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High-cycle rotating bending fatigue test of notched specimens to enhance N-SIF based fatigue assessment approaches

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Abstract

This paper presents the results of a relatively wide experimental campaign to complement test data already available in open literature. Fatigue tension tests of square-section notched specimens were carried out a few years ago to support the validation of fatigue assessment approaches based on Notch Stress Intensity Factor (N-SIF), e.g. the Strain Energy Density approach and the Peak Stress Method, although limiting the run-out up to $2 \cdot 10^6$ cycles. Having in mind the target of ships and offshore structures, experiencing during their lives about 10^8 cycles, a few series of rotating bending fatigue tests of variously notched specimens made by different structural steels were carried out up to $5 \cdot 10^7$ cycles as run-out target. About 100 specimens were prepared and tested, resulting into a rather comprehensive data set including also measurements of local notch geometries. It turned out that the effects of the notch geometry in general and that of the notch radius at tip in particular were quite significant on the fatigue strength. Useful comparisons with numerical finite element simulations applying the Strain Energy Density approach are also outlined in the analysis of the obtained experimental data.

Keywords

fatigue test, rotating bending, high cycle, notched specimens, N-SIF, SED

1 Introduction

Welded structures are typically composed of various joints, with their roots or toes representing critical sites where the stress level is significantly higher than in the far field and cracks originate when subject to cyclic loading. Indeed, the presence of a notch highly reduces static and fatigue strength. In particular, the fatigue strength depends on the geometric features of the weld notch, such as its opening angle, radius and depth.

In the last decades, regulatory bodies required more and more to directly assess the fatigue strength of structural details and, more recently, local approaches have been widely introduced, thanks to the availability of the necessary computational power allowing to model in detail even welded seams. Actually, several local approaches have been proposed in literature to assess fatigue strength, each one having advantages and disadvantages, both in terms of accuracy and efficiency. However, none is universal and comprehensive as testified by a broad literature and by several books on the matter [1], [2], [3], [4], [5], [6], [7], [8] nor it is robust, fast and compliant with the limited design time nowadays required by large welded structures like e.g. a ship.



Evaluating the fatigue strength of a structural detail continue to require skills and experience in performing the analyses according to assessment approaches proposed by rules and to accurately evaluate simulation results, at first by properly selecting the locations where fatigue failure is supposed to occur and then by applying the most appropriate fatigue assessment approach duly considering the availability of fatigue loading and strength data.

Currently, different approaches to evaluate the fatigue behaviour of a welded joint or a component with a notch can be selected by designers, adopting different governing parameters to assess the fatigue behaviour as summarised e.g. in [1], [6], [9]. They can be categorized based on the parameters assumed to govern fatigue phenomena, i.e. stress field, strain field, intensification factors, etc., ranging from the well-known nominal stress approach to the structural and notch stress approaches up to the crack propagation approach. In the majority of cases, the presence of a singularity of the stress field is observed in welded joints where a sharp notch is present both at the root and at the toe [10], [11], [12].

Lazzarin and co-workers have proposed the use of the *Notch Stress Intensity Factor (N-SIF)* as a parameter to predict static and fatigue behaviour of components weakened by notches [13], [14]. The *N-SIF* can be used to describe the singular stress field in way of sharp notches in the same way as at crack tips it is described by *Stress Intensity Factor* in fracture mechanics. Based on Neuber's concept of elementary structural volume [12], it has been also understood that the fatigue behaviour of the component can be described by the strain energy averaged in a small and finite volume of material around the notch tip, being the *N-SIF* and the *Strain Energy Density (SED)* related. Thus, the *N-SIF* approach was developed for assessing fatigue of welded joints and demonstrated that fatigue test results of various geometrically different welded joints realize a fatigue strength curve with reasonable scatter when based on *N-SIF* or related parameters.

Noticeably, as highlighted by [15], Lazzarin and co-workers extended the *N-SIF* approach to estimate the fatigue life of transverse attachments and cruciform joints with failure also from weld root, following the original proposal in [16] suggesting to use the *N-SIF* in loading mode 1 as a parameter governing the crack initiation at notch tip. Indeed, the *N-SIF* explicitly considers the singularity of the stress field and thus the localized material plasticization. Namely, plasticization occurs in the process zone where a fatigue crack enucleates and then propagates. However, by not assuming the presence of an initial fracture as in linear elastic fracture mechanics, the use of the *N-SIF* parameter enables the use of structural analysis tools based on continuum mechanics and in particular numerical methods already nowadays adopted in everyday design practice (i.e. linear Finite Element Analysis or FEA), [17].

While the traditional fatigue assessment approaches based on stress values show some disadvantages in practical engineering applications, the *N-SIF* approach considers a parameter which directly controls the fatigue fracture, strictly depending on the localized geometry and behaviour of the structural detail. However, all local fatigue assessment approaches, including the *N-SIF* approach, need a rather refined FEA mesh with dedicated specific pattern, implying experience and time to create the model, run the simulation and eventually analyse the results properly, which is a substantial drawback, [2], [4], [5].

Recently, based on the *N-SIF* approach, the *SED* approach and the peak stress method (PSM) have been proposed to overcome the a.m. problems in engineering design practice, [18]. The *SED* approach considers the averaged *SED* inside a finite volume around the notch tip as the parameter that controls the fatigue phenomenon for structural details weakened by a sharp notch, as it is the case of welded structural details, see e.g. [15], [19], [20], [21], [22], [23], [24].

The averaged *SED* is a parameter directly related to *N-SIF* that depends on the radius R_0 (critical radius) of a circular volume around the notch tip, [11]. This radius has been demonstrated to be a characteristic property of the material only. Considering cases in which the mode II loading condition of a crack is excluded, i.e. cases in which the notch opening angle is greater than 102.6° , Lazzarin and co-workers proposed to evaluate the radius of the circular sector of the volume in which to evaluate the averaged *SED* under the mode I loading condition, i.e. the crack opening mode, based on **Equation 1**, [11]:

$$R_0 = \left(\frac{\Delta K_{IC}^N}{f_1(0^\circ) \cdot \Delta \sigma_A} \right)^{\frac{1}{1-\lambda_1}} \quad (1)$$

where ΔK_{IC}^N is the critical *N-SIF* evaluated for notches having null opening angle (i.e. cracks) in mode I loading also known as fracture toughness, f_1 is a function that depends only on the opening angle, which is null for a crack, and $\Delta \sigma_A$ is the fatigue strength at $N = 2 \cdot 10^6$ cycles with survival probability $P_S = 97,7\%$, i.e. the FAT value for the structural detail considered. Lazzarin and co-workers, moving from the initial application in [11], eventually proposed and validated the value $R_0 = 0,28$ mm for steel welded structures by collecting more than 900 experimental test data from open literature, obtaining a convincing design SED-N curve valid for both notch types of a weld, i.e. at toe having a nominal opening angle of 135° and at root having null opening angle. They proposed a curve for aluminium alloys too, [21], [22].

The FAT value in terms of averaged *SED* for steel welded structures was found to be $\Delta W = 0.058 \text{ Nmm/mm}^3$ (at $N = 2 \cdot 10^6$ cycles with survival probability $P_S = 97,7\%$) with reasonable scatter, being the ratio with the fatigue strength at $N = 2 \cdot 10^6$ cycles with survival probability $P_S = 2,3\%$ estimated as $T_{\Delta W} = 3.3$, as shown e.g. in Figure 9 of [22]. Later, Fischer

et al. [23], based on a wide dedicated experimental campaign of notched steel specimens, proposed and validated a new value of the radius, $R_0 = 0,32$ mm, and reviewed the energy-based design curves accounting for on this slightly higher value. However, they suggested that, being $R_0 = 0,28$ mm conservative, such latter value may continue be taken for steel welded structures.

To plot and validate S-N curves based on SED, a few test campaigns have been conducted by various researchers. An example is the work proposed in [15] on welded specimens with V-notch and U-notch or the large number of tests available in open literature and summarized by Lazzarin and co-workers since [11] and later on different welded joints primarily subjected to tension. In [24], three web-stiffened corners of ship structural geometries were selected and tested to verify the SED approach. More recently, a summary of available test data of notched specimens has been proposed in [25].

However, to the best of the authors' knowledge, no dedicated test results are available in literature for notched specimens subjected to alternate rotating bending and aimed at fitting SED-N curves. In particular, there are no accessible results for very high cycle levels that can be used also to validate the SED approach in this respect. Owing to the availability of dedicated fatigue tension tests [15] and the limited number of rotating bending fatigue tests of notched specimens in open literature, it was decided to develop a rotating bending test campaign targeting very high cycle life and influence of notch geometry in bending loading. The essential aims of the work can be summarized in the following items:

- To investigate the influence of notch opening angle, notch tip radius, notch depth and material on fatigue strength of notched specimens in rotating bending at high cycle number;
- To obtain S-N curves of notched specimens of typical steel materials used in shipbuilding, encompassing a common high strength steel and a High Strength Low Alloy (HSLA) steel;
- To further validate the SED-N curves proposed for the radius of $R_0 = 0,28$ mm by results obtained from notched specimens in rotating bending and FEA.

