URSI-B EMTS 2025, Bologna, 23–27 June

Analytical evaluation of the effects on the electromagnetic field induced by a moving dielectric slab

Riccardo Onesti(1), Mirco Raffetto(1), and Kirill Zeyde*(1)
(1) University of Genoa, Genoa, Italy, https://unige.it

Abstract

A simple one-dimensional electromagnetic problem in the presence of moving objects is studied. The analytical solution is used to investigate the effects of motion on the electromagnetic field. The results show that for this class of one-dimensional problems the effects of motion are weaker than expected. For this reason the simple problem considered in this work could be of interest for investigating the reliability of inverse scattering procedures or for testing numerical methods.

1 Introduction

The interaction of electromagnetic waves with moving media has been investigated several times, due to its theoretical and practical importance [1], [2], [3], [4], [5]. In this paper we analyse such an interaction in a simple case: a one-dimensional, time-harmonic, electromagnetic problem in the presence of a moving slab with stationary boundaries. The slab is illuminated by a plane wave propagating in direction orthogonal to the boundary planes of the slab.

It should be noted that other authors have addressed similar problems [6], [7]. However, this has been done with completely different analyses and targets. In the following, by using well known results due to Tai and Pyati [1], [2], we deduce the analytic solution of the problem of interest. This will allow us to analyze its properties and, in particular, to evaluate the effects of motion on the electromagnetic field in the media at rest around the slab.

The results show that outside the slab the effects of motion are very small, proportional to the square of the speed normalized to that of light (a small quantity whose square is even smaller). Instead, in other problems involving canonical objects in motion [8], [9], [3], at least one component of the field presents a change proportional to the normalized velocity. The effects of interest are then much weaker in the 1D problem considered in this paper than in other canonical cases. This is a very interesting properties because the obtained small effects are of particular interest for testing inverse scattering procedures aiming at the reconstruction of the speed of moving media [8], [9], [10], or new approaches whose goal is to approximate the weak effects of motion on the scattered field [11].

2 Definition of the problem of interest

The problem considered in this paper is shown in figure 1. A moving slab of thickness d is present between two media: the first of these, identified as medium 1, occupies the region $x<0$, while medium 3 is in the domain $x>d$. The media are isotropic, at rest, and are characterized by effective constitutive parameters $\varepsilon_i, \mu_i \in \mathbb{C}$, $i=1,3$. The slab is made up of medium 2, a lossless material having $\varepsilon_2, \mu_2 \in \mathbb{R}$ when it is at rest. In the adopted reference frame medium 2

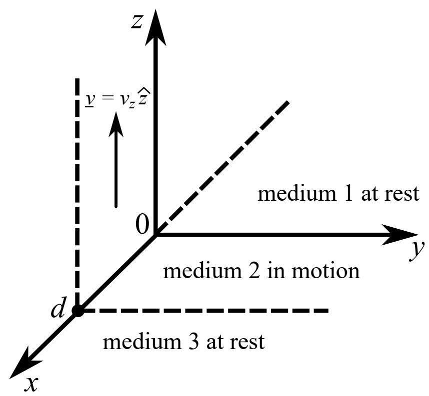

Figure 1. In the problem of interest a lossless slab moves with $\mathbf{v}=v_z\hat{\mathbf{z}}$ and its boundary planes are stationary.

is in uniform motion with velocity $\mathbf{v}=v_z\hat{\mathbf{z}}=\beta c_0\hat{\mathbf{z}}$, $v_z\in\mathbb{R}$. The interfaces between different media are then stationary.

The problem of interest is to find the electromagnetic field $(\mathbf{E},\mathbf{H})$ when the incident field is a progressive wave propagating in medium 1 along the x axis.

If we denote by $(\mathbf{E}_0,\mathbf{H}_0)$ the electromagnetic field obtained under the same illumination when $v_z=0$, the field which represents the effects induced by the motion of the slab is given by $(\mathbf{E}-\mathbf{E}_0,\mathbf{H}-\mathbf{H}_0)$ [11].

