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ORIGINAL ARTICLE

The reasons why maximum diversification is better than minimum risk, including in terms of risk

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Abstract

In well-defined experimental settings, we evaluate the out-of-sample performance of two asset allocation paradigms: minimum risk and maximum diversification. Specifically, for each given risk measure, we compare the optimal minimum risk allocation with the allocation obtained by maximizing a portfolio diversification measure induced by the same risk measure. The experiment is performed in an out-of-sample, long-only framework, accounting for proportional transaction costs and different lengths of both the estimation window and the holding period. The strategies are compared in terms of numerical stability, return, the Sharpe ratio, and risk, as measured through the same risk measures used for calculating the optimal allocation: variance of returns, mean absolute deviation, value at risk, and expected shortfall at significance levels of 1% and 5%. We show that the maximum diversification strategies are highly competitive, if not generally superior, to the risk minimization allocations. This result supports well-known empirical findings of naive investment strategies that are difficult to beat in practice. Risk minimization strategies require highly accurate forecasts of future returns to perform well. Moreover, these strategies exhibit extreme numerical instability, where even infinitesimal variations in the inputs can dramatically alter the optimal allocation. Therefore, implementation costs are high, significantly impairing performance. In contrast, maximum diversification strategies are less sensitive to minor changes in the input parameters, providing stable allocations that are less affected by transaction costs. Furthermore, these strategies do not require accurate predictions of future returns and are effective in controlling investment risk.

Keywords Minimum risk · Maximum diversification · Out-of-sample risk · Out-of-sample performance

Mathematics Subject Classification C02 · G11

Introduction

In finance, the idea that diversification of an investment should reduce its risk is widely accepted. The relationship between risk and diversification is assumed in standard axiomatic risk measures through the sub-additivity property (see Artzner et al. 1999). This property states that the risk of a portfolio is lower than or, at least, equal to the sum of the risks of its individual constituents.

Following the original idea of Markowitz (1952), modern portfolio theory proposed a multitude of models that calculate the optimal portfolio while minimizing a given risk measure. Markowitz used the variance of returns to measure risk. This choice is intuitive and simplifies the computations, as the optimal allocation problem becomes a standard quadratic programming problem. Other authors have proposed calculating the optimal portfolio by minimizing alternative risk measures: for the value at risk, see Gaivoronski and Pflug (2005); for the conditional value at risk, see Krokhmal et al. (2002) and Rockafellar and Uryasev (2000); for the mean absolute deviation, see Moon and Yao (2011); and for the maximum drawdown, see Chekhlov et al. (2005).

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The drawbacks of the standard mean–variance model have been extensively studied in the literature. The Markowitz model is famous for the difficulty of its practical implementation (see Best and Hlouskova (2008) and Hirschberger et al. (2010)) and its poor out-of-sample performance (see DeMiguel et al. (2009) and Gelmini and Uberti (2024)). The literature has identified the extreme numerical instability of the optimal allocation as the principal cause of these issues. Infinitesimal variations in the inputs of the model—namely, expected returns and the covariance matrix—often produce large changes in the optimal allocation, leading to computational inaccuracy and uncertainty. Consequently, the in-sample mean–variance frontier is a biased estimator of the real efficient frontier (see Kan and Smith (2008)). Numerous authors, such as Best and Grauer (1991), Kan and Zhou (2007), and Pflug et al. (2012), consider estimation uncertainty as the primary source of instability in the model. Others identify a structural component of instability in the linear restrictions of the optimization problem (see Fassino et al. (2022)). To improve numerical stability, several alternative proposals have been discussed in the literature: among others, we recall the Bayesian approach (see Frost and Savarino (1986)); the shrinkage approach (see Ledoit and Wolf (2003a)); robust optimization techniques (see Kim et al. (2014); Rosadi et al. (2020)); and lasso techniques (see Brodie et al. (2009)). Although the literature has primarily focused on the issues of the mean–variance model, similar shortcomings affect all risk minimization approaches.

Parallel to these studies, an important stream of empirical research has shown the poor performance of optimization-based allocation strategies (see, among others, DeMiguel et al. (2009), Yuan and Zhou (2022)). The equally weighted portfolio plays a central role in this context. While often termed “naive diversification,” the equally weighted portfolio is the solution to a maximum diversification problem where simple weight-based diversification measures are used, such as the Gini index (see Gini (1921)), the Herfindal–Hirschman index (see Hirschman (1964)), and Shannon entropy (see Shannon (1948)). For a detailed review of diversification measures, refer to Lhabitant (2017).

Numerous researchers have focused on attempting to find non-trivial portfolio allocation strategies capable of outperforming the equally weighted portfolio (see, among others, Kritzman et al. (2010), Kirby and Ostdiek (2012), Ackermann et al. (2017), Bessler et al. (2017), Fugazza et al. (2015), Hanke et al. (2019), Jiang et al. (2019), and Yuan and Zhou (2022)). The debate is still active and far from being concluded.

We begin with the idea that a diversification measure can be induced from a given risk measure. These approaches to diversification measurement represent a rapidly emerging research topic. Among the various contributions, we recall the papers by Cesarone and Colucci (2018), Cesarone et al. (2023), Embrechts et al. (2009), and Tasche (2006), along with the remarkable and recent approach based on the diversification quotient introduced in Han et al. (2023) and Han et al. (2024) via an axiomatic foundation, and the Geometric Portfolio Diversification Measure (GPDM) (see Torrente and Uberti (2024)).

For a given risk measure, we calculate two alternative optimal asset allocations. The first allocation is obtained by minimizing the risk measure. The second allocation is calculated by maximizing the diversification induced by the same risk measure. Of course, by construction, maximum diversification is sub-optimal in terms of risk in-sample, but it proves to be highly competitive in an out-of-sample framework. The risk measures considered are the variance of returns, the mean absolute deviation, the value at risk (VaR), and the expected shortfall (CVaR) at the 1% and 5% significance levels. The out-of-sample performance of the allocations is compared in terms of risk (using the same risk measures), return, the Sharpe ratio, and average numerical stability. We implement a comprehensive empirical experiment in a rolling window framework, with varying lengths for the estimation window and holding period. Proportional transaction costs are also considered. The experiment is repeated across five databases differing in the number of assets (ranging from 10 to 69), the type of assets (namely, stocks or portfolios of stocks), and the period under analysis.

The results are robust and do not depend on the number of scenarios. In an out-of-sample framework, maximum diversification approaches are highly competitive compared to risk minimization techniques, and several factors contribute to this. The optimally diversified portfolios exhibit more stable allocations, are easier to implement, and are less affected by transaction costs. The most unexpected result is that, in most cases, the maximum diversified portfolio is also less risky than the corresponding minimum risk portfolio. The literature reports other instances of such counterintuitive results (see, for example, Jagannathan and Ma (2003)). Of course, this outcome is limited to the out-of-sample framework, as we have noted that the risk minimization approach, by definition, provides the optimal strategy in-sample. In-sample, a risk measure only describes certain features of the distribution of past returns—such as dispersion or the thickness of the left tail—unless the past return distribution serves as a reliable proxy for the future distribution. The risk of an investment lies in the uncertainty of future returns. An approach based on risk minimization requires highly accurate predictions of future returns to perform effectively. In contrast, maximum diversification strategies do not rely on such predictions. This occurs because it is easier to bet on the future level of portfolio diversification than to predict the distribution of future returns. Consequently, significant risk reduction is achieved as a secondary byproduct of diversification.

The remainder of this paper is organized as follows. "Background Material" section presents the preliminary notions for understanding diversification induced by a risk measure. "The Numerical Experiments" section describes the entire experiment and is divided into two subsections, which cover the general experiment and the databases. "Empirical Results" section provides a detailed report of the application on one database, while the results relative to the other databases are collected in Appendix C. "Conclusions" section concludes the paper.

Background Material

In this section, we introduce risk measures (see "Risk Measures" section) and recall the notions of *Geometric Portfolio Diversification Measures* and *Maximum Diversification* strategy (see "Maximum Diversification Strategy" section).

Risk Measures

Alternative axiomatic characterizations of *risk measures* are provided in numerous different papers (see, among others, Acerbi and Tasche (2002), Artzner et al. (1999), Rachev et al. (2008), Rockafellar et al. (2006)). Mathematically, risk measures are defined as mappings from a set of random variables to the real numbers. A risk measure that is monotone, sub-additive, translation-invariant, and positively homogeneous is called *coherent* (see Artzner et al. (1999)); *deviation* risk measures are normalized, strictly positive, sub-additive, shift-invariant, and positively homogeneous (see Rockafellar et al. (2006)); and *convex* risk measures are normalized, strictly positive, shift-invariant, and convex (see Föllmer and Schied (2002)). The above enumeration is not exhaustive. A more detailed list of the axiomatic definitions of risk measures proposed in the literature is beyond the scope of the present paper. In Appendix A, we recall the definitions of the risk measures (*variance*, *mean absolute deviation* (MAD), *value at risk* (VaR), *expected shortfall* or *conditional value at risk* (CVaR)) that will be used throughout the paper.

Maximum Diversification Strategy

The notion of *Geometric Portfolio Diversification Measures* (GPDMs) refers to a class of functions that, starting from a given risk measure, induces a portfolio diversification measure (see Torrente and Uberti (2024) and Torrente and Uberti (2024)). The optimization problem of maximizing the GPDM admits a unique maximum portfolio, which is attained at the *Risk-adjusted Geometric Diversified Portfolio* (RAGDP) when all long-short portfolios are considered (see Torrente and Uberti (2024, Proposition 3.2)). If the optimization problem is restricted to the set of long-only portfolios, the unique solution is called the *Maximum Diversification* (MD) strategy (see Definition 3). In Appendix B, these concepts are recalled in detail.

The Numerical Experiments

In this section, we provide a comprehensive empirical study to evaluate the performance of the MD strategy (see "Maximum Diversification Strategy" section). In particular, we perform a direct comparison of the MD strategy, defined using different well-known risk measures (see "Risk Measures" section), with the classical *risk minimization* (mr) strategy, aimed at constructing a portfolio that minimizes the overall risk. Mathematically, the mr strategy is obtained by minimizing the risk measure over the set of long-only portfolios $\mathbb{W}_n$, which embodies the core idea of allocating capital across assets in a way that reduces overall exposure to risk.

For the reader's convenience, we outline here the two optimization problems defining the MD and mr strategies.