2 Plan of fatigue tests

2.1 Test series, geometry, and manufacturing of specimens

The tests were carried out on specimens of two different steels. A total of 50 and 51 specimens with different notches were tested for material S355 and material S460 (HSLA) respectively, whose mechanical characterization was verified in compliance with specifications of ISO EN 10025 Standards.

Initially, specimens were divided into six series, according to material and notch angle (i.e. two materials and three notch opening angles). More than 15 specimens were prepared for each series. Series were named according to material and opening angle: i.e. S355-0°, S355-90°, S355-135°, for material S355, and S460-0°, S460-90°, S460-135° for material S460. Instead, when analysing experimental results, the test series have been regrouped based on the different geometric features of the notch, including not only the opening angle but also both, the notch depth and the notch tip radius. According to recommendations of the International Institute of Welding (IIW, [3]), each new group was planned to consist of about 10 failed specimens at least, being appropriate for regression analysis. Though, a few groups did not reach the target because of the limited number of certain notch geometries. Eventually, the data were reorganized to create meaningful plots to evaluate the influence of notch depth and notch radius too.

The geometry of the specimens and the parameters indicating the value of the geometric features of the notch are shown in Fig. 1. Steel specimens were created by trimming and machining steel bars obtained from 15 mm thick plates using a computer numerical control machine (CNC). It is worth noting that, while length and diameter of the cylindrical specimens were obtained with relatively high precision (± 0.01 mm), notch geometries at the midspan of the specimens presented slightly different geometric features due to unavoidable manufacturing tolerances in fabricating the notch. However, these differences were expected and considered when planning the test campaign, looking forward to having a sufficient scatter for the appraisal of the effect of the notch geometry onto the fatigue strength.

All specimens presented a midspan notch and two threaded holes at ends to permit the specimen installation on the rotating bending test machines, as shown in Fig. 1.

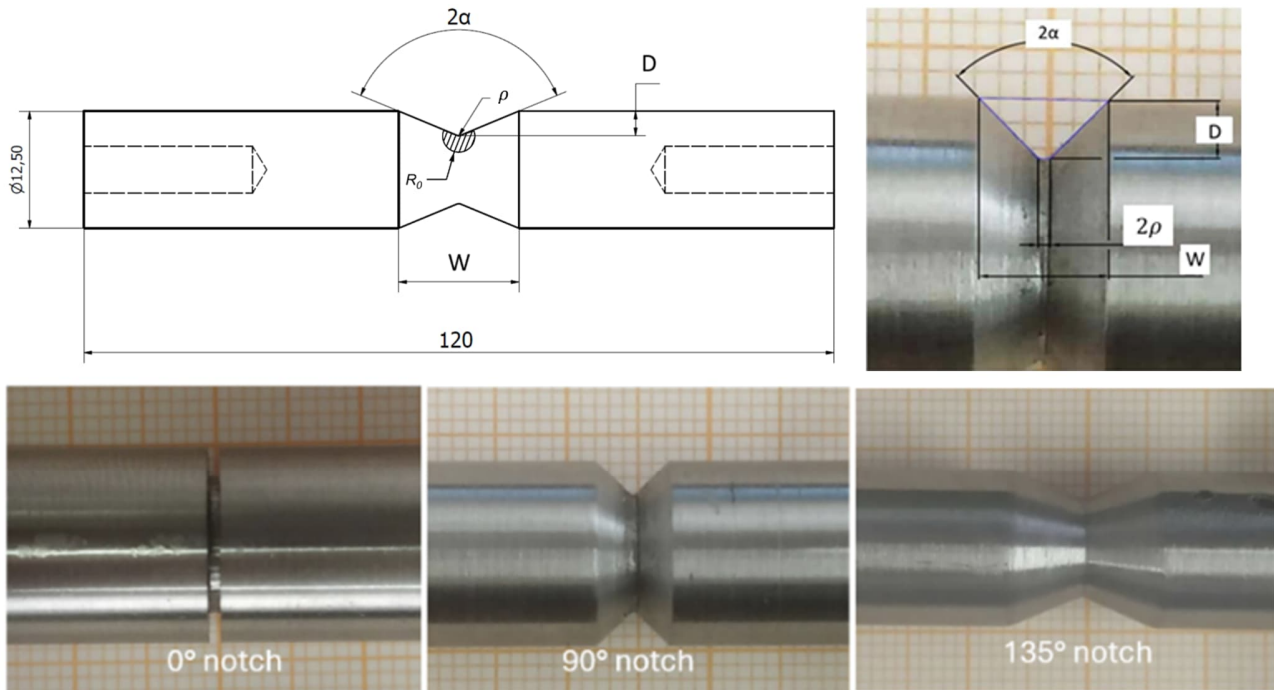


Fig. 1 Main dimensions of specimen and definition of measured parameters of the notch

2.2 Measurements of the specimens

Before starting the test of each series, the geometric features of the notch of each specimen were measured. In order to investigate how these features influence the fatigue strength, it was considered appropriate to collect: the opening angle 2α , the notch depth D , its radius ρ (measuring the width at the tip), and the width W of notch. The values considered on each specimen are defined in Fig. 1.

To collect the geometrical features, different equipment was used. At first, a calliper (accuracy 0,02 mm) and, at second, a CAD software to measure the values on digital images of the specimens, as shown in Fig. 1, were used. To measure opening angle, depth, and radius, four measurements on perpendicular radial sections were conducted on each specimen and the average of the four measurements was computed and then considered in the experimental results analysis. After subdividing the data into test series, an evaluation of measurements uncertainty was carried out by simply considering average and standard deviation. It has been observed that the scatter with respect to the average is less than 5% in all series and all measurements, which is considered satisfactory.

2.3 Implementation of long-life fatigue tests

In the following, the results obtained from the fatigue tests of the previously described notched specimens are reported, allowing understanding the behaviour of fatigue strength of steels commonly used in shipbuilding under rotating bending at high cycle number in relation to the geometrical features of a notch. The tests were conducted according to recommendations of IIW [3]. As suggested in [26], also for the present tests, fifty million cycles were selected as the run-out limit, since one hundred million cycles is a typical target life for ship and offshore structures or some civil structures. A rotating bending fatigue test (stress ratio $R = -1$) was performed. After taking the geometric measurements, the nominal acting load at notch of the specimen was evaluated according to Equation 2:

$$\pm\sigma = \frac{32}{\pi} \cdot \frac{f \cdot a}{(\varnothing - 2D_{average})^3} \quad (2)$$

Where f are the point forces applied on the specimens, at distance a from the support of the bearings, \varnothing is the diameter of the specimen, and $D_{average}$ is the average value of the notch depth that was measured on specimens in 4 radial sections as earlier mentioned. See also Fig. 2.

As test run-out was set to $5 \cdot 10^7$ cycles, tests were stopped upon reaching the run-out limit and subsequently restarted at a higher load level according to [3]. The tests were conducted at room temperature and an average machine frequency of approximately 43 Hz was recorded. Frequency variations, such as those caused by electric motors slippage, overheating or disturbances were also recorded.

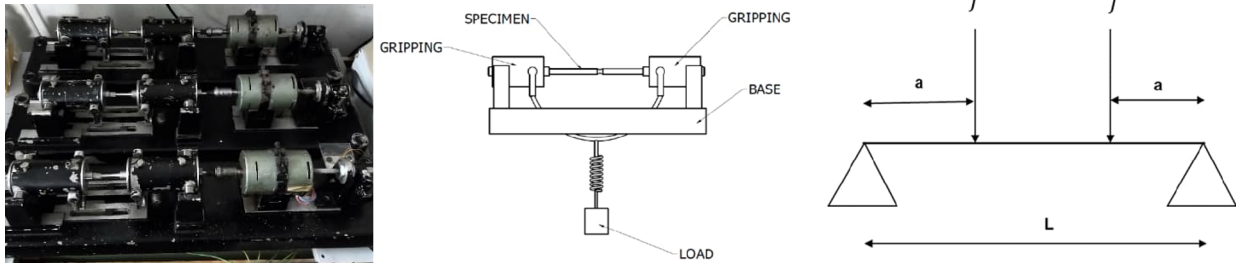


Fig. 2 Three out of six rotating bending machines used for the tests and nominal stress evaluation

3 Results of fatigue tests

The results of fatigue tests were analysed according to recommendations of IIW excluding run-out specimens, which were much more than expected, and following formulations reported in [3] and in [27]. As anticipated, the data were reordered to plot the S-N curves for each different geometric feature of the notch. The obtained curves are presented in the following. At first, the results were reordered to analyse the influence of the notch opening angle (0° , 90° and 135°) for both, S355 and S460 materials. Thereafter, in order to analyse the influence of notch tip radius, fatigue tests data have been grouped by the same material and opening angle but different tip radius. Finally, the results of fatigue tests were reordered grouping the same material, opening angle, and tip radius, but different depths of the notch.