3 Mathematical formulation

For the moving slab we adopted the same conventions of Tai [1]. It is well known that, in the reference frame in which media 1 and 3 are at rest, medium 2 is perceived as bianisotropic when $v_z\neq 0$. Its constitutive relations are [1]:

$$\mathbf{D}=\underline{\underline{\varepsilon}}\mathbf{E}+\Omega v_z(\hat{\mathbf{z}}\times\mathbf{H}) \tag{1}$$

$$\mathbf{B} = -\Omega v_z(\hat{\mathbf{z}} \times \mathbf{E}) + \underline{\underline{\mu}}\mathbf{H}, \tag{2}$$

where

$$\underline{\underline{\varepsilon}} = \varepsilon_2 \underline{\underline{M}}, \quad \underline{\underline{\mu}} = \mu_2 \underline{\underline{M}}, \quad n_i = c_0\sqrt{\mu_i \varepsilon_i},\ i = 1,2,3, \tag{3}$$

$$\Omega = \frac{n_2^2 - 1}{c_0^2(1 - n_2^2\beta^2)}, \tag{4}$$

$$\underline{\underline{M}} = \begin{bmatrix} 1+\Omega v_z^2 & 0 & 0 \\ 0 & 1+\Omega v_z^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

The solution for the case $v_z = 0$ is well known [4] (section 9.2.1); we simply recall the most important points for the next developments. For a general polarization of the incident field we have:

$$E_{0x} = 0 = H_{0x}, \quad \forall x \in \mathbb{R}, \tag{6}$$

$$E_{0\alpha}|_{medium\ i} = A_{0pi\alpha}e^{-jk_{i0}x} + A_{0ri\alpha}e^{jk_{i0}x},\ i = 1,2,3, \tag{7}$$

$$H_{0\gamma}|_{medium\ i} = \pm Y_{i0}A_{0pi\alpha}e^{-jk_{i0}x} \mp Y_{i0}A_{0ri\alpha}e^{jk_{i0}x},\ i = 1,2,3, \tag{8}$$

where $\alpha = y,z$ and $\gamma = z$ and the signs are the upper ones (respectively, $\gamma = y$ and lower signs) when $\alpha = y$ (respectively, $\alpha = z$). As usual $k_{i0} = \omega\sqrt{\mu_i\varepsilon_i} = \frac{\omega}{c_0}n_i$, $Y_{i0} = \sqrt{\frac{\varepsilon_i}{\mu_i}} = \frac{k_{i0}}{\omega\mu_i} = \frac{\omega\varepsilon_i}{k_{i0}}$. Moreover, $A_{0p1\alpha}$, $\alpha = y,z$, will be known coefficients given by the incident field and $A_{0r3\alpha} = 0$, $\alpha = y,z$, since there is no regressive wave in medium 3.

Thus $(\mathbf{E}_0, \mathbf{H}_0)$ is determined by the unknown coefficients $\mathbf{x}_{0\alpha} = (A_{0r1\alpha}, A_{0p2\alpha}, A_{0r2\alpha}, A_{0p3\alpha})$, $\alpha = y,z$. They can be found by solving two uncoupled linear algebraic systems of equations: the first one is obtained from the continuity of E_y and H_z at the interfaces $x = 0$ and $x = d$, while the second can be deduced from the continuity of E_z and H_y at the same planes.

The systems have the following form (the known coefficients are written in the right-hand side), for $\alpha = y,z$:

$$\begin{cases} -A_{0r1\alpha} + A_{0p2\alpha} + A_{0r2\alpha} = A_{0p1\alpha} \\ \pm Y_{10}A_{0r1\alpha} \pm Y_{20}A_{0p2\alpha} \mp Y_{20}A_{0r2\alpha} = \pm Y_{10}A_{0p1\alpha} \\ A_{0p2\alpha}e^{-jk_{20}d} + A_{0r2\alpha}e^{jk_{20}d} - A_{0p3\alpha}e^{-jk_{30}d} = 0 \\ \pm Y_{20}A_{0p2\alpha}e^{-jk_{20}d} \mp Y_{20}A_{0r2\alpha}e^{jk_{20}d} \\ \qquad \mp Y_{30}A_{0p3\alpha}e^{-jk_{30}d} = 0. \end{cases} \tag{9}$$