Optimization problem for *MD strategy*

$$\max_{w \in \mathbb{W}_n} \Phi_{\rho(X)}(w, A) \qquad (1)$$

Optimization problem for *mr strategy*

$$\min_{w \in \mathbb{W}_n} \rho(w, A) \qquad (2)$$

In the numerical experiments, these two active approaches (MD and mr strategies) are also compared with two standard benchmark strategies: the *equally weighted* portfolio (denoted by $1/n$) and the *buy & hold* allocation (denoted by b&h). The results of the comparison are presented in "Empirical Results" section.

Description of the Numerical Experiments

The numerical experiment is performed in a rolling window setting. The length of the estimation window is w_{in} days (weeks or months) and the holding period is w_{out} days (weeks or months). We recall that n denotes the number of risky assets. The risk measures listed in "Risk Measures" section are considered individually.

At the first computation, the first $w_{\text{in}} \times n$ entries of the return matrix are used to estimate the parameters and solve the optimal allocation problems (1) and (2). The performance of the MD and mr strategies is computed for the returns from $w_{\text{in}} + 1$ to $w_{\text{in}} + w_{\text{out}}$. This procedure is then iterated, considering at each step new time slots: at each iteration, the estimation window and the holding period are obtained from the previous ones by shifting the windows w_{out} periods ahead.

To compare the performance of the MD and mr strategies at each iteration, turnover is introduced. The turnover of a strategy is computed as the average absolute difference between the allocation at the end of one holding period and the optimal allocation at the beginning of the subsequent period. The allocation at the end of one holding period differs from the optimal allocation at the beginning of the same period due to price variation. This explains why the equally weighted portfolio has a small but positive turnover. Transaction costs are calculated proportionally to the turnover, using a proportionality coefficient of fifty basis points, as in DeMiguel et al. (2009). The effective returns of a strategy are then obtained by subtracting the implementation costs from the gross returns.

The entire numerical experiment is performed on various datasets (S&P500 sector, DAX, MIB, ESX, and FTSE), which differ in terms of the number of assets and the period under analysis; for a detailed description, see "Datasets" section. Furthermore, in each case, the entire procedure is repeated for the following lengths of the estimation window and the holding period: $w_{\text{in}} = 20, 60, 120, 240$ and $w_{\text{out}} = 1, 5, 20, 60$ for daily data ($w_{\text{out}} = 1, 4, 12$ and $w_{\text{out}} = 1, 3$ for weekly and monthly data, respectively).

Both the parameters w_{in} and w_{out} play a fundamental role in the application. On the one hand, a short estimation window considers only the most recent data, which are usually the most relevant for achieving good predictive performance. It is intuitive that old data fail to reflect actual market conditions. Conversely, risk measures require estimation based on time series of adequate length. The length of the estimation window may depend on the portfolio's size n. In the mean–variance framework, the minimal technical requirement is $w_{\text{in}} \geq n$, which ensures that the covariance matrix is full-rank and invertible. The literature on the topic is extensive; we cite, among others, Bickel and Levina (2008) and Ledoit and Wolf (2003b). More generally, a sufficiently long time series is necessary to obtain a meaningful estimation of a risk measure. The value of w_{in} also affects the numerical stability of the allocation, with direct consequences on turnover. When w_{in} is short, a single observation can strongly impact the estimation of the parameters. Consequently, the optimal allocation may vary significantly from one investing period to the next. The choice of w_{out} also affects the results. The longer the holding period, the less transaction costs erode performance. In this case, the improvement in stability is obtained simply by increasing the length of the holding period and reducing the number of portfolio rebalancings. Both w_{in} and w_{out} are relevant for implementing a profitable active investment strategy.

In each numerical experiment, the MD and mr strategies are compared in terms of return, the Sharpe ratio, risk (using the risk measures presented in "Risk Measures" section and Appendix A), and *average (numerical) stability* (aveSt). Computationally, the average stability is calculated by dividing the turnover of each strategy by w_{out}, obtaining a value that does not depend on the length of the holding period. This allows for direct comparison across cases with different values of w_{out}. Moreover, we replace the usual notion of turnover with the aveSt to highlight the following point. Each investment portfolio is subject to a variation in composition from the beginning to the end of the holding period due to price variation, except for the passive buy-and-hold strategy. Price variation is common to each strategy under comparison. Therefore, even if turnover is appropriate for calculating the effective proportional transaction costs, what truly differentiates the active strategies in terms of implementation costs is their own numerical stability.

Datasets

The real data application is performed on the returns of five different financial indexes. For simplicity, assets without a complete time series over the chosen period have been removed from the databases. Consequently, the number n of effective constituents of the financial indexes is smaller than the nominal one; it ranges from 10 for the S&P500 sector to 69 for the FTSE, allowing the strategies to be tested across different portfolio sizes. To further verify the robustness of our findings, the databases also differ in terms of the referring period, for the time horizon, reflecting different periods of analysis (daily, weekly, or monthly returns), and for the assets, which may be individual stocks or portfolios of stocks. The datasets are as follows:

1. S&P500 sector ($n = 10$) comprises daily returns from 01/03/1990 to 09/17/2020 for ten sectoral portfolios of the S&P500 index, obtained following the Global Industry Classification Standard (GICS): Material, Information-Technology, Healthcare, Energy, Utilities, Financials, Consumer-Staples, Consumer-Discretionary, Industrials, Telecommunications.
2. S&P500 sector ($n = 10$)—weekly—comprises weekly returns from 01/03/1990 to 09/17/2020 for ten sectoral portfolios of the S&P500 dataset.
3. S&P500 sector ($n = 10$)—monthly—comprises monthly returns from 01/03/1990 to 09/17/2020 for ten sectoral portfolios of the S&P500 dataset.
4. DAX ($n = 23$) is the stock index of major German blue-chip companies trading on the Frankfurt Stock Exchange. We consider daily returns from 01/05/2005 to 09/17/2020.
5. MIB ($n = 30$) contains daily returns from 01/04/2010 to 04/11/2023 of the most traded stocks on the Borsa Italiana, the Italian national stock exchange.
6. MIB ($n = 30$)—weekly—contains weeky returns from 01/04/2010 to 04/11/2023 of the most traded stocks on the Borsa Italiana.
7. MIB ($n = 30$)—monthly—contains monthly returns from 01/04/2010 to 04/11/2023 of the most traded stocks on the Borsa Italiana.
8. ESX ($n = 38$) is a Eurozone stock index comprising 50 European blue-chip companies considered as leaders in their respective sectors. We consider daily returns from 01/05/2005 to 09/17/2020.
9. FTSE ($n = 69$) is the Financial Times Stock Exchange 100 Index, composed of the 100 companies listed on the London Stock Exchange with the highest market capitalization. We consider daily returns from 01/05/2005 to 09/17/2020.

Table 1 Performance of the buy & hold (b&h) strategy computed on the MIB dataset

	b&h
aveSt	0
stDev	0.0156
SR	0.0029
VaR (1%)	0.0446
VaR (5%)	0.0258
CVaR (1%)	0.0470
CVaR (5%)	0.0284
MAD	0.0110

Empirical Results

The results of the empirical applications and the associated comments are robust with respect to the databases. Consequently, in "Results on the MIB Dataset" section, we present the detailed results and corresponding comments only for one database, the MIB dataset. The tables summarizing the results for all other databases (see "Datasets" section) are included in Appendix C. Specific comments and peculiarities of the findings that depend on the individual dataset are summarized in "Comments on the Other Databases" section.

Results on the MIB Dataset

In the following, we discuss in detail the application to the MIB dataset (daily returns). Table 1 reports the performance of the buy & hold strategy (b&h). The overall results are summarized in Tables 2, 3, 4, 5, 6, and 7, which present the numerical experiment for all possible combinations of $w_{\text{in}} = 20, 60, 120, 240$ and $w_{\text{out}} = 1, 5, 20, 60$. The MD strategy is computed by solving the maximization problem (1), where the following risk measures are considered: variance, mean absolute deviation (MAD), value at risk (VaR), and expected shortfall (CVaR) at significance levels of 1% and 5% (see "Risk Measures" section). Similarly, the mr strategy is implemented by solving the minimization problem (2), based on the aforementioned risk measures. Each table covers a specific risk measure: Table 2 for variance; Table 3 for MAD; Tables 4 and 5 for VaR at 1% and 5%, respectively; and Tables 6 and 7 for CVaR at 1% and 5%, respectively.

We do not comment on each individual table (i.e., the behavior of the MD and mr strategies under a specific risk measure). The results are qualitatively highly consistent across Tables 2, 3, 4, 5, 6, and 7, showing that the choice of risk measure does not appear to be fundamental. Rather, we discuss the overall behavior of the strategies under comparison concerning the performance evaluation measures. We also examine the effects when w_{in} and w_{out} vary.

Regarding average stability (aveSt), the interpretation is straightforward: a null or infinitesimal value of aveSt characterizes almost passive investment strategies, namely b&h (see Table 1) and $1/n$, as opposed to active strategies, namely MD and mr. Among the active strategies, we note that the MD strategies consistently exhibit lower aveSt values than the mr strategies, which is observed for all w_{in} and w_{out} values and all risk measures. This implies that, among the active strategies, the impact of transaction costs is always lower for the MD strategies than for the mr strategies. As expected, active strategies are not competitive with passive strategies in terms of transaction costs.