For each test series of the specimens, before plotting S-N curves, a revision of data was conducted with the purpose of excluding from the analysis each incorrect result, mostly due to misalignment of specimens on the test machine. In figures of section #3, the data indicated with black marks were excluded from the statistical analysis because misalignments were noted, leading to overheating of the machine during the test. This process allowed reducing the scatter ratio T_N of the S-N curves (still imposing the slope $m = 3$ in the statistics as usual in fatigue data analysis).

Moreover, for the test series in Tab. 1, where all the measured geometric features of the notch of the specimens are reported, it was found appropriate to investigate the slope of the S-N regressions. While the slope proposed by standard guidelines ($m = 3$) was at first used, the one from regression analysis was later estimated using suggestions stated in [27], section 8.2.

Series	Number of specimens	Steel	Opening Angle 2α [deg]	Depth D [mm]	Radius R [mm]	Width W [mm]
S355- 0°	14	S355	0°	2,02 ($\pm 0,10$)	0,37 ($\pm 0,02$)	0,75 ($\pm 0,02$)
S355- 90°	14	S355	90° ($\pm 0,82$)	2,42 ($\pm 0,12$)	0,12 ($\pm 0,05$)	5,19 ($\pm 0,13$)
S355- 135°	13	S355	135° ($\pm 0,37$)	2,64 ($\pm 0,11$)	0,14 ($\pm 0,04$)	12,50 ($\pm 0,40$)
S460- 0°	15	S460	0°	2,03 ($\pm 0,07$)	0,36 ($\pm 0,04$)	0,72 ($\pm 0,04$)
S460- 90°	16	S460	90° ($\pm 0,77$)	2,40 ($\pm 0,16$)	0,14 ($\pm 0,04$)	5,23 ($\pm 0,22$)
S460- 135°	15	S460	135° ($\pm 1,16$)	2,56 ($\pm 0,16$)	0,12 ($\pm 0,05$)	12,25 ($\pm 0,77$)

Tab. 1 Notch geometry of specimens made in S355 and S460 materials (average and scatter)

3.1 Influence of notch opening angle

The influence of the opening angle was analysed first. Fig. 3 to Fig. 8 show the S-N curves for the test series, statistically analysed imposing the slope value $m = 3$ and then evaluating it as earlier mentioned.

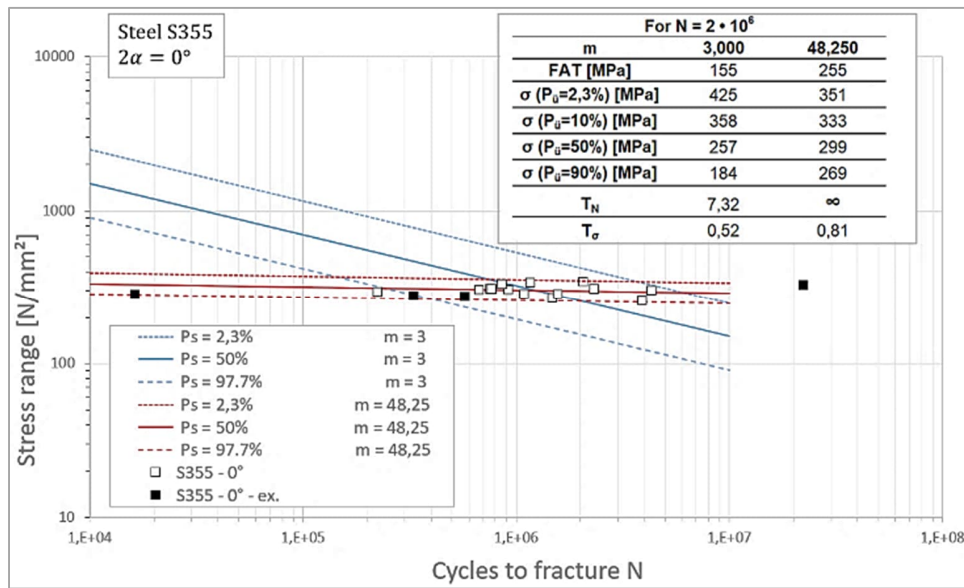


Fig. 3 S-N curves for steel S355 and opening angles $2\alpha = 0^\circ$ with different slopes.

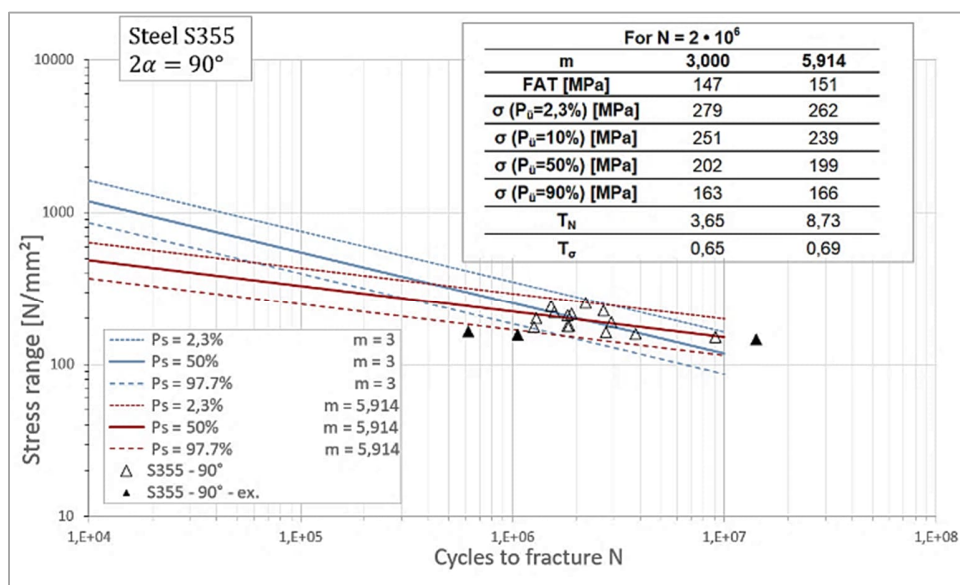


Fig. 4 S-N curves for steel S355 and opening angles $2\alpha = 90^\circ$ with different slopes.

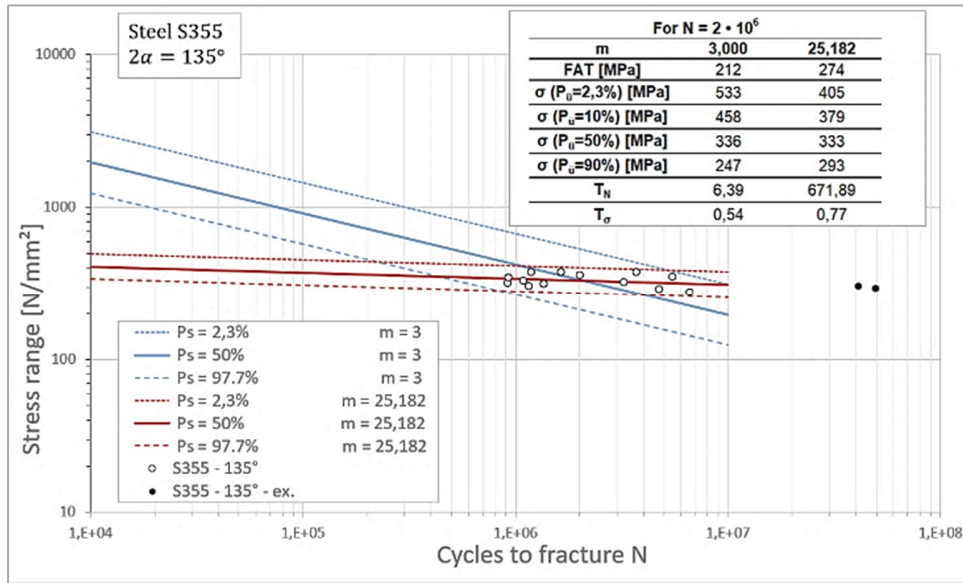


Fig. 5 S-N curves for steel S355 and opening angles $2\alpha = 135^\circ$ with different slopes.

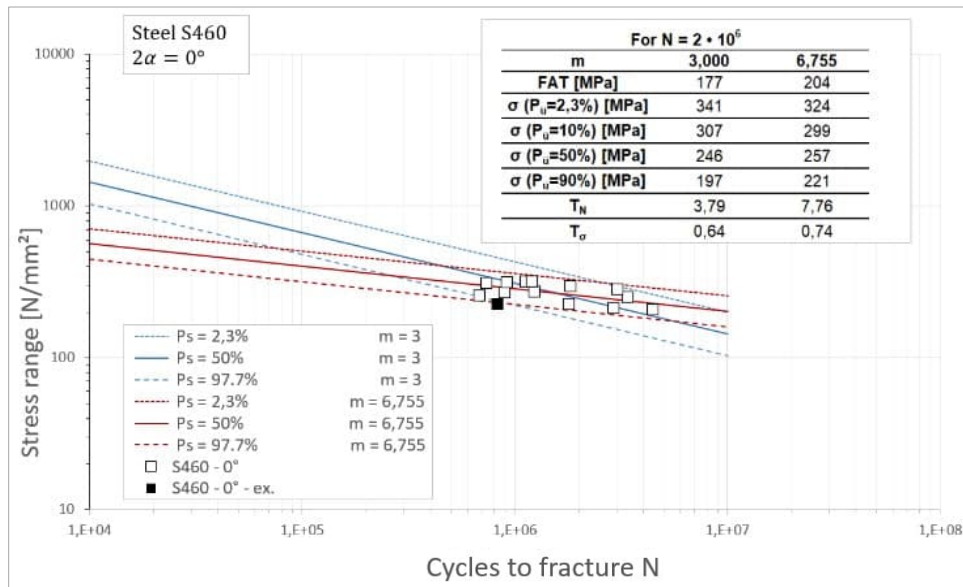


Fig. 6 S-N curves for steel S460 and opening angles $2\alpha = 0^\circ$ with different slopes.