The above system of equations can be written as

$$\underline{\underline{A}}_{0\alpha}\mathbf{x}_{0\alpha} = \mathbf{b}_\alpha, \quad \alpha = y,z. \tag{10}$$

From equation (9) we deduce:

$$\underline{\underline{A}}_{0\alpha} = \begin{bmatrix} -1 & 1 & 1 & 0 \\ \pm Y_{10} & \pm Y_{20} & \mp Y_{20} & 0 \\ 0 & e^{-jk_{20}d} & e^{jk_{20}d} & -e^{-jk_{30}d} \\ 0 & \pm Y_{20}e^{-jk_{20}d} & \mp Y_{20}e^{jk_{20}d} & \mp Y_{30}e^{jk_{30}d} \end{bmatrix}. \tag{11}$$

When $v_z \neq 0$ the expressions of $\mathbf{E}$ and $\mathbf{H}$ in media 1 and 3, which remain at rest, retain the same form. Then we consider again the expressions defined in equations (6), (7) and (8). However, we drop the subscript "0" from all symbols. Since we consider the same incident field, we have $A_{p1\alpha} = A_{0p1\alpha}$, $\alpha = y,z$. Moreover, for the same reason as above, $A_{r3\alpha} = 0$, $\alpha = y,z$.

The wavenumber k_2 of the plane wave in medium 2 when $v_z \neq 0$ is given by the following dispersion equation [2]:

$$k_2^2(1+\Omega v_z^2) = \frac{\omega^2}{c_0^2}n_2^2(1+\Omega v_z^2)^2 - \omega^2\Omega^2 v_z^2. \tag{12}$$

After a few calculations one gets

$$k_2 = \frac{\omega}{c_0}\sqrt{\frac{n_2^2 - \beta^2}{1 - \beta^2}}, \tag{13}$$

which gives $k_2 > 0$ for all cases of interest. The negative value of k_2 has been neglected on purpose because the two signs are explicitly considered in the propagation factors, as usual (see, for example equations (14) and (15)).

Equation (13) together with the following expressions

$$\mathbf{E}|_{medium\ 2} = \mathbf{A}_{p2}e^{-jk_2x} + \mathbf{A}_{r2}e^{jk_2x}, \tag{14}$$

$$\mathbf{H}|_{medium\ 2} = \mathbf{B}_{p2}e^{-jk_2x} + \mathbf{B}_{r2}e^{jk_2x}, \tag{15}$$

$$\mathbf{A}_{p2} = -\frac{\omega\Omega v_z}{(1+\Omega v_z^2)k_2}A_{p2z}\hat{\mathbf{x}} + A_{p2y}\hat{\mathbf{y}} + A_{p2z}\hat{\mathbf{z}}, \tag{16}$$

$$\mathbf{A}_{r2} = \frac{\omega\Omega v_z}{(1+\Omega v_z^2)k_2}A_{r2z}\hat{\mathbf{x}} + A_{r2y}\hat{\mathbf{y}} + A_{r2z}\hat{\mathbf{z}}, \tag{17}$$

$$\mathbf{B}_{p2} = -\frac{\Omega v_z}{(1+\Omega v_z^2)\mu_2}A_{p2y}\hat{\mathbf{x}} - Y_{2z}A_{p2z}\hat{\mathbf{y}} + Y_{2y}A_{p2y}\hat{\mathbf{z}}, \tag{18}$$

$$\mathbf{B}_{r2} = -\frac{\Omega v_z}{(1+\Omega v_z^2)\mu_2}A_{r2y}\hat{\mathbf{x}} + Y_{2z}A_{r2z}\hat{\mathbf{y}} - Y_{2y}A_{r2y}\hat{\mathbf{z}}, \tag{19}$$

$$Y_{2y} = \frac{k_2}{\omega\mu_2} > 0, \tag{20}$$

$$Y_{2z} = \frac{\omega\varepsilon_2}{k_2} > 0, \tag{21}$$

define the field of interest. As a matter of fact, the field having the four free complex parameters A_{p2y}, A_{p2z}, A_{r2y} and A_{r2z}, satisfies Maxwell's curl equations and the constitutive relations (1) and (2), as one can directly verify.