The standard deviation (stDev) of returns is higher for the benchmark passive strategies, b&h (see Table 1) and

Table 2 Performance of the equally weighted portfolio ($1/n$), the maximum diversification (MD) and the minimum risk (mr) strategies computed with respect to the variance on the MIB dataset

Variance			$w_{in}=20$		$w_{in}=60$		$w_{in}=120$		$w_{in}=240$	
w_{out}		$1/n$	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2×10^{-16}	–	0.2299	0.1328	0.0826	0.0670	0.0430	0.0350	0.0232
	stDev	0.0146	–	0.0111	0.0108	0.0109	0.0105	0.0108	0.0104	0.0109
	SR	0.0057	–	-0.0695	-0.0297	-0.0135	0.0013	0.0075	0.0133	0.0085
	VaR (1%)	0.0406	–	0.0331	0.0314	0.0311	0.0291	0.0306	0.0295	0.0304
	VaR (5%)	0.0245	–	0.0184	0.0182	0.0177	0.0174	0.0171	0.0171	0.0174
	CVaR (1%)	0.0431	-	0.0353	0.0335	0.0324	0.0308	0.0315	0.0308	0.0326
	CVaR (5%)	0.0266	–	0.0185	0.0193	0.0181	0.0178	0.0189	0.0174	0.0175
	MAD	0.0102	–	0.0078	0.0075	0.0075	0.0072	0.0073	0.0070	0.0073
5	aveSt	0.0048	–	0.1188	0.0672	0.0458	0.0347	0.0243	0.0190	0.0138
	stDev	0.0146	–	0.0113	0.0110	0.0110	0.0106	0.0109	0.0105	0.0110
	SR	0.0041	–	-0.0289	-0.0014	0.0012	0.0154	0.0148	0.0195	0.0115
	VaR (1%)	0.0406	–	0.0331	0.0310	0.0313	0.0293	0.0308	0.0296	0.0305
	VaR (5%)	0.0245	–	0.0186	0.0181	0.0176	0.0173	0.0171	0.0172	0.0176
	CVaR (1%)	0.0431	–	0.0334	0.0327	0.0325	0.0308	0.0327	0.0297	0.0320
	CVaR (5%)	0.0266	–	0.0190	0.0202	0.0179	0.0193	0.0178	0.0181	0.0194
	MAD	0.0102	–	0.0079	0.0076	0.0076	0.0072	0.0074	0.0071	0.0073
20	aveSt	0.0026	–	0.0572	0.0327	0.0248	0.0183	0.0139	0.0105	0.0079
	stDev	0.0146	–	0.0115	0.0112	0.0111	0.0108	0.0110	0.0106	0.0111
	SR	0.0048	–	0.0017	0.0145	0.0119	0.0240	0.0212	0.0235	0.0151
	VaR (1%)	0.0406	–	0.0332	0.0303	0.0313	0.0298	0.0306	0.0298	0.0302
	VaR (5%)	0.0245	–	0.0187	0.0182	0.0175	0.0175	0.0174	0.0172	0.0178
	CVaR (1%)	0.0431	–	0.0334	0.0308	0.0324	0.0314	0.0332	0.0324	0.0305
	CVaR (5%)	0.0266	–	0.0196	0.0200	0.0187	0.0176	0.0192	0.0192	0.0193
	MAD	0.0102	–	0.0080	0.0077	0.0076	0.0073	0.0074	0.0071	0.0073
60	aveSt	0.0016	–	0.0185	0.0176	0.0145	0.0109	0.0085	0.0066	0.0053
	stDev	0.0146	–	0.0118	0.0113	0.0113	0.0109	0.0111	0.0108	0.0112
	SR	0.0052	–	0.0141	0.0180	0.0068	0.0205	0.0144	0.0204	0.0128
	VaR (1%)	0.0406	–	0.0338	0.0303	0.0322	0.0302	0.0313	0.0300	0.0311
	VaR (5%)	0.0245	–	0.0188	0.0183	0.0183	0.0177	0.0177	0.0174	0.0180
	CVaR (1%)	0.0431	–	0.0341	0.0308	0.0324	0.0313	0.0332	0.0323	0.0333
	CVaR (5%)	0.0266	–	0.0203	0.0199	0.0187	0.0203	0.0192	0.0191	0.0193
	MAD	0.0102	–	0.0081	0.0078	0.0077	0.0074	0.0075	0.0072	0.0074

$1/n$. Active strategies can significantly reduce risk, which is a central finding throughout the paper: active strategies consistently outperform in terms of risk. In all combinations of w_{in} and w_{out}, across all databases and risk measures, this result is consistently confirmed. Only the mr strategy, in a very limited number of scenarios with too short estimation window, does not exhibit a risk reduction compared to the passive benchmarks. Moreover, the risk reduction is obtained independently of the risk measures used to calculate the optimal allocation and evaluate the results. In other words, optimizing the allocation concerning one risk measure—either by minimizing the risk or maximizing the induced diversification—leads to a reduction in out-of-sample risk for both the MD and mr strategies across the remaining risk measures. The most interesting fact about the out-of-sample risk reduction is that in the majority of the cases (across all databases, scenarios, and risk measures), the MD strategies outperform the mr strategies in terms of risk.

Consequently, when comparing the strategies based on the Sharpe ratio (SR), the MD strategies are frequently preferable among the active strategies. The Sharpe ratio is the most widely used risk-adjusted performance measure in both academic and practical contexts. The MD and the mr strategies do not incorporate returns in the optimization process. In this framework, the only way to affect the numerator of the Sharpe ratio is by reducing transaction costs. This can only be achieved by increasing the stability of the active allocation strategies. We note that maximum diversification is highly competitive with passive benchmarks in terms of the Sharpe ratio, including for short estimation windows.

One further aspect to underline is that the MD approaches seem to perform very well even with short estimation

Table 3 Performance of the equally weighted portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed with respect to the MAD on the MIB dataset

MAD			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2×10^{-16}	0.4840	0.2053	0.2260	0.0713	0.1379	0.0361	0.0859	0.0186
	stDev	0.0146	0.0118	0.0110	0.0109	0.0109	0.0105	0.0108	0.0104	0.0108
	SR	0.0057	-0.1782	-0.0658	-0.0691	-0.0118	-0.0294	0.0071	-0.0100	0.0105
	VaR (1%)	0.0406	0.0382	0.0326	0.0318	0.0317	0.0296	0.0307	0.0292	0.0311
	VaR (5%)	0.0245	0.0215	0.0186	0.0186	0.0177	0.0175	0.0172	0.0172	0.0173
	CVaR (1%)	0.0431	0.0385	0.0327	0.0318	0.0330	0.0312	0.0330	0.0311	0.0323
	CVaR (5%)	0.0266	0.0233	0.0203	0.0199	0.0202	0.0193	0.0198	0.0184	0.0190
	MAD	0.0102	0.0083	0.0077	0.0076	0.0074	0.0072	0.0073	0.0070	0.0072
5	aveSt	0.0048	0.1791	0.0980	0.0830	0.0350	0.0491	0.0181	0.0286	0.0099
	stDev	0.0146	0.0121	0.0111	0.0111	0.0109	0.0106	0.0108	0.0105	0.0109
	SR	0.0041	-0.0539	-0.0241	-0.0057	0.0030	0.0109	0.0142	0.0152	0.0135
	VaR (1%)	0.0406	0.0368	0.0331	0.0313	0.0315	0.0294	0.0308	0.0291	0.0310
	VaR (5%)	0.0245	0.0202	0.0182	0.0180	0.0177	0.0171	0.0173	0.0170	0.0173
	CVaR (1%)	0.0431	0.0392	0.0332	0.0333	0.0335	0.0295	0.0318	0.0308	0.0319
	CVaR (5%)	0.0266	0.0219	0.0204	0.0193	0.0178	0.0174	0.0187	0.0178	0.0186
	MAD	0.0102	0.0085	0.0078	0.0076	0.0075	0.0073	0.0073	0.0070	0.0072
20	aveSt	0.0026	0.0699	0.0477	0.0368	0.0184	0.0221	0.0099	0.0124	0.0055
	stDev	0.0146	0.0120	0.0113	0.0112	0.0110	0.0108	0.0109	0.0106	0.0109
	SR	0.0048	0.0023	0.0021	0.0148	0.0129	0.0235	0.0184	0.0207	0.0151
	VaR (1%)	0.0406	0.0336	0.0334	0.0304	0.0315	0.0296	0.0311	0.0291	0.0313
	VaR (5%)	0.0245	0.0192	0.0183	0.0181	0.0177	0.0171	0.0173	0.0170	0.0174
	CVaR (1%)	0.0431	0.0352	0.0348	0.0326	0.0339	0.0320	0.0320	0.0303	0.0316
	CVaR (5%)	0.0266	0.0193	0.0207	0.0187	0.0201	0.0183	0.0181	0.0192	0.0181
	MAD	0.0102	0.0084	0.0079	0.0076	0.0075	0.0073	0.0073	0.0071	0.0072
60	aveSt	0.0016	0.0244	0.0162	0.0183	0.0107	0.0124	0.0062	0.0074	0.0037
	stDev	0.0146	0.0127	0.0117	0.0113	0.0112	0.0109	0.0110	0.0108	0.0110
	SR	0.0052	-0.0008	0.0124	0.0189	0.0138	0.0226	0.0173	0.0181	0.0150
	VaR (1%)	0.0406	0.0359	0.0334	0.0309	0.0320	0.0303	0.0316	0.0300	0.0317
	VaR (5%)	0.0245	0.0200	0.0189	0.0182	0.0178	0.0175	0.0173	0.0172	0.0174
	CVaR (1%)	0.0431	0.0368	0.0358	0.0325	0.0339	0.0320	0.0320	0.0303	0.0343
	CVaR (5%)	0.0266	0.0219	0.0197	0.0186	0.0201	0.0182	0.0181	0.0192	0.0180
	MAD	0.0102	0.0086	0.0081	0.0078	0.0076	0.0074	0.0074	0.0072	0.0073

windows, whereas risk minimization requires longer estimation windows. This trend is consistently observed across all tables. The MD approach delivers very strong results even starting from short values of w_{in}, while the mr strategies become competitive when the length of the estimation window increases. This is not merely a matter of numerical stability or related transaction costs. The mr strategies require longer estimation windows to effectively reduce risk. In contrast, the risk reduction achieved through diversification in the MD strategies is significant even starting from short estimation windows, with only marginal benefits from increasing w_{in}. The empirical evidence thus suggests adopting maximum diversification when w_{in} is short and, eventually, minimum risk when w_{in} is long. Reviewing the entire set of results of the application, it is challenging to identify a single scenario where the minimum risk approach is explicitly preferable.

The question of estimation window length is central to these applications. A deeper analysis of this point is beyond the scope of the present research. We only underline that, in the present framework—which is very common in empirical financial applications—it may be difficult to determine whether the increased performance associated with longer estimation windows depends on an improved quality of the estimated parameters or a reduced level of transaction costs due to increased stability. While the latter effect is certain, the former is difficult to evaluate and measure.