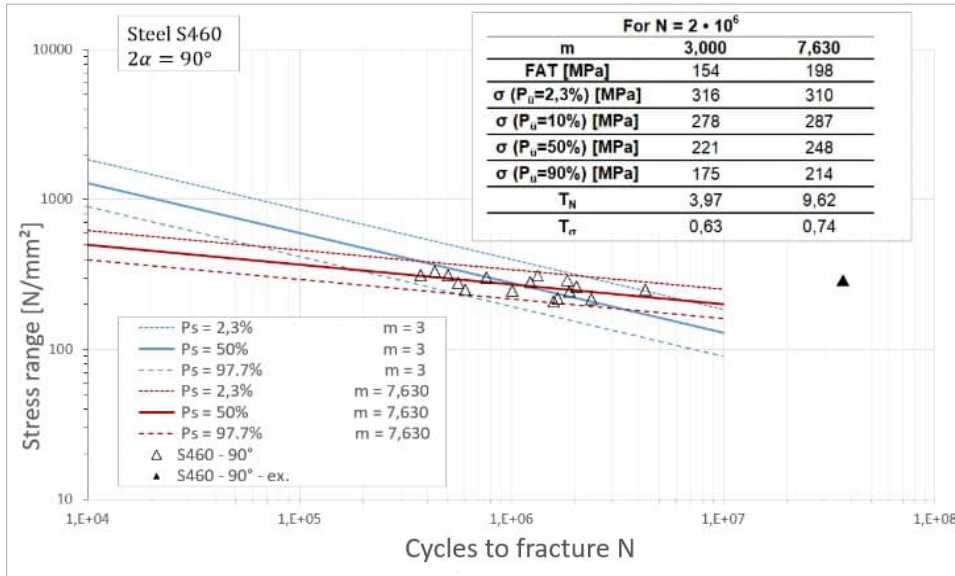


Fig. 7 S-N curves for steel S460 and opening angles $2\alpha = 90^\circ$ with different slopes.

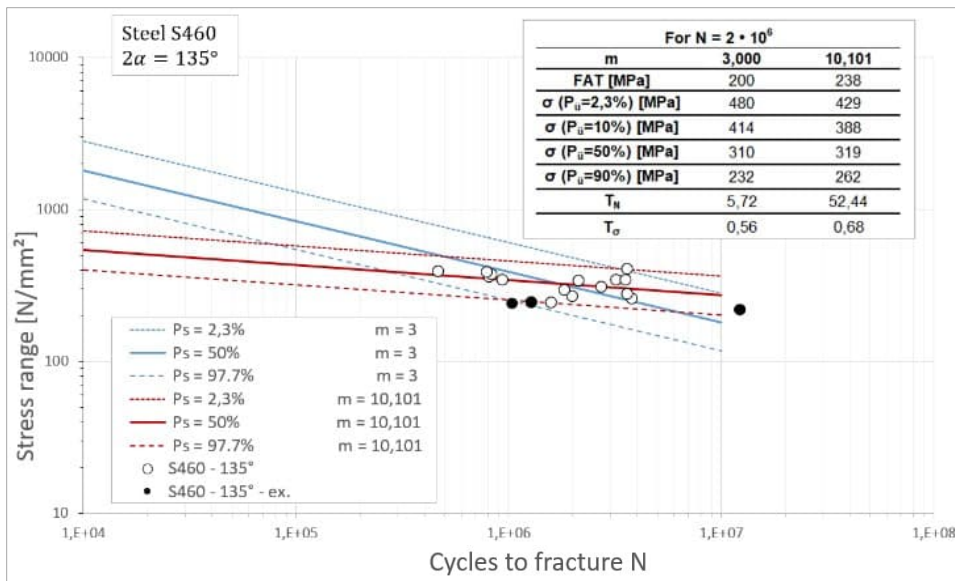


Fig. 8 S-N curves for steel S460 and opening angles $2\alpha = 135^\circ$ with different slopes.

In order to underline how the opening angle influences the fatigue strength for the same material for test series in Tab. 1, S-N curves with slope $m = 3$ are reported in a single plot. The fitted S-N curves are shown in Fig. 9 (for the sake of clarity only at probability of survival $P_s = 97,7\%$).

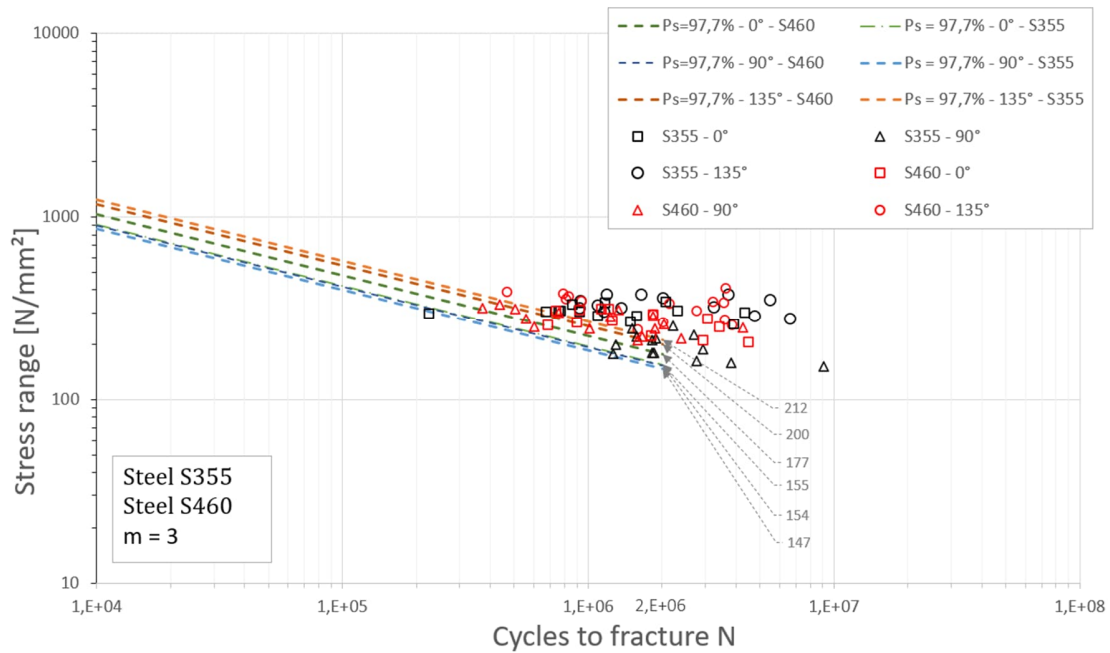


Fig. 9 S-N curves for steel S355 and S460, different opening angles and same slope $m = 3$.

3.2 Influence of notch tip radius

Due to the manufacturing processes, specimens with the same opening angle did not have the same tip radius. Thus, the data were reorganized in order to regroup the specimens having the same tip radius, material and opening angle. Six groups of specimens (and test data) with different tip radii were identified and recollected in Tab. 2. The aim was to arrange each series in such a way that contains more than 10 specimens, in compliance with IIW guidelines, but due to the limited number of specimens each group contain 6, 7 or 8 specimens only. However, analysis was carried out as well to obtain the searched information. Fig. 10 to Fig. 12 show S-N curves plotted for different tip radii with slope $m = 3$, material S355 and different opening angles, whereas Fig. 13 to Fig. 15 show the S-N curves plotted for different tip radii, material S460 and different opening angles.

Series	Number of specimens considered	Steel	Opening Angle 2α [deg]	Depth P [mm]	Radius R [mm]	Width L [mm]
S355-0°-37	7	S355	0°	2,00	0,37	0,74
S355-0°-39	7	S355	0°	2,03	0,38	0,77
S355-90°-10	8	S355	90°	2,48	0,10	5,15
S355-90°-14	6	S355	90°	2,37	0,14	5,24
S355-135°-13	7	S355	135°	2,65	0,13	12,58
S355-135°-16	6	S355	135°	2,68	0,16	12,73
S460-0°-35	6	S460	0°	2,00	0,35	0,69
S460-0°-37	7	S460	0°	2,06	0,37	0,74
S460-90°-13	9	S460	90°	2,47	0,13	5,34
S460-90°-16	7	S460	90°	2,34	0,16	5,18
S460-135°-11	8	S460	135°	2,61	0,11	12,82
S460-135°-16	7	S460	135°	2,52	0,16	11,92

Tab. 2 Geometric features of the notch of the specimens groups for different tip's radius.