The field $(\mathbf{E}, \mathbf{H})$ when $v_z \neq 0$ can be found by enforcing the continuity of the tangential components of $\mathbf{E}$ and $\mathbf{H}$ at the interfaces $x = 0$ and $x = d$, as explained above. The x components then do not play any role in the determination of the solution. Taking account of equations (14)-(19) we can then deduce

$$E_\alpha|_{medium\ 2} = A_{p2\alpha}e^{-jk_2x} + A_{r2\alpha}e^{jk_2x}, \tag{22}$$

$$H_\gamma|_{medium\ 2} = \pm Y_{2\alpha}A_{p2\alpha}e^{-jk_2x} \mp Y_{2\alpha}A_{r2\alpha}e^{jk_2x}, \tag{23}$$

where, once again, $\alpha = y,z$ and $\gamma = z$ and the signs are the upper ones (respectively, $\gamma = y$ and lower signs) when $\alpha = y$

(respectively, $\alpha = z$). The reader should notice the strong analogies and the few differences of the expressions (7)-(8) and (22)-(23).

For $v_z \neq 0$ the unknown coefficients $\mathbf{x}_\alpha = (A_{r1\alpha}, A_{p2\alpha}, A_{r2\alpha}, A_{p3\alpha})$, $\alpha = y,z$, and then $(\mathbf{E},\mathbf{H})$, are determined by the following algebraic systems, for $\alpha = y,z$:

$$\begin{cases} -A_{r1\alpha} + A_{p2\alpha} + A_{r2\alpha} = A_{p1\alpha} = A_{0p1\alpha} \\ \pm Y_{10} A_{r1\alpha} \pm Y_{2\alpha} A_{p2\alpha} \mp Y_{2\alpha} A_{r2\alpha} = \pm Y_{10} A_{p1\alpha} = \\ \qquad \pm Y_{10} A_{0p1\alpha} \\ A_{p2\alpha} e^{-jk_2 d} + A_{r2\alpha} e^{jk_2 d} - A_{p3\alpha} e^{-jk_{30} d} = 0 \\ \pm Y_{2\alpha} A_{p2\alpha} e^{-jk_2 d} \mp Y_{2\alpha} A_{r2\alpha} e^{jk_2 d} \\ \qquad \mp Y_{30} A_{p3\alpha} e^{-jk_{30} d} = 0. \end{cases} \tag{24}$$

With matrix and vector notation, the above equation becomes

$$\underline{\underline{A}}_\alpha \mathbf{x}_\alpha = \mathbf{b}_\alpha, \quad \alpha = y,z. \tag{25}$$

The vectors $\mathbf{b}_\alpha$, $\alpha = y,z$, are unchanged. The structure of the matrices $\underline{\underline{A}}_\alpha$ is the same as that of $\underline{\underline{A}}_{0\alpha}$ but the last three numbers of the second and third columns are different. In particular, we have:

$$\underline{\underline{A}}_\alpha = \begin{bmatrix} -1 & 1 & 1 & 0 \\ \pm Y_{10} & \pm Y_{2\alpha} & \mp Y_{2\alpha} & 0 \\ 0 & e^{-jk_2 d} & e^{jk_2 d} & -e^{-jk_{30} d} \\ 0 & \pm Y_{2\alpha} e^{-jk_2 d} & \mp Y_{2\alpha} e^{jk_2 d} & \mp Y_{30} e^{jk_{30} d} \end{bmatrix}. \tag{26}$$

4 Estimate of the effects of motion

The velocity of a moving medium is usually determined in a very reliable way from the Doppler frequency shift [4] (p. 955) between incident and reflected waves. However, for moving media with stationary boundaries the frequency shift is not present and, in these cases, the estimate of the velocity of moving objects is difficult [8], [9]. Inverse scattering techniques can be used to solve these challenging problems. They exploit the properties of the difference field $(\mathbf{E} - \mathbf{E}_0, \mathbf{H} - \mathbf{H}_0)$ outside the scatterers. The most difficult problems are those of major interest for engineers because for non-relativistic velocity values ($|v_z| << c_0, |\beta| << 1$) the effects of motion are usually very small [9].