Our experiment, unlike standard ones, also introduces the parameter w_{out} to change the length of the holding period. This possibility was suggested by different authors, such as Chan et al. (1999) and Jagannathan and Ma (2003). A longer

Table 4 Performance of the equally weighted portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed with respect to the VaR (1%) on the MIB dataset

VaR 1%			$w_{in}=20$		$w_{in}=60$		$w_{in}=120$		$w_{in}=240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2×10^{-16}	0.7343	0.2361	0.4817	0.0775	0.4221	0.0414	0.3492	0.0216
	stDev	0.0146	0.0121	0.0110	0.0118	0.0111	0.0114	0.0109	0.0110	0.0109
	SR	0.0057	-0.2738	-0.0762	-0.1697	-0.0092	-0.1575	0.0063	-0.1307	0.0059
	VaR (1%)	0.0406	0.0363	0.0327	0.0345	0.0317	0.0342	0.0321	0.0333	0.0312
	VaR (5%)	0.0245	0.0233	0.0192	0.0211	0.0192	0.0206	0.0178	0.0204	0.0180
	CVaR (1%)	0.0431	0.0383	0.0337	0.0357	0.0339	0.0345	0.0346	0.0356	0.0323
	CVaR (5%)	0.0266	0.0234	0.0198	0.0219	0.0202	0.0210	0.0188	0.0213	0.0192
	MAD	0.0102	0.0087	0.0079	0.0082	0.0078	0.0078	0.0075	0.0075	0.0073
5	aveSt	0.0048	0.2082	0.1199	0.1221	0.0507	0.0973	0.0285	0.0773	0.0166
	stDev	0.0146	0.0124	0.0113	0.0119	0.0113	0.0115	0.0110	0.0111	0.0110
	SR	0.0041	-0.0505	-0.0312	-0.0182	0.0022	-0.0222	0.0099	-0.0166	0.0065
	VaR (1%)	0.0406	0.0345	0.0327	0.0325	0.0318	0.0323	0.0322	0.0314	0.0310
	VaR (5%)	0.0245	0.0209	0.0192	0.0192	0.0189	0.0189	0.0178	0.0190	0.0179
	CVaR (1%)	0.0431	0.0365	0.0332	0.0330	0.0322	0.0327	0.0338	0.0328	0.0311
	CVaR (5%)	0.0266	0.0217	0.0206	0.0199	0.0197	0.0195	0.0199	0.0204	0.0205
	MAD	0.0102	0.0088	0.0080	0.0081	0.0079	0.0078	0.0075	0.0075	0.0074
20	aveSt	0.0026	0.0731	0.0600	0.0447	0.0314	0.0318	0.0179	0.0247	0.0105
	stDev	0.0146	0.0129	0.0116	0.0125	0.0115	0.0114	0.0112	0.0110	0.0111
	SR	0.0048	-0.0039	-0.0045	0.0012	0.0101	0.0050	0.0114	0.0080	0.0070
	VaR (1%)	0.0406	0.0351	0.0340	0.0354	0.0325	0.0323	0.0317	0.0309	0.0317
	VaR (5%)	0.0245	0.0210	0.0191	0.0198	0.0189	0.0187	0.0180	0.0186	0.0181
	CVaR (1%)	0.0431	0.0364	0.0353	0.0371	0.0327	0.0324	0.0325	0.0322	0.0334
	CVaR (5%)	0.0266	0.0218	0.0217	0.0203	0.0191	0.0195	0.0189	0.0197	0.0200
	MAD	0.0102	0.0089	0.0081	0.0083	0.0079	0.0078	0.0076	0.0074	0.0074
60	aveSt	0.0016	0.0234	0.0198	0.0206	0.0186	0.0143	0.0105	0.0119	0.0070
	stDev	0.0146	0.0137	0.0120	0.0119	0.0114	0.0114	0.0112	0.0112	0.0112
	SR	0.0052	0.0041	0.0154	0.0050	0.0057	0.0044	0.0090	0.0086	0.0081
	VaR (1%)	0.0406	0.0384	0.0336	0.0341	0.0337	0.0335	0.0314	0.0320	0.0323
	VaR (5%)	0.0245	0.0212	0.0192	0.0198	0.0189	0.0187	0.0182	0.0187	0.0182
	CVaR (1%)	0.0431	0.0385	0.0354	0.0343	0.0354	0.0349	0.0325	0.0322	0.0334
	CVaR (5%)	0.0266	0.0213	0.0195	0.0203	0.0192	0.0195	0.0189	0.0198	0.0200
	MAD	0.0102	0.0092	0.0083	0.0083	0.0079	0.0078	0.0076	0.0075	0.0075

holding period certainly reduces the implementation costs of a strategy by limiting the frequency of rebalancing. Since we use daily observations, the parameter w_{out} also serves to make the experiment more realistic. A daily asset allocation corresponds to the rebalancing frequency of a trading system, whereas standard allocation strategies rebalance portfolios weekly, monthly, or less frequently.

One final point concerns the behavior of individual risk measures. It is impossible to identify one risk measure that outperforms the others. Each risk measure, when used to either minimize risk or maximize induced diversification, produces a generally observable risk reduction, independently of the risk measure used to quantify risk. As is typically the case in out-of-sample exercises, the optimal risk reduction for a given risk measure is not necessarily obtained when that same measure is used to calculate the allocations.

Finally, to summarize our main findings, we fix one scenario: $w_{in}=240$, $w_{out}=60$, and CVaR (5%) (see Table 7). We provide a graphical representation of this scenario, simplifying the intuition and interpretation of the figures reported in Tables 2, 3, 4, 5, 6, and 7. Figures 1 and 2 highlight the three primary findings of the research. Figure 1 compares the evolution over time of the optimal allocations. It is evident that the MD strategy is more stable than the mr strategy. This aspect is quantitatively measured by aveSt. A stable allocation reduces the influence of transaction costs on the implementation of the investment strategy, thereby

Table 5 Performance of the equally weighted portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed with respect to the VaR (5%) on the MIB dataset

VaR 5%			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2×10^{-16}	0.6216	0.2313	0.4713	0.0809	0.4011	0.0420	0.3210	0.0217
	stDev	0.0146	0.0119	0.0110	0.0111	0.0108	0.0108	0.0107	0.0106	0.0108
	SR	0.0057	-0.2387	-0.0767	-0.1774	-0.0114	-0.1441	0.0053	-0.1173	0.0084
	VaR (1%)	0.0406	0.0341	0.0324	0.0337	0.0319	0.0322	0.0307	0.0316	0.0307
	VaR (5%)	0.0245	0.0231	0.0193	0.0205	0.0180	0.0195	0.0174	0.0190	0.0174
	CVaR (1%)	0.0431	0.0364	0.0344	0.0356	0.0325	0.0328	0.0312	0.0338	0.0323
	CVaR (5%)	0.0266	0.0243	0.0202	0.0227	0.0198	0.0210	0.0189	0.0208	0.0188
	MAD	0.0102	0.0085	0.0079	0.0078	0.0075	0.0075	0.0073	0.0072	0.0071
5	aveSt	0.0048	0.1925	0.1184	0.1242	0.0447	0.0971	0.0243	0.0722	0.0128
	stDev	0.0146	0.0120	0.0113	0.0115	0.0110	0.0109	0.0108	0.0107	0.0108
	SR	0.0041	-0.0553	-0.0323	-0.0202	0.0029	-0.0160	0.0129	-0.0072	0.0122
	VaR (1%)	0.0406	0.0324	0.0337	0.0323	0.0322	0.0310	0.0306	0.0299	0.0306
	VaR (5%)	0.0245	0.0206	0.0192	0.0189	0.0180	0.0183	0.0174	0.0178	0.0174
	CVaR (1%)	0.0431	0.0336	0.0347	0.0330	0.0329	0.0313	0.0315	0.0322	0.0322
	CVaR (5%)	0.0266	0.0220	0.0217	0.0206	0.0194	0.0195	0.0181	0.0178	0.0186
	MAD	0.0102	0.0086	0.0080	0.0079	0.0075	0.0075	0.0073	0.0072	0.0072
20	aveSt	0.0026	0.0727	0.0587	0.0445	0.0244	0.0307	0.0136	0.0230	0.0072
	stDev	0.0146	0.0125	0.0116	0.0115	0.0111	0.0110	0.0109	0.0107	0.0108
	SR	0.0048	-0.0082	-0.0037	0.0086	0.0137	0.0133	0.0191	0.0120	0.0133
	VaR (1%)	0.0406	0.0352	0.0338	0.0327	0.0314	0.0307	0.0308	0.0304	0.0311
	VaR (5%)	0.0245	0.0207	0.0191	0.0186	0.0179	0.0180	0.0175	0.0177	0.0174
	CVaR (1%)	0.0431	0.0375	0.0347	0.0336	0.0320	0.0312	0.0322	0.0305	0.0324
	CVaR (5%)	0.0266	0.0231	0.0211	0.0191	0.0184	0.0204	0.0186	0.0194	0.0189
	MAD	0.0102	0.0087	0.0081	0.0079	0.0076	0.0075	0.0074	0.0073	0.0072
60	aveSt	0.0016	0.0236	0.0195	0.0195	0.0134	0.0142	0.0084	0.0101	0.0047
	stDev	0.0146	0.0132	0.0120	0.0116	0.0111	0.0110	0.0109	0.0107	0.0109
	SR	0.0052	0.0107	0.0165	0.0090	0.0148	0.0189	0.0137	0.0161	0.0138
	VaR (1%)	0.0406	0.0361	0.0336	0.0331	0.0319	0.0307	0.0315	0.0301	0.0315
	VaR (5%)	0.0245	0.0208	0.0191	0.0190	0.0180	0.0180	0.0176	0.0177	0.0173
	CVaR (1%)	0.0431	0.0384	0.0360	0.0335	0.0320	0.0312	0.0322	0.0305	0.0324
	CVaR (5%)	0.0266	0.0227	0.0200	0.0191	0.0183	0.0204	0.0186	0.0194	0.0188
	MAD	0.0102	0.0091	0.0083	0.0081	0.0076	0.0076	0.0074	0.0073	0.0073

significantly impacting returns. Since expected returns are not considered by either the MD or mr strategies, acting on costs is the sole way to influence returns.

Figure 2 compares the distributions of the out-of-sample returns. To obtain a smooth representation of these distributions, we used the kernel smoothing function *ksdensity* included in MathWorks (2021). The main observation is that both the MD and mr approaches symmetrically reduce the dispersion of the out-of-sample return distributions, affecting both the left and right tails. In this case, as in the majority of scenarios, the dispersion of returns is smaller for the MD strategy than for the mr strategy. This fact is reflected in Tables 2, 3, 4, 5, 6, and 7 by the values of stDev and MAD. Both the MD and mr strategies reduce risk compared to the equally weighted (1/*n*) and b&hold (b&h) strategies (see Table 1).

Figure 2, Panel (b) focuses on the left tails of the return distributions to visualize the effect of risk reduction when asymmetric risk measures are considered. In particular, we observe that the MD and mr strategies are associated with return distributions characterized by a thinner left tail than the equally weighted and the buy & hold allocations, hence they are superior in terms of VaR and CVaR. Again, the MD approach outperforms the mr strategy also in terms of the thickness of the left tail—namely, in terms of the probability of extreme losses.