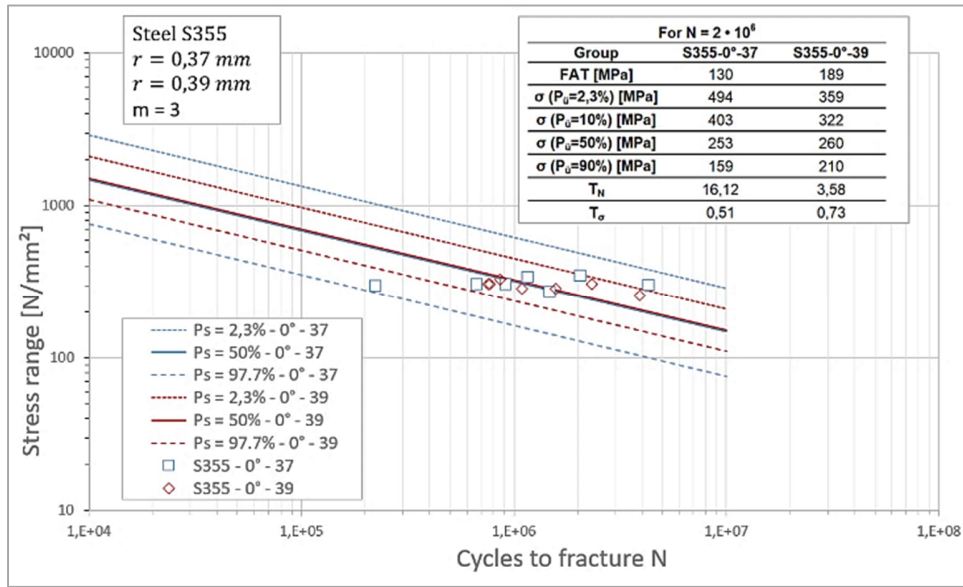


Fig. 10 S-N curves for different tip radius, same opening angle $2\alpha = 0^\circ$ and material S355.

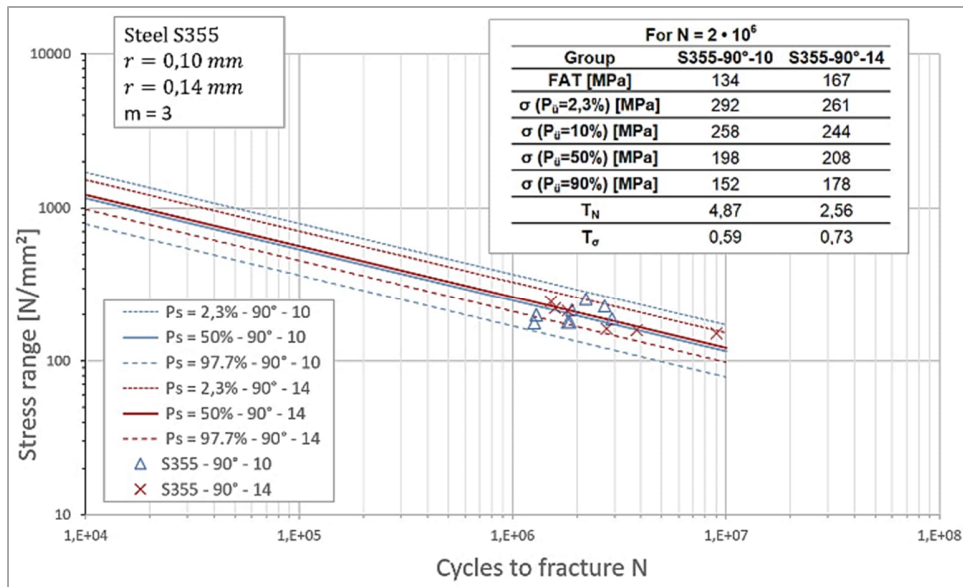


Fig. 11 S-N curves for different tip radius, same opening angle $2\alpha = 90^\circ$ and material S355.

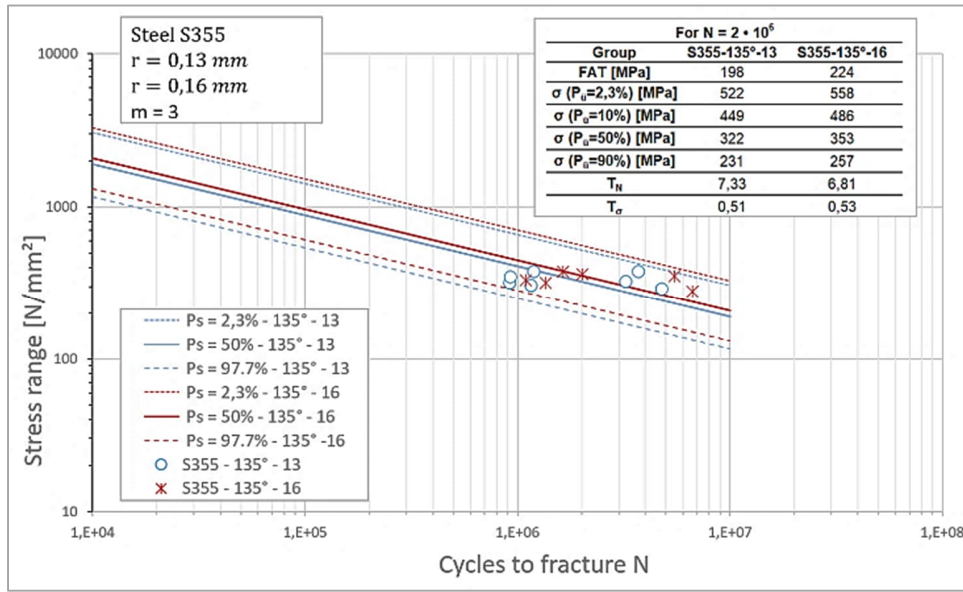


Fig. 12 S-N curves for different tip radius, same opening angle $2\alpha = 135^\circ$ and material S355.

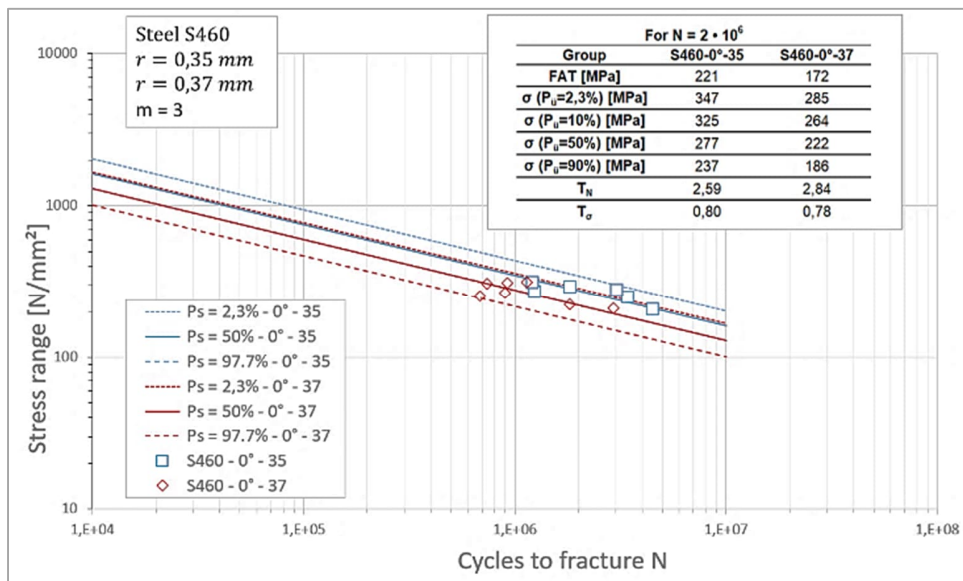


Fig. 13 S-N curves for different tip radius, same opening angle $2\alpha = 0^\circ$ and material S460.

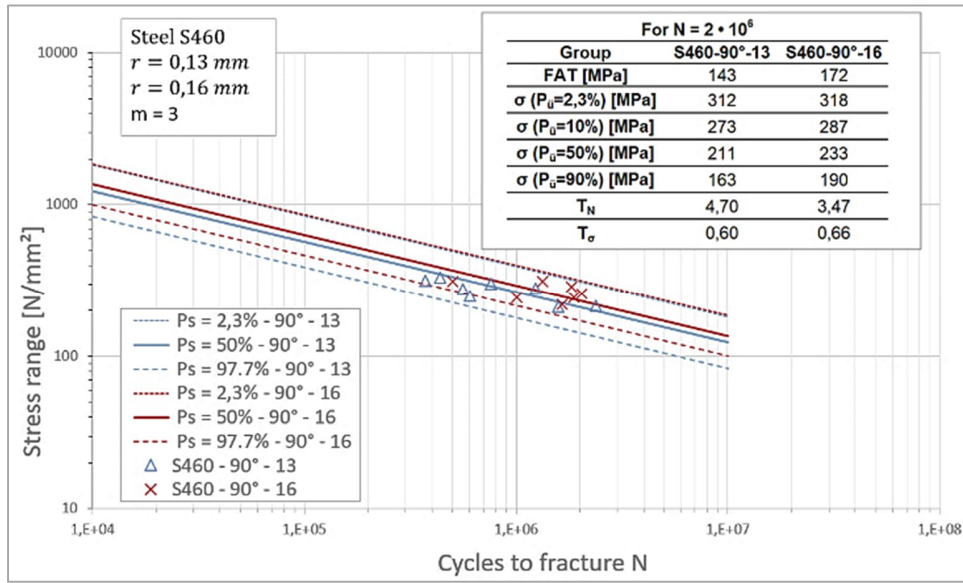


Fig. 14 S-N curves for different tip radius, same opening angle $2\alpha = 90^\circ$ and material S460.