For the indicated reasons, for the class of problems considered in this work, we are particularly interested in the behavior of the difference field in media 1 and 3, when $|\beta|$ is much smaller than 1.

From equations (6), (7) and (8) and the considerations below equation (11) one deduces

$$E_x - E_{0x} = 0 = H_x - H_{0x}, \quad \forall x \in (-\infty, 0) \cup (d, +\infty), \tag{27}$$

$$E_\alpha - E_{0\alpha}|_{medium\ 1} = (A_{r1\alpha} - A_{0r1\alpha}) e^{jk_{10} x}, \ \alpha = y,z, \tag{28}$$

$$E_\alpha - E_{0\alpha}|_{medium\ 3} = (A_{p3\alpha} - A_{0p3\alpha}) e^{-jk_{30} x}, \ \alpha = y,z, \tag{29}$$

$$H_\gamma - H_{0\gamma}|_{medium\ 1} = \mp Y_{10} (A_{r1\alpha} - A_{0r1\alpha}) e^{jk_{10} x}, \tag{30}$$

$$H_\gamma - H_{0\gamma}|_{medium\ 3} = \pm Y_{30} (A_{p1\alpha} - A_{0p1\alpha}) e^{-jk_{30} x}, \tag{31}$$

where, as usual, $\gamma = z$ and the signs are the upper ones (respectively, $\gamma = y$ and lower signs) when $\alpha = y$ (respectively, $\alpha = z$).

It is now clear that the difference vectors $\mathbf{x}_\alpha - \mathbf{x}_{0\alpha}$ control the magnitude of the difference field outside the slab. In order to estimate the difference vectors we have to find some properties of $\underline{\underline{A}}_{0\alpha}^{-1}$ and $\underline{\underline{A}}_\alpha^{-1}$. We can compute $\underline{\underline{A}}_{0\alpha}^{-1}$ because the analytic solution when the slab is at rest is well known [4]. As for $\underline{\underline{A}}_\alpha$, $\alpha = y,z$, the difference matrix $\underline{\underline{A}}_\alpha - \underline{\underline{A}}_{0\alpha}$ has the first row and the first and last columns equal to zero, as one can easily check from equations (11) and (26). For space reason in the following formula we show just the non-zero entries of the difference matrix; in particular the last three numbers of the second and third columns are:

$$\begin{bmatrix} \pm(Y_{2\alpha} - Y_{20}) & \mp(Y_{2\alpha} - Y_{20}) \\ e^{-jk_2 d} - e^{-jk_{20} d} & e^{jk_2 d} - e^{jk_{20} d} \\ \pm(Y_{2\alpha} e^{-jk_2 d} - Y_{20} e^{-jk_{20} d}) & \mp(Y_{2\alpha} e^{jk_2 d} - Y_{20} e^{jk_{20} d}) \end{bmatrix}. \tag{32}$$

We have to estimate the entries appearing in the submatrix shown in equation (32) to complete our analysis. The crucial work is that on the wavenumber k_2 as a function of β. By equation (13), $k_2(\beta)|_{\beta=0} = k_{20}$. Moreover, $k_2(\beta)$ depends on β^2 and we can find a neighborhood of $\beta = 0$ (by the way, the values of interest for engineering applications) in which $k_2(\beta)$ is an analytic function of β. Finally, due to the presence of just β^2 terms, it is an even function of β. Thus, after a few calculations one can find the following second-degree Maclaurin polynomial, which provides a very good approximation for the indicated values of β:

$$k_2(\beta) \simeq k_{20} + k_{20} \frac{n_2^2 - 1}{2n_2^2} \beta^2. \tag{33}$$

With this result one easily gets:

$$Y_{2y} = \frac{k_2}{\omega \mu_2} \simeq \frac{k_{20}}{\omega \mu_2} + \frac{k_{20}}{\omega \mu_2} \frac{n_2^2 - 1}{2n_2^2} \beta^2 = Y_{20} + Y_{20} \frac{n_2^2 - 1}{2n_2^2} \beta^2, \tag{34}$$

$$Y_{2z} = \frac{\omega \varepsilon_2}{k_2} \simeq Y_{20} - Y_{20} \frac{n_2^2 - 1}{2n_2^2} \beta^2, \tag{35}$$

$$e^{\pm jk_2 d} \simeq e^{\pm jk_{20} d} e^{\pm jk_{20} \frac{n_2^2 - 1}{2n_2^2} \beta^2 d}. \tag{36}$$