Table 6 Performance of the equally weighted portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed with respect to the CVaR (1%) on the MIB dataset

CVaR 1%			$w_{in}=20$		$w_{in}=60$		$w_{in}=120$		$w_{in}=240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2×10^{-16}	0.7646	0.2399	0.6433	0.0927	0.5884	0.0553	0.5344	0.0308
	stDev	0.0146	0.0121	0.0110	0.0125	0.0111	0.0117	0.0109	0.0116	0.0109
	SR	0.0057	-0.2869	-0.0780	-0.2399	-0.0159	-0.2267	0.0001	-0.2076	0.0020
	VaR (1%)	0.0406	0.0368	0.0327	0.0376	0.0317	0.0364	0.0322	0.0351	0.0311
	VaR (5%)	0.0245	0.0239	0.0193	0.0229	0.0192	0.0226	0.0178	0.0220	0.0180
	CVaR (1%)	0.0431	0.0386	0.0342	0.0394	0.0337	0.0369	0.0347	0.0374	0.0329
	CVaR (5%)	0.0266	0.0261	0.0203	0.0244	0.0199	0.0239	0.0189	0.0224	0.0197
	MAD	0.0102	0.0087	0.0079	0.0084	0.0078	0.0082	0.0075	0.0080	0.0073
5	aveSt	0.0048	0.2028	0.1199	0.1529	0.0519	0.1360	0.0308	0.1223	0.0186
	stDev	0.0146	0.0125	0.0113	0.0119	0.0113	0.0119	0.0110	0.0117	0.0110
	SR	0.0041	-0.0543	-0.0313	-0.0374	0.0019	-0.0342	0.0090	-0.0400	0.0058
	VaR (1%)	0.0406	0.0348	0.0327	0.0340	0.0318	0.0332	0.0321	0.0341	0.0311
	VaR (5%)	0.0245	0.0210	0.0192	0.0200	0.0189	0.0199	0.0178	0.0202	0.0180
	CVaR (1%)	0.0431	0.0374	0.0333	0.0360	0.0323	0.0343	0.0331	0.0352	0.0329
	CVaR (5%)	0.0266	0.0233	0.0206	0.0220	0.0198	0.0200	0.0193	0.0225	0.0196
	MAD	0.0102	0.0088	0.0080	0.0083	0.0079	0.0082	0.0075	0.0080	0.0074
20	aveSt	0.0026	0.0667	0.0597	0.0466	0.0314	0.0393	0.0181	0.0329	0.0107
	stDev	0.0146	0.0125	0.0116	0.0121	0.0115	0.0120	0.0112	0.0117	0.0111
	SR	0.0048	-0.0104	-0.0042	0.0016	0.0103	0.0058	0.0112	0.0006	0.0073
	VaR (1%)	0.0406	0.0359	0.0339	0.0341	0.0325	0.0349	0.0317	0.0336	0.0315
	VaR (5%)	0.0245	0.0208	0.0192	0.0201	0.0189	0.0194	0.0180	0.0191	0.0180
	CVaR (1%)	0.0431	0.0365	0.0346	0.0343	0.0348	0.0349	0.0330	0.0349	0.0338
	CVaR (5%)	0.0266	0.0231	0.0210	0.0207	0.0212	0.0212	0.0195	0.0213	0.0206
	MAD	0.0102	0.0088	0.0081	0.0084	0.0079	0.0082	0.0076	0.0079	0.0074
60	aveSt	0.0016	0.0221	0.0198	0.0189	0.0186	0.0158	0.0105	0.0124	0.0070
	stDev	0.0146	0.0130	0.0119	0.0122	0.0114	0.0120	0.0112	0.0119	0.0112
	SR	0.0052	0.0015	0.0158	0.0072	0.0055	0.0061	0.0093	0.0081	0.0087
	VaR (1%)	0.0406	0.0376	0.0335	0.0356	0.0336	0.0349	0.0314	0.0339	0.0321
	VaR (5%)	0.0245	0.0209	0.0192	0.0199	0.0189	0.0196	0.0183	0.0193	0.0182
	CVaR (1%)	0.0431	0.0376	0.0353	0.0370	0.0348	0.0349	0.0330	0.0349	0.0338
	CVaR (5%)	0.0266	0.0210	0.0194	0.0206	0.0212	0.0212	0.0195	0.0213	0.0206
	MAD	0.0102	0.0091	0.0083	0.0085	0.0079	0.0082	0.0076	0.0080	0.0075

Comments on the Other Databases

The results of the experiment on the S&P500, DAX, MIB (weekly and monthly data), ESX, and FTSE databases are resumed in Appendix C. The main findings are similar to those detailed in "Results on the MIB Dataset" section, where the analysis is performed on the MIB database. Hence, this section briefly describes the few relevant differences across the different databases.

When variance is used as the risk measure, the mr strategy coincides with the global minimum variance portfolio. If the portfolio size n exceeds w_{in}, the covariance matrix is singular and the optimization problem is not well-defined. In particular, the uniqueness of the mean–variance solution is lost (see Pappas et al. (2010)). For this reason, the corresponding values in the tables are missing in these cases (see Table 15, case $w_{in}=20$, column mr and Table 21, cases $w_{in}=20$ and $w_{in}=60$, column mr). In the application on the S&P500 sector database, the differences between the MD and mr strategies appear less pronounced than in the other applications. In this experiment, the MD portfolio is highly competitive, but in our view, it is not definitively superior to the mr strategy. Numerous possible explanations exist for this outcome. First, this database contains the fewest constituents, likely limiting the available diversification opportunities. Second, the asset classes consist of portfolios of stocks, hence the mr approach benefits from the fact

Table 7 Performance of the equally weighted portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed with respect to the CVaR (5%) on the MIB dataset

CVaR 5%			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2×10^{-16}	0.7165	0.2393	0.5895	0.0983	0.5119	0.0570	0.4635	0.0353
	stDev	0.0146	0.0121	0.0110	0.0120	0.0108	0.0120	0.0107	0.0119	0.0108
	SR	0.0057	-0.2707	-0.0807	-0.2327	-0.0190	-0.1837	-0.0018	-0.1701	0.0026
	VaR (1%)	0.0406	0.0372	0.0325	0.0375	0.0319	0.0359	0.0307	0.0368	0.0308
	VaR (5%)	0.0245	0.0237	0.0193	0.0233	0.0181	0.0225	0.0175	0.0222	0.0174
	CVaR (1%)	0.0431	0.0386	0.0344	0.0376	0.0343	0.0377	0.0311	0.0377	0.0327
	CVaR (5%)	0.0266	0.0244	0.0201	0.0252	0.0190	0.0243	0.0188	0.0236	0.0193
	MAD	0.0102	0.0087	0.0078	0.0085	0.0075	0.0085	0.0073	0.0082	0.0072
5	aveSt	0.0048	0.1910	0.1182	0.1415	0.0468	0.1265	0.0281	0.1126	0.0187
	stDev	0.0146	0.0125	0.0113	0.0119	0.0110	0.0122	0.0108	0.0120	0.0108
	SR	0.0041	-0.0508	-0.0319	-0.0379	0.0019	-0.0326	0.0104	-0.0307	0.0102
	VaR (1%)	0.0406	0.0353	0.0337	0.0347	0.0322	0.0348	0.0307	0.0348	0.0307
	VaR (5%)	0.0245	0.0213	0.0192	0.0205	0.0179	0.0208	0.0174	0.0204	0.0174
	CVaR (1%)	0.0431	0.0380	0.0348	0.0359	0.0335	0.0357	0.0311	0.0370	0.0319
	CVaR (5%)	0.0266	0.0221	0.0217	0.0211	0.0201	0.0229	0.0177	0.0210	0.0183
	MAD	0.0102	0.0088	0.0080	0.0084	0.0075	0.0085	0.0073	0.0082	0.0072
20	aveSt	0.0026	0.0643	0.0586	0.0444	0.0244	0.0363	0.0140	0.0328	0.0087
	stDev	0.0146	0.0127	0.0115	0.0122	0.0111	0.0124	0.0109	0.0122	0.0108
	SR	0.0048	-0.0084	-0.0038	0.0053	0.0133	0.0042	0.0190	-0.0005	0.0130
	VaR (1%)	0.0406	0.0357	0.0338	0.0356	0.0314	0.0360	0.0306	0.0362	0.0311
	VaR (5%)	0.0245	0.0211	0.0191	0.0201	0.0179	0.0202	0.0174	0.0200	0.0174
	CVaR (1%)	0.0431	0.0376	0.0339	0.0358	0.0323	0.0387	0.0308	0.0387	0.0332
	CVaR (5%)	0.0266	0.0226	0.0203	0.0222	0.0187	0.0218	0.0199	0.0201	0.0197
	MAD	0.0102	0.0089	0.0081	0.0085	0.0076	0.0085	0.0074	0.0083	0.0072
60	aveSt	0.0016	0.0210	0.0194	0.0177	0.0133	0.0155	0.0084	0.0127	0.0050
	stDev	0.0146	0.0136	0.0120	0.0123	0.0111	0.0125	0.0109	0.0124	0.0109
	SR	0.0052	0.0016	0.0165	0.0026	0.0146	0.0142	0.0130	0.0119	0.0133
	VaR (1%)	0.0406	0.0368	0.0336	0.0351	0.0319	0.0349	0.0315	0.0362	0.0316
	VaR (5%)	0.0245	0.0220	0.0191	0.0199	0.0181	0.0200	0.0175	0.0198	0.0173
	CVaR (1%)	0.0431	0.0391	0.0337	0.0359	0.0323	0.0359	0.0336	0.0387	0.0332
	CVaR (5%)	0.0266	0.0235	0.0204	0.0222	0.0187	0.0217	0.0199	0.0201	0.0197
	MAD	0.0102	0.0094	0.0083	0.0085	0.0076	0.0085	0.0074	0.0083	0.0073

that these asset classes are already internally diversified. In contrast, the correlations within the sector portfolios are typically positive and high, which penalizes the maximum diversification approach.

Tables 2, 3, 4, 5, 6, and 7 report the results for the MIB dataset with weekly frequency. Since the data are collected weekly, the out-of-sample holding periods are set to $w_{out} = 1, 4, 12$, to avoid excessively long evaluation horizons. While broadly consistent with the findings of "Results on the MIB Dataset" section, these experiments seem to suggest that, for relatively long holding periods, the results tend to be slightly more balanced, offering a modest advantage to the risk minimization approach.