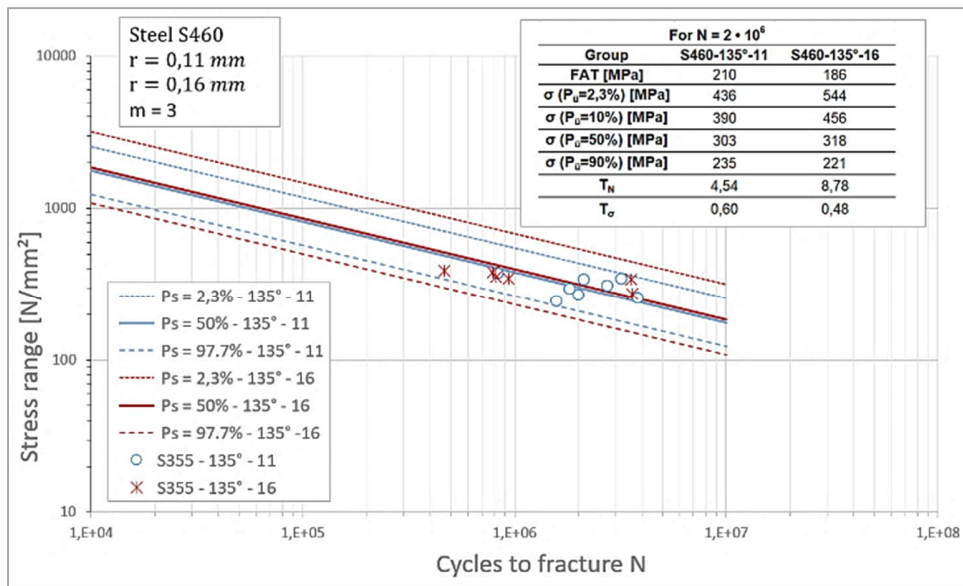


Fig. 15 S-N curves for different tip radius, same opening angle $2\alpha = 135^\circ$ and material S460.

3.3 Discussion

In the following a few hints about the tests results are provided.

3.3.1 Slope of S-N curves

From test results shown in Fig. 3 to Fig. 8 grouping specimens by same material and notch opening angle, it has been noted that the slope value of data is significantly higher than the one proposed by standards. The difference between the slopes is in accordance with the results of the fatigue tests conducted in [26] on the same materials but for smooth specimens without any notch. Hence, even for notched specimens, the experimental slope is quite higher than $m = 3$ in high cycle

fatigue. On the other hand, it should be considered that most of failed specimens survived several millions of cycles, possibly biasing the slope estimates.

3.3.2 Influence of notch opening angle

As shown in Fig. 9, the S-N curves of specimens with a notch opening angle of 135° show a fatigue strength higher than the fatigue strength of specimens with 0° and 90° notch angles. Thus, it has been confirmed that larger opening angles reduce the notch effect, as for the results obtained from butt joints in [15]. This result was obtained for both materials, S355 and S460.

However, Fig. 9 show that, for both tested materials, the series of specimens with a null opening angle present a fatigue strength slightly higher than the series with an opening angle of 90°. This result was not expected as the specimens with a relatively small notch opening angle, ideally a crack, should have a lower fatigue strength than the specimens with a larger notch opening angle due to the higher stress singularity. To understand these peculiar results, the influence of the other geometric features of the notch were analysed. It is worth noting that the actual notch shape of the specimens having null opening angle was more similar to a lack of penetration at weld root of fillet welds than to a real crack in base material because of the gap between the two faces of the notch. See null opening angle notch in Fig. 1.

3.3.3 Influence of notch tip radius and depth

In Fig. 10 to Fig. 12, the 50% regression lines of S355 specimens with larger radius (red curves) are above those of specimens with small radius (blue curves) as expected. Fig. 10 shows the S-N curves for series S355-0°-37 and S355-0°-39 with a fixed slope $m = 3$, where the numbers added to the series' names refer to the measured notch radius at tip (in 10 μm). In this case, due to the reduced difference between the radii, the fatigue strength at 2 million cycles at $P_s = 50\%$ differ only by $\approx 2\%$. Instead, for series S355-90°-10 and S355-90°-14, reported in Fig. 11, the difference of the radii is slightly greater compared to the previous case and the fatigue strength at 2 million cycles at $P_s = 50\%$ differ by $\approx 5\%$. Equivalent results can be observed for series S355-135°-13 and S355-135°-16 in Fig. 12, where difference is approximately $\approx 10\%$.

It then seems that the tip radius effect is approximately linear but depending on the notch opening angle. However, the above-mentioned analysis shall be regarded as an attempt to interpretate the experiments and taken with utmost care, owing the limitation of available data as well as the humble accuracy of notch geometry measurements. Though, in accordance with both, the results of butt-welded joints in [15] and the results of notched bars by Schwaiger and Mühlenbeck [28], the S-N curves of the series in Tab. 2 confirm again that a large notch tip radius reduces the notch effect and increases the fatigue strength of specimens.

The previous comments are not completely applicable to all curves presented in Fig. 13 - Fig. 15 of series in Tab. 2 relevant to S460 specimens, as outlined in the following.

In the case of comparison of series S460-90°-13 vs. S460-90°-16 and S460-135°-11 vs. S460-135°-16, the results are as expected, indicating that the specimens with a larger notch tip radius exhibit a higher fatigue limit strength than the specimens with a smaller notch tip radius. On the contrary, in Fig. 13, the S-N curves of specimens with a larger radius (series S460-0°-37) are not above those of specimens with a smaller one (series S460-0°-35). Again, however, relatively small difference of radii and features of null opening angle notches should be considered.

To ascertain the cause of the discrepancy between observed and expected results, it was attempted to examine the value of the notch depth for each series. It was observed that the specimens in Tab. 2 belonging to series S460-0°-37 had a notch depth greater by 0,06 mm on average compared to the notch depth of series S460-0°-35. Consequently, despite the reduced notch tip radius, the specimens belonging to series S460-0°-35 had a different strength section than the specimens belonging to series S460-0°-37 resulting in a relatively higher stress level enhanced by the stress singularity. It is guessed there are here two different competing effects, that of the notch radius and that of the notch depth but a definitive conclusion is hard to reach because of the very limited geometrical differences in the order of the measurement accuracy.

The effect of the notch tip radius and of the notch depth is evident comparing series S355-0°-37 vs. S355-0°-39 and S355-135°-13 vs. S355-135°-16. In such instances, where the values indicated in Tab. 2 are referred, the difference in notch depth is found to be lower (0.03 mm only) than the difference in notch tip width (0.04 mm). This suggests that, despite the small variation, the notch tip radius may be the geometric feature influencing most the fatigue strength of specimens.

The analysis of the notch depth values suggests that:

- Both for S355 and S460 steels, the specimens with a notch opening angle of 90° have a notch depth larger than the specimens with a null notch opening angle. The difference between the notch depth is 0,40 mm. Thus, the

- specimens with a null notch opening angle have a cross-section relatively higher than the specimens with a notch opening angle of 90° ;
- The specimens belonging to series in Fig. 11, Fig. 14, and Fig. 15 exhibited a correlation between a large notch tip radius and a reduced notch depth. Consequently, due to the limited number of specimens tested having the same notch geometrical features and to the relatively low accuracy of the geometric measurements, it was not feasible to analyse the influence on fatigue strength of the same notch tip radius and different notch depth independently.

4 Numerical analyses and N-SIF based approaches

Following the conclusion of experimental tests, some numerical analyses have been performed aiming at further validating fatigue assessment approaches based on *N-SIF* parameters in very high cycle bending conditions.

Many studies have been conducted to compare the nominal stress approach with local approaches and to validate approaches such as the structural stress approach, the effective notch stress approach as well as the *N-SIF*-based approaches as summarized e.g. in [1] [2], [4], [5]. This made local stress approaches more and more popular in design offices. However, local approaches based on the *N-SIF* concept, such as the averaged strain energy density approach (SED) and the peak stress method (*PSM*), are not applied in practical engineering, particularly in the shipbuilding industry, because rules and guidelines do not currently address them [29]. Actually, literature has demonstrated that the *N-SIF* approach, based on the notch stress intensity factor proposed by Gross and Mendelson [30], after Williams [31], is a successful method for evaluating the fatigue strength of a welded detail, see e.g. [16], [19], [21], [22], [23], [24], [32]. This approach allows the fatigue assessment of notched components to overcome at the root the need to define a conventional value of stress, being the stress field singular at notch tip. It was demonstrated in [10] that the use of the *N-SIF* enables the analytical description of the infinite stress field at V-notches, in particular for toes and roots of welded joints [12], [15].