Then, neglecting all fourth-order terms one deduces

$$Y_{2\alpha} - Y_{20} \simeq \pm Y_{20} \frac{n_2^2 - 1}{2n_2^2} \beta^2, \quad \alpha = y,z, \tag{37}$$

$$e^{\pm jk_2 d} - e^{\pm jk_{20} d} \simeq \tag{38}$$
$$e^{\pm jk_{20} d} (e^{\pm jk_{20} \frac{n_2^2 - 1}{2n_2^2} \beta^2 d} - 1) = \pm jk_{20} d \frac{n_2^2 - 1}{2n_2^2} e^{\pm jk_{20} d} \beta^2,$$

$$Y_{2\alpha} e^{\pm jk_2 d} - Y_{20} e^{\pm jk_{20} d} \simeq \tag{39}$$
$$\pm Y_{20} \frac{n_2^2 - 1}{2n_2} e^{\pm jk_{20} d} (jk_{20} d + 1) \beta^2, \quad \alpha = y,z.$$

As a consequence, by using equations (32), (37), (38) and (39), the magnitudes of all entries of $\underline{\underline{A}}_\alpha - \underline{\underline{A}}_{0\alpha}$ are controlled by β^2. Then, with a very good approximation for small values of $|\beta|$

$$\underline{\underline{A}}_\alpha = \underline{\underline{A}}_{0\alpha} - \beta^2 \underline{\underline{C}} \tag{40}$$

for some matrix $\underline{\underline{C}}$ independent of β (the structure of this matrix is by definition the same as that of $\underline{\underline{A}}_\alpha - \underline{\underline{A}}_{0\alpha}$).

With this form of the matrix $\underline{\underline{A}}_\alpha$, from equation (25) we get

$$\mathbf{b}_\alpha = \underline{\underline{A}}_\alpha \mathbf{x}_\alpha = (\underline{\underline{A}}_{0\alpha} - \beta^2 \underline{\underline{C}})\mathbf{x}_\alpha \quad \alpha = y, z, \tag{41}$$

and then, by the invertibility of $\underline{\underline{A}}_{0\alpha}$

$$\mathbf{x}_{0\alpha} = \underline{\underline{A}}_{0\alpha}^{-1} \mathbf{b}_\alpha = (\underline{\underline{I}} - \beta^2 \underline{\underline{A}}_{0\alpha}^{-1} \underline{\underline{C}})\mathbf{x}_\alpha \quad \alpha = y, z. \tag{42}$$

The matrix $\underline{\underline{A}}_{0\alpha}^{-1} \underline{\underline{C}}$ is independent of β, too, and then one can find an upper bound for $|\beta|$ which guarantees $\lim_{n\to\infty} (\beta^2 \underline{\underline{A}}_{0\alpha}^{-1} \underline{\underline{C}})^n = 0$. This is not a restrictive condition for engineering applications.

Then we can apply the Neumann series [12] (p. 126) to find the inverse matrix of $(\underline{\underline{I}} - \beta^2 \underline{\underline{A}}_{0\alpha}^{-1} \underline{\underline{C}})$.

We get

$$(\underline{\underline{I}} - \beta^2 \underline{\underline{A}}_{0\alpha}^{-1} \underline{\underline{C}})^{-1} = \underline{\underline{I}} + (\beta^2 \underline{\underline{A}}_{0\alpha}^{-1} \underline{\underline{C}}) + \\ + (\beta^2 \underline{\underline{A}}_{0\alpha}^{-1} \underline{\underline{C}})^2 + \ldots \simeq \underline{\underline{I}} + \beta^2 \underline{\underline{A}}_{0\alpha}^{-1} \underline{\underline{C}}. \tag{43}$$

and then

$$\mathbf{x}_\alpha \simeq \mathbf{x}_{0\alpha} + \beta^2 \underline{\underline{A}}_{0\alpha}^{-1} \underline{\underline{C}} \mathbf{x}_{0\alpha}, \quad \alpha = y, z, \tag{44}$$

so that

$$\mathbf{x}_\alpha - \mathbf{x}_{0\alpha} \simeq \beta^2 \underline{\underline{A}}_{0\alpha}^{-1} \underline{\underline{C}} \mathbf{x}_{0\alpha}, \quad \alpha = y, z. \tag{45}$$

Together with our previous deductions, this clearly shows that all components of the difference field outside the moving slab have magnitudes controlled by β^2. Thus, the effects of motion outside the slab are very weak, being of the second order.