For larger values of *n* (see Tables 21, 22, 23, 24, 25, and 26), the maximum diversification approach generally outperforms the minimum risk strategy. We argue that this could depend on the number of effectively distinct stocks available for the allocation. The application evidence indicates that the MD strategy performs well for large values of *n*, whereas increasing *n* penalizes the mr strategy. In general, mr seems to require longer estimation windows—not only to ensure a full-rank covariance matrix in the mean–variance case but also for the other risk measures. In our experiment, the minimum value for w_{in} is 20. To calculate the VaR at 1% and VaR at 5% and obtain two different values, we approximate the out-of-sample return distribution using the kernel smoothing function

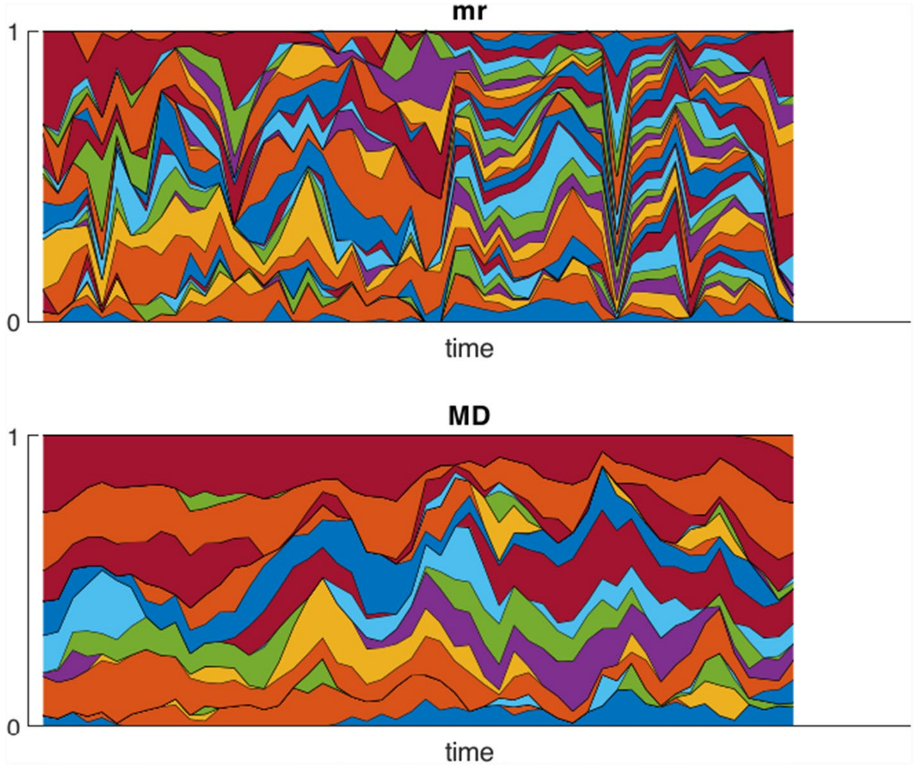

Fig. 1 Evolution over time of the optimal portfolio for the mr strategy (top panel) and the MD strategy (bottom panel), $w_{\text{in}} = 240$, $w_{\text{out}} = 60$, risk measure CVaR (5%)

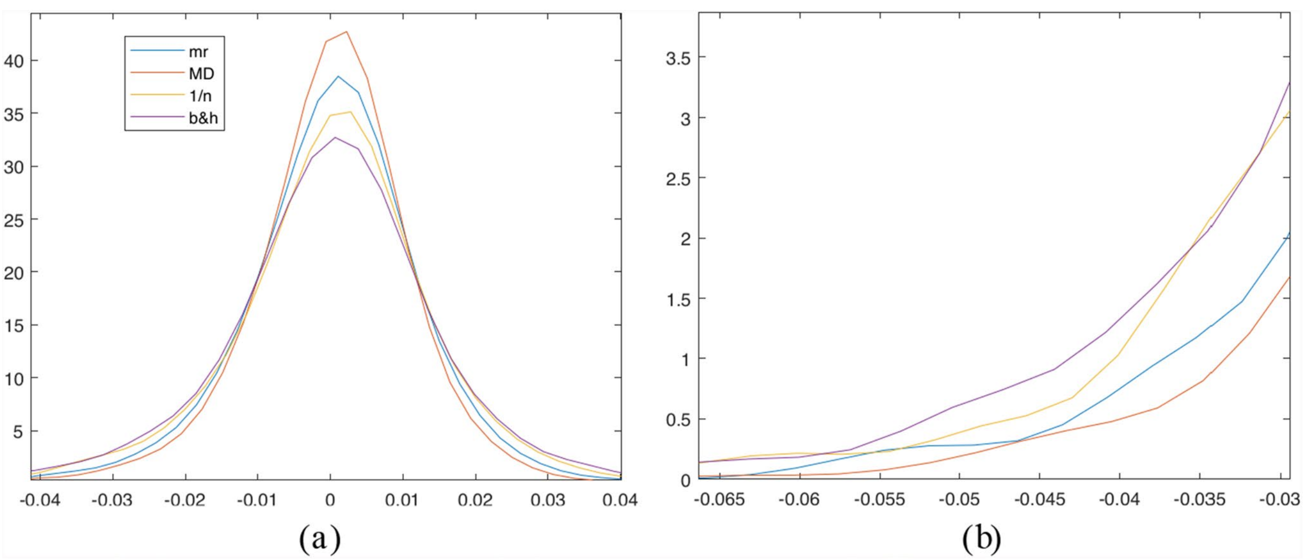

Fig. 2 **a** Distributions of the out-of-sample returns for the four strategies: 1/*n*, b&h, mr and MD both implemented with $w_{\text{in}} = 240$, $w_{\text{out}} = 60$ and the CVaR (5%). **b** A focus on the left tail of the return distributions

ksdensity included in MathWorks (2021). Percentiles are then calculated from the approximated distribution. The same procedure is applied for the CVaR at 1% and CVaR at 5%. We do not take into account the number of portfolio constituents when comparing the results, for several reasons. First, the number of assets in a portfolio is too crude to measure portfolio diversification, as it does not account for the presence of strongly correlated assets. Second, one could argue that a maximally diversified portfolio is well diversified regardless of the number of its constituents, based on the risk reduction results. If a significant risk reduction relative to passive benchmarks is present, it is entirely attributable to diversification, since no other allocation principle has been applied.

Conclusions

The in-sample optimality of an asset allocation strategy is of limited value if the strategy's practical implementation is challenging and results in poor performance. This is typically the case with minimum risk allocation strategies. The essence of risk lies in the uncertainty surrounding future returns, not in the shape of the distribution of past returns. While it is natural to assume that an investor is willing to allocate wealth to minimize risk, the key issue is how to reach this goal in real-life investments. This paper suggests that, in an out-of-sample framework, very good results can be obtained by applying a seemingly sub-optimal criterion that is easier to implement. More specifically, it is easier to control the future level of portfolio diversification than its risk. Therefore, an investor who seeks to effectively reduce risk should prefer to invest in a diversified portfolio, without concern for the value of future returns, which are very challenging to predict. Risk reduction is thus obtained as a secondary product of diversification. Optimal diversified portfolios are also preferable for their numerical stability, low transaction costs, risk-adjusted performance, and, in general, the simplicity of implementation. However, interesting problems remain, such as determining the level of accuracy in future return forecasts required for minimum risk approaches to become competitive in practice or investigating risk minimization approaches under highly accurate future return forecasts.

Appendix A: Risk Measures

We introduce risk measures in the usual axiomatic approach, in which the usual probability space $(\Omega, \mathcal{A}, P)$ is considered and the set of functions $X : \Omega \to \mathbb{R}$, treated as random variables, belong to the linear space $\mathcal{L}^2(\Omega, \mathcal{A}, P)$. We recall here the definitions of the risk measures that are used throughout the paper.

(i) The *variance*

$$\mathrm{Var}(X) := E[(X-\mu)]^2$$

where $\mu = E(X)$.

(ii) The *mean absolute deviation*

$$\mathrm{MAD}(X) := \frac{1}{n}\sum_{i=1}^{n} |X(i) - \mu|$$

where $\mu = E(X)$.

(iii) The *value at risk*, see Jorion (2007), at level $\alpha \in (0.1)$

$$\mathrm{V@R}_\alpha(X) = \inf\{x \in \mathbb{R} \ : \ \mathbb{P}(X \le x) \ge 1-\alpha\}, \qquad X \in L^0.$$

(iv) The *expected shortfall* or *conditional value at risk*, see Acerbi and Tasche (2002) and Inui and Kijima (2005), at level $\alpha \in (0.1)$

$$\mathrm{CV@R}_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha \mathrm{VaR}_\beta(X)d\beta, \qquad X \in L^1.$$

The variance and the mean absolute deviation are standard statistical measures of dispersion of a given distribution, in our case of the returns. The value at risk and the expected shortfall are quantiles that describe the left tail of the returns, where the extreme losses are contained.

Appendix B: Maximum Diversification Strategy

This section is based on Torrente and Uberti (2024) and Torrente and Uberti (2024). Let $n \ge 2$ be the number of assets, $\Gamma_n = \{w = (w_1, \dots, w_n) \in \mathbb{R}^n \ : \ \sum_{j=1}^n w_j = 1\}$ and $\mathbb{W}_n = \{w = (w_1, \dots, w_n) \in \mathbb{R}^n_{\ge 0} \ : \ \sum_{j=1}^n w_j = 1\}$ be the set of long-short and long-only portfolios respectively. A portfolio is uniquely identified by the vector w, where the j-th entry w_j is the percentage of wealth invested in the j-th asset. The single asset portfolio with the allocation concentrated in the j-th asset is denoted by $e^j \in \mathbb{W}_n$, where $\{e^1, \dots, e^n\}$ is the standard basis of $\mathbb{R}^n$. Further, we let X_j, for each $j = 1, \dots, n$, be the return of the the j-th asset for a fixed period, and consider the tuple of n possibly correlated random variables $X = (X_1, \dots, X_n)^t$. Using the notations defined above, we assume $X_j \in \mathcal{L}^2(\Omega, \mathcal{A}, P)$. Let $\rho : \mathcal{L}^2(\Omega, \mathcal{A}, P) \to \mathbb{R} \cup \{+\infty\}$ be a chosen risk measure. Throughout the paper we make the standing assumption that $\rho(X_j) > 0$ for each $j = 1, \dots, n$ and, in order to simplify the notation, we denote $\rho(X) = (\rho(X_1), \dots, \rho(X_n))^t$. Further, we let A_j be the realizations of the random variable X_j, i.e. the m returns of the j-th asset, for each $j = 1, \dots, n$, and we denote with $A = (A_1, ..., A_n) \in \mathrm{Mat}_{m\times n}(\mathbb{R})$ the usual matrix of portfolio returns.