Accounting for the above, various authors investigated the behaviour of *N-SIFs* in relation to the implementation of finite element models to estimate it. These studies demonstrated that evaluating *N-SIF* parameters by means of finite element analyses necessitates very refined mesh (up to 10^{-5} mm), [14], [17], which involves time-consuming processes for building up models and analysing results by a suitably skilled analyst, similarly to the crack propagation approach. In more recent times, alternative methodologies have been proposed to overcome the issues of calculating *N-SIF* by finite elements analysis such as the averaged strain energy density approach, or SED, and the peak stress method, or *PSM*, [18], [33]. The strain energy density is a parameter intimately correlated with *N-SIF*, as demonstrated by [11], allowing the evaluation of *N-SIFs* by relatively coarser meshes. A number of studies have been conducted to assess mesh sensitivity within finite elements models, considering element size, its formulation, mesh pattern, given boundary and loading conditions at the tip of a sharp notch. The analysis of components with sharp notches led to the conclusion that the value of the SED averaged over a finite volume of material could be considered as the parameter controlling fatigue failure equivalently to *N-SIF*, see e.g. [17], [24]. Actually, Lazzarin and co-workers showed that the SED evaluated in a relatively limited volume is practically mesh independent when finite element analysis is applied. In particular, it was demonstrated in [33] that an extremely refined finite element mesh is not required for the evaluation of averaged SED, as it is for *N-SIF*, because the parameter can be in principle evaluated only via the nodal forces and displacements. It was noticed that, being SED the ratio between scalar quantities such as energy and volume, it is a parameter independent of the opening angle of the notch or of stress components direction. Eventually, the SED approach proposes to evaluate the strain energy density for steel material with linear-elastic behaviour, averaged in a circular control volume with the centre at notch tip and the radius equal to $R_0 = 0,28$ mm. The fatigue strength limit was evaluated from experimental tests on welded joints, see e.g. [11], [15], [33].

The results of the test series in rotating bending summarized in the first part of this paper were converted in terms of averaged SED using finite element analysis (FEA). FEA were conducted on a parametric model of the specimens in order to obtain the exact notch geometry of each specimen. Each specimen was in fact modelled with variable notch opening angle and depth, matching the measured values, being the tip radius null in the model geometry as required by the SED approach, [17], [22].

Considering specimens' symmetry, only half specimen was modelled and proper boundary conditions were applied on the symmetry plane. See Fig. 16 where the FEA models are outlined. Mesh pattern was initially prepared in a radial section and thereafter the solid mesh was obtained revolving the meshed surface using 90 subdivisions over the 180° (semi-) revolution. The SED control volume of radius $R_0 = 0,28$ mm was placed as required with its center at notch tip. Another concentric volume with radius $R_0 = 0,50$ mm was created surrounding the control volume in order to help the software meshing algorithm in the transition of the element size from the smallest ones inside the control volume to those of the remaining part of the specimen model.

It is worth noting that some sensitivity analysis on mesh refinement was carried out. Basically, the behaviour of the SED when varying element size and the order of the finite element interpolation functions was analysed and variations do indeed lead to a rather limited difference in the results of the strain energy density in way of the notch. Thus, confirming conclusions already reported e.g. in [17], [33]. Also stress and strain results at nodes were tentatively used to obtain the SED, resulting in a limited scatter (generally less than 2% in all cases).

Based on these results, approximately 900 elements within the $R_0 = 0,28 \text{ mm}$ control volume and a total of 1800 – 2400 elements in the $R_0 = 0,50 \text{ mm}$ transition volume were eventually used to reduce the computation effort. Three-dimensional solid isoparametric elements with linear shape-functions were used to realise the mesh. Cubic and prismatic elements, with 8 and 6 nodes, were used in the control volume to investigate whether the element type, size and pattern influence the estimated value of the averaged SED and no significant differences were noted as mentioned. Out of the control volume, only cubic elements with 8 nodes were always used.

The averaged SED was evaluated within the control volume with radius $R_0 = 0,28 \text{ mm}$ around the notch tip and it was obtained from the FEA by averaging the scalar product of the stress tensor $\vec{\sigma}$ and strain tensor $\vec{\epsilon}$ computed at integration points of the elements, as suggested in [17], being the default option of the applied software.

Noticeably, despite the rather small control volume, the meshing process is relatively smooth and can be largely automated; in case of very complex geometries, it can be supported by sub-modelling techniques. Thus, allowing rapid implementation of the SED fatigue assessment approach in everyday design practice.

The data of series presented in Tab. 1 are now shown in Fig. 17, plotted ranging from 10^4 to $2 \cdot 10^7$ cycles only to better compare the results with existing curves proposed in literature, [15], [22], [23], [32]. An imposed slope of $m = 1.5$ was used, in agreement with slope $m = 3$ of curves based on stress range.

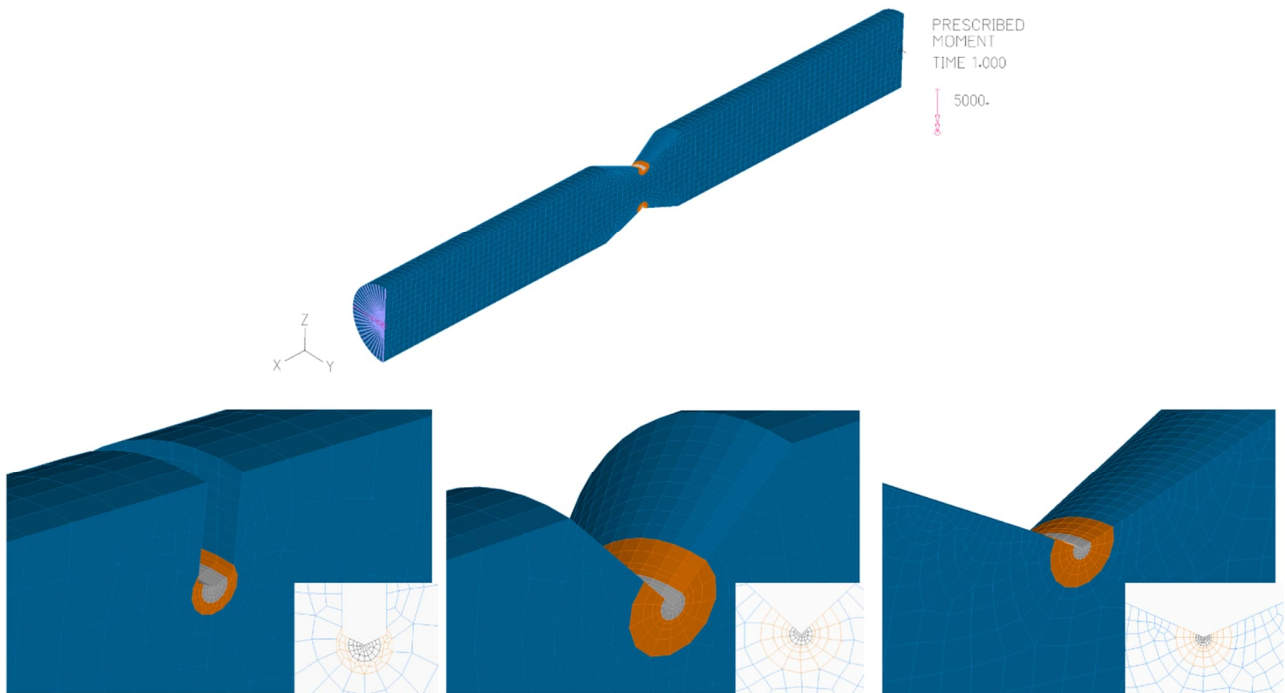


Fig. 16 parametric FEA model examples (loading conditions and local mesh pattern are shown)

The a.m. values of averaged SED evaluated by FEA on actual individual model specimens' geometries have been used to gain SED-N curves from tests results and are shown in Fig. 18. In the same figure, a comparison of the SED-N curve proposed by Berto and Lazzarin in [22] for welded joints with the data from tested specimens is also reported. Considering that the tested specimens mostly failed in way of the knee point of the typical S-N curves, i.e. at approximately few millions cycles where a slope change is usually assumed, the agreement is considered satisfactory. Indeed, experimental data substantially fall within the scatter band of the SED-N curve proposed in literature as shown in Fig. 17.