It is important to observe that in other canonical problems involving media in motion a first order effect is always present [11], [3] to the best of authors' knowledge. The particular features of the simple problem considered in this work could be of interest for investigating the reliability of inverse scattering procedures aiming at the reconstruction of velocity profiles [8], [10] or the capabilities of new approaches to approximate the difference field in the presence of moving media [11].

References

[1] C. T. Tai, "The dyadic Green"s function for a moving isotropic medium", *IEEE Transactions on Antennas and Propagation*, March 1965, vol. 13, no. 2, pp. 322-323, doi: 10.1109/TAP.1965.1138414.

[2] V. P. Pyati, "Reflection and refraction of electromagnetic waves by a moving dielectric medium", *Journal of Applied Physics*, Febuary 1967, vol. 38, no. 2, pp. 652-655, doi: 10.1063/1.1709390.

[3] D. De Zutter, "Scattering by a rotating dielectric sphere", *IEEE Transactions on Antennas and Propagation*, September 1980, vol. 28, no. 5, pp. 643-651, doi: doi=10.1109/TAP.1980.1142392.

[4] J. G. Van Bladel, "Electromagnetic Fields", =IEEE Press, 2007, 2nd edition, Piscataway, NJ, USA.

[5] M. Brignone and M. Raffetto, "Well posedness and finite element approximability of two-dimensional time-harmonic electromagnetic problems involving non-conducting moving objects with stationary boundaries", *ESAIM: Mathematical Modelling and Numerical Analysis*, July-August 2015, vol. 49, no. 4, pp. 1157-1192, doi: 10.1051/m2an/2015006.

[6] A. Kashaninejad-Rad, A. Abdolali, and M. M. Salary, "Interaction of electromagnetic waves with a moving slab: Fundamental dyadic method", *Progress In Electromagnetics Research B*, 2014, vol. 60, pp. 1-13.

[7] K. Zheng, X. Liu, Z. Mu and G. Wei, "Analysis of scattering fields from moving multilayered dielectric slab illuminated by an impulse source, *IEEE Antennas and Wireless Propagation Letters*, 2017, vol. 16, pp. 2130-2133.

[8] M. Pastorino, M. Raffetto, and A. Randazzo, "Electromagnetic inverse scattering of axially moving cylindrical targets", *IEEE Transactions on Geoscience and Remote Sensing*, March 2015, vol. 53, no. 3, pp. 1452-1462, doi: 10.1109/TGRS.2014.2342933.

[9] M. Brignone, G. L. Gragnani, M. Pastorino, M. Raffetto, and A. Randazzo, "Noise limitations on the recovery of average values of velocity profiles in pipelines by simple imaging systems, *IEEE Geoscience and Remote Sensing Letters*, 2016, vol. 13, no. 9, pp. 1340-1344, doi: 10.1109/LGRS.2016.2584703.

[10] P. Kalarickel Ramakrishnan, M. R. Clemente Vargas, and M. Raffetto, "Electromagnetic inverse scattering of rotating axisymmetric objects, *IEEE Access*, 2021, vol. 9, pp. 168185-168192, doi: 10.1109/ACCESS.2021.3136356.

[11] M. Raffetto, M. R. Clemente Vargas, and K. Zeyde, "A new born-approximation approach to compute the effects of motion on the solution of electromagnetic problems involving moving materials with stationary boundaries", *IET Science, Measurement & Technology*, 2024, vol. 18, no. 5, pp. 245-257, doi: 10.1049/smt2.12184.

[12] C. D. Meyer, "Matrix analysis and applied linear algebra", *SIAM*, 2000.