Definition 1 (Risk Adjusted Distance) The *Risk Adjusted Distance (RAD)* with respect to the risk measure ρ is the map $d_{\rho(X)} : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}$ is defined by

$$d_{\rho(X)}(x, y) = \left(\sum_{j=1}^{n} \frac{1}{\rho(X_j)}(x_j - y_j)^2\right)^{\frac{1}{2}}, \quad \forall x, y \in \mathbb{R}^n.$$

Definition 2 (Geometric Portfolio diversification Measure) The *Geometric Portfolio Diversification Measure (GPDM)* with respect to the risk measure ρ is the map $\Phi_{\rho(X)} : \Gamma_n \times \mathrm{Mat}_{m\times n}(\mathbb{R}) \setminus \{\mathbf{0}_{m\times n}\} \to \mathbb{R}_{\ge 0}$ defined by

$$\Phi_{\rho(X)}(w,A) := (\mathrm{rank}(A)-1)\left(1-\frac{f_{\rho(X)}(w)}{\max_{j=1,\dots,n} f_{\rho(X)}(e^j)}\right) \tag{B1}$$

for each $w \in \Gamma_n$ and $A \in \mathrm{Mat}_{m\times n} \setminus \{\mathbf{0}_{m\times n}\}$, where $f_{\rho(X)} : \Gamma_n \to \mathbb{R}_{\geq 0}$ is:

$$f_{\rho(X)}(w) := \sum_{j=1}^{n}\sum_{k=j+1}^{n}\left(d^2_{\rho(X)}(w,e^j) - d^2_{\rho(X)}(w,e^k)\right)^2 \tag{B2}$$

for each $w \in \Gamma_n$.

Since $f_{\rho(X)}(e^j) > 0$, for each $j \in \{1,\dots,n\}$, it is fairly straightforward to notice that $\Phi_{\rho(X)}$ is well-defined on Γ_n for any choice of the risk measure ρ. Further, from the convexity of $f_{\rho(X)}$ (see Torrente and Uberti (2024, Lemma 3.1, item (iii))), it follows that $f_{\rho(X)}(w) \leq \max_{j=1,\dots,n} f_{\rho(X)}(e^j)$, for each $w \in \mathbb{W}_n \subset \Gamma_n$, hence $\Phi_{\rho(X)}(w,A) \geq 0$ for any $(w,A) \in \mathbb{W}_n \times \mathrm{Mat}_{m\times n}(\mathbb{R}) \setminus \{\mathbf{0}_{m\times n}\}$.

Using again the convexity of $f_{\rho(X)}$ we observe that $\Phi_{\rho(X)}(\cdot,A)$ has a unique maximum in Γ_n (see Torrente and Uberti (2024, Proposition 3.2)), and analogously, when restricting to long-only portfolios, the optimization problem $\max_{w\in\mathbb{W}_n} \Phi_{\rho(X)}(w,A)$ admits a unique maximum, called the *Maximum Diversification* (MD) strategy.

Definition 3 Let $\Phi_{\rho(X)}$ be the GPDM with respect to $d_{\rho(X)}$ and $A \in \mathrm{Mat}_{m\times n}(\mathbb{R})$ be a nonzero matrix. The unique

$$w^* = \mathrm{argmax}_{w\in\mathbb{W}_n} \Phi_{\rho(X)}(w,A)$$

is called the *Maximum Diversification* (MD) strategy (with respect to $d_{\rho(X)}$).

Remark 1 By Definitions 2 and 3 it immediately follows that

$$w^* = \mathrm{argmax}_{w\in\mathbb{W}_n} \Phi_{\rho(X)}(w,A) = \mathrm{argmin}_{w\in\mathbb{W}_n} f_{\rho(X)}(w).$$

Appendix C: Results for all the Datasets

In this section we collect the results of our experiments on the datasets considered in the paper (S&P500, DAX, MIB, ESX, FTSE). The performance of the buy & hold strategy for S&P500 (daily returns), DAX, ESX, FTSE is reported in Appendix C.1, whereas the results on the datasets S&P500 (daily returns), MIB—weekly, and FTSE are detailed in Appendix C.2, C.3 and 8.4, respectively. Due to space constraints, the tables with the results on the datasets MIB—monthly, DAX, S&P500—weekly, S&P500—monthly and ESX are available to the reader upon request.

C.1 Results for the Buy&Hold Strategy

See Table 8.

Table 8 Performance of the buy&hold strategy computed on the datasets S&P500, DAX, ESX and FTSE

	S&P500	DAX	ESX	FTSE
aveSt	0	0	0	0
stDev	0.0115	0.0136	0.0139	0.0117
SR	0.0252	0.0208	0.0021	0.0049
VaR (1%)	0.0325	0.0395	0.0412	0.0342
VaR (5%)	0.0177	0.0216	0.0218	0.0182
CVaR (1%)	0.0326	0.0403	0.0428	0.0360
CVaR (5%)	0.0177	0.0226	0.0228	0.0184
MAD	0.0076	0.0093	0.0094	0.0078

C.2 Dataset S&P500

See Tables 9, 10, 11, 12, 13, 14.

Table 9 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the variance on the S&P500 dataset

variance			$w_{in}=20$		$w_{in}=60$		$w_{in}=120$		$w_{in}=240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	1×10^{-16}	0.2237	0.1282	0.0791	0.0445	0.0409	0.0231	0.0209	0.0121
	stDev	0.0110	0.0087	0.0091	0.0085	0.0091	0.0084	0.0090	0.0083	0.0090
	SR	0.0242	-0.1043	-0.0419	-0.0227	0.0004	0.0019	0.0125	0.0157	0.0224
	VaR (1%)	0.0313	0.0253	0.0264	0.0248	0.0263	0.0245	0.0261	0.0241	0.0255
	VaR (5%)	0.0166	0.0144	0.0143	0.0131	0.0138	0.0127	0.0137	0.0126	0.0135
	CVaR (1%)	0.0333	0.0257	0.0269	0.0255	0.0276	0.0248	0.0274	0.0258	0.0257
	CVaR (5%)	0.0186	0.0156	0.0157	0.0150	0.0143	0.0146	0.0141	0.0135	0.0148
	MAD	0.0072	0.0060	0.0061	0.0057	0.0060	0.0056	0.0060	0.0055	0.0059
5	aveSt	0.0023	0.1079	0.0638	0.0400	0.0238	0.0211	0.0128	0.0111	0.0071
	stDev	0.0110	0.0088	0.0092	0.0085	0.0091	0.0084	0.0091	0.0083	0.0090
	SR	0.0232	-0.0371	-0.0084	0.0009	0.0111	0.0138	0.0179	0.0215	0.0249
	VaR (1%)	0.0313	0.0249	0.0262	0.0245	0.0262	0.0244	0.0260	0.0241	0.0255
	VaR (5%)	0.0166	0.0140	0.0141	0.0130	0.0138	0.0127	0.0137	0.0125	0.0135
	CVaR (1%)	0.0334	0.0250	0.0282	0.0254	0.0265	0.0248	0.0263	0.0241	0.0269
	CVaR (5%)	0.0187	0.0152	0.0149	0.0137	0.0155	0.0132	0.0153	0.0141	0.0139
	MAD	0.0072	0.0060	0.0061	0.0057	0.0060	0.0056	0.0060	0.0055	0.0059
20	aveSt	0.0013	0.0503	0.0324	0.0206	0.0137	0.0115	0.0078	0.0063	0.0044
	stDev	0.0110	0.0089	0.0093	0.0085	0.0092	0.0085	0.0091	0.0084	0.0090
	SR	0.0237	-0.0036	0.0075	0.0133	0.0172	0.0193	0.0206	0.0250	0.0268
	VaR (1%)	0.0313	0.0253	0.0261	0.0245	0.0262	0.0241	0.0258	0.0240	0.0254
	VaR (5%)	0.0166	0.0138	0.0141	0.0128	0.0138	0.0126	0.0136	0.0124	0.0135
	CVaR (1%)	0.0333	0.0266	0.0282	0.0253	0.0277	0.0257	0.0270	0.0251	0.0256
	CVaR (5%)	0.0186	0.0141	0.0148	0.0148	0.0143	0.0132	0.0158	0.0125	0.0147
	MAD	0.0072	0.0060	0.0062	0.0057	0.0060	0.0056	0.0060	0.0055	0.0059
60	aveSt	0.0008	0.0182	0.0122	0.0118	0.0088	0.0071	0.0053	0.0042	0.0032
	stDev	0.0110	0.0093	0.0093	0.0086	0.0092	0.0087	0.0091	0.0085	0.0090
	SR	0.0239	0.0149	0.0179	0.0188	0.0211	0.0222	0.0232	0.0255	0.0281
	VaR (1%)	0.0313	0.0258	0.0263	0.0245	0.0266	0.0243	0.0259	0.0241	0.0254
	VaR (5%)	0.0166	0.0136	0.0142	0.0130	0.0138	0.0128	0.0138	0.0126	0.0136
	CVaR (1%)	0.0333	0.0269	0.0266	0.0263	0.0268	0.0250	0.0261	0.0252	0.0257
	CVaR (5%)	0.0186	0.0142	0.0154	0.0135	0.0160	0.0135	0.0152	0.0141	0.0148
	MAD	0.0072	0.0061	0.0062	0.0058	0.0061	0.0057	0.0060	0.0056	0.0059

Table 10 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the MAD on the S&P500 dataset

