo, F., Llena-Nozal, A., Mercier, J., & Tjadens, F. (2011). *Help Wanted?: Providing and Paying for Long-Term Care*. OECD Health Policy Studies. OECD.

Crainich, D., Eeckhoudt, L., & Courtois, O. L. (2017). Health and portfolio choices: A diffidence approach. *European Journal of Operational Research*, *259*(1), 273–279.

Davidoff, T. (2010). Home equity commitment and long-term care insurance demand. *Journal of Public Economics*, *94*(1–2), 44–49.

De Donder, P., Leroux, M.-L., & Salanié, F. (2023). Advantageous selection without moral hazard. *Journal of Risk and Uncertainty*, *67*(1), 21–43.

Doerpinghaus, H. I., & Gustavson, S. G. (2002). Long-term care insurance purchase patterns. *Risk Management and Insurance Review*, *5*(1), 31–43.

Dong, J., Smieliauskas, F., & Konetzka, R. T. (2018). Effects of long-term care insurance on financial well-being. *The Geneva Papers on Risk and Insurance - Issues and Practice*, *44*(2), 277–302.

Einav, L., & Finkelstein, A. (2023). Empirical analyses of selection and welfare in insurance markets: a self-indulgent survey. *The Geneva Risk and Insurance Review*, *48*(2), 167–191.

Eling, M., & Ghavibazoo, O. (2018). Research on long-term care insurance: status quo and directions for future research. *The Geneva Papers on Risk and Insurance - Issues and Practice*, *44*(2), 303–356.

Eling, M., Ghavibazoo, O., & Hanewald, K. (2021). Willingness to take financial risks and insurance holdings: A European survey. *Journal of Behavioral and Experimental Economics*, *95*, 101781.

Eurostat. (2022). Disability statistics—Elderly needs for help or assistance. Technical report, Eurostat.

Eurostat. (2023). Healthy life years statistics. Techreport, Eurostat.

Eurostat. (2024). Household composition statistics. Technical report, Eurostat.

Hashiguchii, T. C. O., & Llena-Nozali, A. (2020). The effectiveness of social protection for long-term care in old age. OECD Health Working Papers 117, OECD.

He, Y., & Gokhale, J. (2022). Projecting Medicaida's long-term care expenditures. Technical report, Penn Warton Budget Model.

Kotschy, R., & Bloom, D. E. (2022). A comparative perspective on long-term care systems. *International Social Security Review*, *75*(3–4), 47–69.

Lambregts, T. R., & Schut, F. T. (2020). Displaced, disliked and misunderstood: A systematic review of the reasons for low uptake of long-term care insurance and life annuities. *The Journal of the Economics of Ageing*, *17*, 100236.

Lockwood, L. M. (2018). Incidental bequests and the choice to self-insure late-life risks. *American Economic Review*, *108*(9), 2513–2550.

NAIC. (2023). Long-term care insurance experience reports for 2022. Technical report, National Association of Insurance Commissioners.

Nordman, E. C. (2016). The state of long-term care insurance: The market, challenges and future innovations. Technical report, National Insurance of Insurance Commissioner.

OECD. (2024). *Is care affordable for older people?* OECD.

Pauly, M. V. (1990). The rational nonpurchase of long-term-care insurance. *Journal of Political Economy*, *98*(1), 153–168.

Rieger-Fels, M. (2024). Why do people buy insurance? A modern answer to an old question. *Risk Management and Insurance Review*, *27*(1), 89–114.

Sloan, F. A., & Norton, E. C. (1997). Adverse selection; bequests, crowding out, and private demand for insurance: Evidence from the long-term care insurance market. *Journal of Risk and Uncertainty*, *15*(3), 201–219.

Zweifel, P., & Strowe, W. (1996). Long-term care insurance and bequests as instruments for shaping inter-generational relationships. *Journal of Risk and Uncertainty*, *12*(1), 65–76.

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