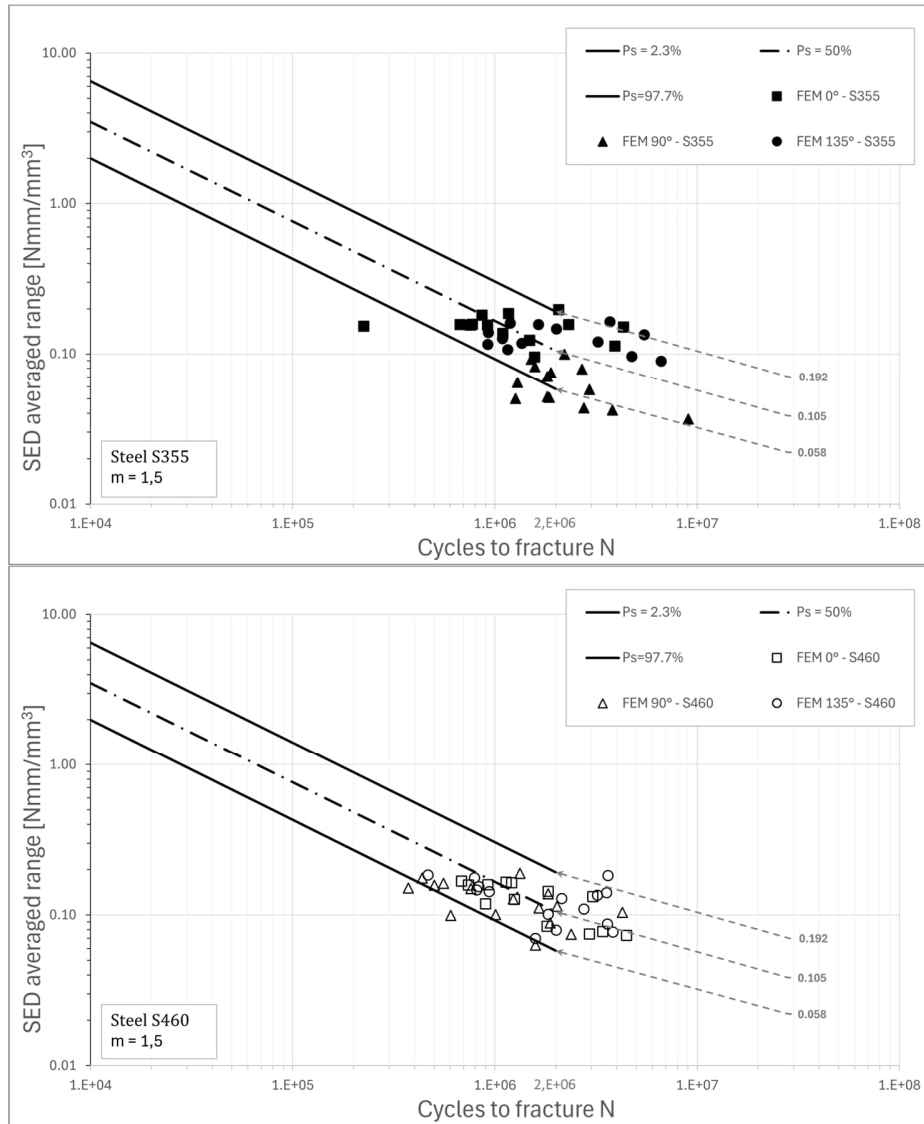


Fig. 17 averaged SED range estimated by FEA modelling actual specimens' geometry and loading compared with the SED-N curve proposed in [15], [22] and [23] (top: S355 steel- bottom: S460 steel).

The fact that the experimental data of the specimens with notch opening angle of 90° are slightly below those of the specimens with null notch opening angle is likely due to the relatively large scatter of the former data set. Possibly, the notch shape of the specimens with null notch opening angle influenced the results, as earlier mentioned. See Fig. 1 and also refer to the discussion of experimental results in section #3.3. This is reflected in the obtained SED-N curves too.

The plots in Fig. 18 highlight the ability of the SED approach to capture the influence of notch opening angle and of notch depth on fatigue strength. It appears that influence of the notch opening angle and of the notch depth shown in the SED-N plot in Fig. 18 is in line with conclusions already discussed for the S-N ones in Fig. 3 to Fig. 8, i.e. the SED is a fatigue governing parameter well able to deal with such geometrical features. The effect of the material may be guessed from the difference found in the curves of Fig. 18, even if a clear trend is not easy to grasp as the material effect is limited.

It is worth remembering again that the SED approach is intrinsically able to account for all stress components, i.e. for multiaxial fatigue, and that it is not a conventional value used to overcome the singularity of the stress field like any local stress range definition, e.g. according to the structural stress or the notch stress approaches. Rather, SED is a physical parameter assumed to govern fatigue strength at very local level and therefore it is really measuring the damage of the material and assessing whether a crack is originating in the notch.

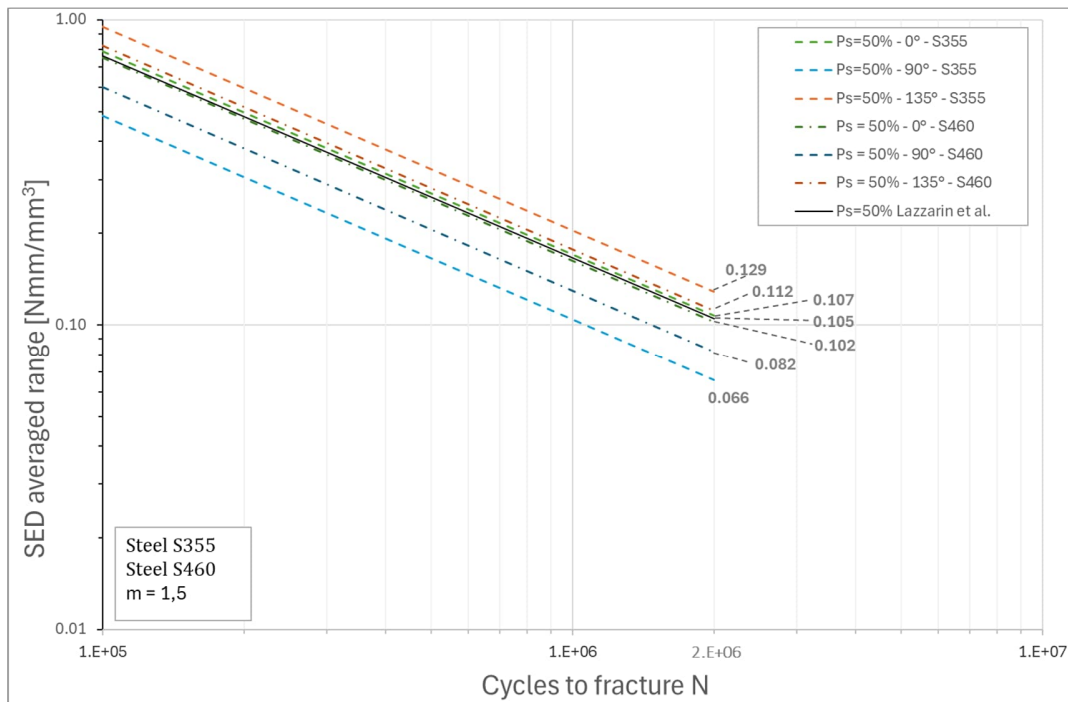


Fig. 18 S-N curves based on averaged SED range estimated by FEA modelling actual specimens' geometry (for the sake of clarity only curves at probability of survival $P_S = 50\%$ are shown).

5 Conclusions

The results obtained by the rotating bending tests reported in this paper allow to get insights to study the fatigue behaviour of notched structural components, such as welded joints. In particular, the results have evidenced/confirmed the following:

- As reported in [26], [15] and [23], the S-N curves for notched specimens show higher slopes than those proposed by standards ($m = 3 \div 5 \div 7$), somehow biased also by the very high number of cycles reached by the tested specimens.
- Geometric features of notches influence fatigue strength: in particular, a large notch opening angle is certainly beneficial as it has a lower stress intensity while effects of other geometric features seem influencing the fatigue behaviour to a lesser extent.
- It was confirmed that the notch tip radius seems to affect the fatigue strength not negligibly. Though, such effect was not crystal clear from the test due to limited radius variation among notches of tested specimens.
- It is confirmed that the notch depth is a geometric feature influencing the fatigue strength as well and its effect was quantified. However, it was difficult to definitely exploit experimental results in this respect.
- The fatigue test results of the series presented in Tab. 1 have highlighted that, in presence of a notch, the material is not the most important parameter influencing fatigue strength. Though, results of specimens of series S355-90° and S460-90° demonstrated that, for a given notch geometry, the material with the higher yielding strength also exhibited a higher fatigue limit.
- The S-N curves based on averaged strain energy density obtained by the conversion of the test data generally confirm the SED-N curves proposed by [22], [15] and [23]. The curves could be reliably used to evaluate the number of cycles to failure from SED values averaged in the control volume obtained from FEA.
- The effect of the opening angle of the notch is evident from Fig. 18 and a trend can be guessed, even if to be taken with utmost care owing the uncertainties involved in the experimental data at very high cycles number.

Further studies and analyses are planned to exploit the obtained results and, in particular, to validate a robust, reliable and rapid procedure to be used by structural designers in the framework of scantling design of naval vessels. Indeed, a standard routine to locally refine the mesh of existing finite element models in way of structural details is currently under development aiming to easily and rapidly apply the SED approach. Possibly, in the future, such a routine can

be automatically implemented in selected locations of the finite element models usually adopted in ship structural design whenever the stress level exceeds a certain threshold and/or geometrical parameters of structural joints so require.

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