MAD			$w_{\text{in}} = 20$		$w_{\text{in}} = 60$		$w_{\text{in}} = 120$		$w_{\text{in}} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	1×10^{-16}	0.3557	0.1069	0.1942	0.0360	0.1419	0.0181	0.1207	0.0092
	stDev	0.0110	0.0089	0.0095	0.0086	0.0094	0.0085	0.0094	0.0084	0.0093
	SR	0.0242	-0.1764	-0.0287	-0.0906	0.0076	-0.0560	0.0169	-0.0430	0.0238
	VaR (1%)	0.0313	0.0267	0.0272	0.0258	0.0272	0.0251	0.0271	0.0248	0.0267
	VaR (5%)	0.0166	0.0153	0.0147	0.0139	0.0144	0.0135	0.0144	0.0133	0.0141
	CVaR (1%)	0.0333	0.0273	0.0275	0.0268	0.0286	0.0272	0.0286	0.0253	0.0276
	CVaR (5%)	0.0186	0.0171	0.0163	0.0142	0.0150	0.0147	0.0151	0.0149	0.0142
	MAD	0.0072	0.0061	0.0063	0.0058	0.0062	0.0057	0.0062	0.0056	0.0061
5	aveSt	0.0023	0.1216	0.0493	0.0589	0.0173	0.0392	0.0091	0.0288	0.0049
	stDev	0.0110	0.0090	0.0095	0.0086	0.0094	0.0085	0.0094	0.0085	0.0093
	SR	0.0232	-0.0457	0.0004	-0.0108	0.0171	0.0026	0.0213	0.0109	0.0258
	VaR (1%)	0.0313	0.0258	0.0270	0.0251	0.0271	0.0246	0.0271	0.0241	0.0267
	VaR (5%)	0.0166	0.0142	0.0145	0.0133	0.0142	0.0130	0.0143	0.0127	0.0141
	CVaR (1%)	0.0334	0.0260	0.0288	0.0269	0.0278	0.0264	0.0282	0.0253	0.0275
	CVaR (5%)	0.0187	0.0142	0.0153	0.0143	0.0143	0.0139	0.0147	0.0148	0.0163
	MAD	0.0072	0.0061	0.0063	0.0058	0.0062	0.0057	0.0062	0.0056	0.0061
20	aveSt	0.0013	0.0503	0.0249	0.0233	0.0095	0.0155	0.0052	0.0100	0.0029
	stDev	0.0110	0.0090	0.0096	0.0087	0.0095	0.0086	0.0095	0.0085	0.0094
	SR	0.0237	-0.0046	0.0134	0.0105	0.0213	0.0170	0.0234	0.0226	0.0271
	VaR (1%)	0.0313	0.0259	0.0270	0.0249	0.0271	0.0245	0.0269	0.0240	0.0266
	VaR (5%)	0.0166	0.0139	0.0144	0.0132	0.0143	0.0129	0.0143	0.0127	0.0141
	CVaR (1%)	0.0333	0.0277	0.0282	0.0251	0.0283	0.0252	0.0288	0.0257	0.0276
	CVaR (5%)	0.0186	0.0142	0.0147	0.0141	0.0146	0.0146	0.0153	0.0128	0.0142
	MAD	0.0072	0.0061	0.0063	0.0058	0.0062	0.0057	0.0062	0.0056	0.0061
60	aveSt	0.0008	0.0176	0.0093	0.0123	0.0062	0.0080	0.0035	0.0051	0.0020
	stDev	0.0110	0.0093	0.0096	0.0088	0.0095	0.0087	0.0095	0.0086	0.0094
	SR	0.0239	0.0146	0.0209	0.0181	0.0222	0.0214	0.0242	0.0237	0.0274
	VaR (1%)	0.0313	0.0266	0.0272	0.0248	0.0274	0.0244	0.0271	0.0243	0.0266
	VaR (5%)	0.0166	0.0139	0.0146	0.0132	0.0144	0.0130	0.0144	0.0128	0.0142
	CVaR (1%)	0.0333	0.0276	0.0292	0.0256	0.0283	0.0257	0.0274	0.0250	0.0284
	CVaR (5%)	0.0186	0.0152	0.0155	0.0143	0.0148	0.0145	0.0163	0.0142	0.0149
	MAD	0.0072	0.0062	0.0064	0.0058	0.0063	0.0058	0.0062	0.0056	0.0061

Table 11 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the VaR (1%) on the S&P500 dataset

VaR (1%)			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	1×10^{-16}	0.5584	0.1272	0.3587	0.0364	0.3400	0.0185	0.2667	0.0091
	stDev	0.0110	0.0092	0.0095	0.0089	0.0095	0.0087	0.0094	0.0087	0.0094
	SR	0.0242	-0.2726	-0.0370	-0.1778	0.0065	-0.1656	0.0165	-0.1201	0.0250
	VaR (1%)	0.0313	0.0286	0.0275	0.0266	0.0275	0.0264	0.0272	0.0256	0.0268
	VaR (5%)	0.0166	0.0169	0.0148	0.0153	0.0145	0.0148	0.0143	0.0142	0.0141
	CVaR (1%)	0.0333	0.0297	0.0289	0.0279	0.0291	0.0267	0.0276	0.0261	0.0289
	CVaR (5%)	0.0186	0.0173	0.0152	0.0173	0.0153	0.0156	0.0143	0.0143	0.0152
	MAD	0.0072	0.0063	0.0063	0.0061	0.0063	0.0059	0.0062	0.0058	0.0061
5	aveSt	0.0023	0.1593	0.0632	0.0935	0.0238	0.0800	0.0130	0.0626	0.0074
	stDev	0.0110	0.0093	0.0095	0.0090	0.0095	0.0087	0.0095	0.0087	0.0094
	SR	0.0232	-0.0581	-0.0055	-0.0267	0.0115	-0.0175	0.0191	-0.0059	0.0258
	VaR (1%)	0.0313	0.0271	0.0274	0.0255	0.0274	0.0249	0.0271	0.0248	0.0269
	VaR (5%)	0.0166	0.0151	0.0145	0.0140	0.0144	0.0135	0.0143	0.0133	0.0141
	CVaR (1%)	0.0334	0.0280	0.0282	0.0257	0.0280	0.0259	0.0293	0.0250	0.0270
	CVaR (5%)	0.0187	0.0161	0.0148	0.0154	0.0146	0.0151	0.0156	0.0143	0.0158
	MAD	0.0072	0.0064	0.0063	0.0061	0.0063	0.0059	0.0062	0.0058	0.0061
20	aveSt	0.0013	0.0576	0.0315	0.0341	0.0151	0.0264	0.0084	0.0196	0.0050
	stDev	0.0110	0.0092	0.0096	0.0089	0.0096	0.0088	0.0095	0.0089	0.0094
	SR	0.0237	-0.0077	0.0114	0.0062	0.0158	0.0159	0.0215	0.0196	0.0272
	VaR (1%)	0.0313	0.0264	0.0272	0.0253	0.0273	0.0245	0.0268	0.0247	0.0268
	VaR (5%)	0.0166	0.0144	0.0146	0.0137	0.0145	0.0133	0.0144	0.0131	0.0141
	CVaR (1%)	0.0333	0.0272	0.0272	0.0264	0.0295	0.0252	0.0290	0.0248	0.0282
	CVaR (5%)	0.0186	0.0154	0.0159	0.0140	0.0158	0.0144	0.0154	0.0138	0.0149
	MAD	0.0072	0.0063	0.0064	0.0060	0.0063	0.0059	0.0062	0.0058	0.0061
60	aveSt	0.0008	0.0208	0.0114	0.0169	0.0096	0.0126	0.0054	0.0090	0.0034
	stDev	0.0110	0.0094	0.0096	0.0091	0.0096	0.0090	0.0096	0.0090	0.0095
	SR	0.0239	0.0177	0.0205	0.0148	0.0220	0.0203	0.0231	0.0265	0.0289
	VaR (1%)	0.0313	0.0260	0.0270	0.0258	0.0275	0.0249	0.0269	0.0252	0.0270
	VaR (5%)	0.0166	0.0143	0.0145	0.0136	0.0146	0.0134	0.0145	0.0131	0.0141
	CVaR (1%)	0.0333	0.0263	0.0275	0.0270	0.0293	0.0269	0.0291	0.0263	0.0276
	CVaR (5%)	0.0186	0.0145	0.0163	0.0156	0.0157	0.0147	0.0157	0.0148	0.0143
	MAD	0.0072	0.0064	0.0064	0.0061	0.0064	0.0060	0.0063	0.0059	0.0061

Table 12 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the VaR (5%) on the S&P500 dataset

VaR (5%)			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	1×10^{-16}	0.4719	0.1232	0.3745	0.0415	0.3347	0.0203	0.2933	0.0102
	stDev	0.0110	0.0090	0.0094	0.0087	0.0094	0.0087	0.0094	0.0087	0.0093
	SR	0.0242	-0.2317	-0.0340	-0.1908	0.0040	-0.1634	0.0155	-0.1322	0.0234
	VaR (1%)	0.0313	0.0277	0.0272	0.0260	0.0271	0.0263	0.0269	0.0260	0.0266
	VaR (5%)	0.0166	0.0164	0.0148	0.0151	0.0144	0.0148	0.0143	0.0141	0.0141
	CVaR (1%)	0.0333	0.0290	0.0273	0.0277	0.0289	0.0269	0.0284	0.0275	0.0276
	CVaR (5%)	0.0186	0.0168	0.0161	0.0155	0.0152	0.0164	0.0151	0.0141	0.0143
	MAD	0.0072	0.0062	0.0063	0.0059	0.0062	0.0059	0.0062	0.0057	0.0061
5	aveSt	0.0023	0.1442	0.0618	0.0934	0.0226	0.0761	0.0119	0.0650	0.0063
	stDev	0.0110	0.0091	0.0095	0.0088	0.0094	0.0087	0.0094	0.0087	0.0094
	SR	0.0232	-0.0566	-0.0048	-0.0301	0.0135	-0.0171	0.0191	-0.0083	0.0251
	VaR (1%)	0.0313	0.0263	0.0271	0.0256	0.0270	0.0252	0.0270	0.0247	0.0265
	VaR (5%)	0.0166	0.0148	0.0145	0.0139	0.0143	0.0134	0.0143	0.0132	0.0142
	CVaR (1%)	0.0334	0.0273	0.0286	0.0256	0.0287	0.0268	0.0287	0.0261	0.0284
	CVaR (5%)	0.0187	0.0156	0.0151	0.0153	0.0152	0.0143	0.0153	0.0146	0.0151
	MAD	0.0072	0.0063	0.0063	0.0060	0.0062	0.0058	0.0062	0.0057	0.0061
20	aveSt	0.0013	0.0559	0.0314	0.0327	0.0128	0.0247	0.0070	0.0187	0.0038
	stDev	0.0110	0.0092	0.0096	0.0087	0.0095	0.0087	0.0094	0.0086	0.0094
	SR	0.0237	-0.0047	0.0105	0.0059	0.0195	0.0135	0.0221	0.0172	0.0270
	VaR (1%)	0.0313	0.0262	0.0270	0.0252	0.0271	0.0248	0.0268	0.0246	0.0265
	VaR (5%)	0.0166	0.0143	0.0146	0.0134	0.0144	0.0132	0.0143	0.0129	0.0141
	CVaR (1%)	0.0333	0.0278	0.0289	0.0260	0.0287	0.0251	0.0287	0.0254	0.0276
	CVaR (5%)	0.0186	0.0155	0.0154	0.0143	0.0152	0.0148	0.0153	0.0149	0.0143
	MAD	0.0072	0.0063	0.0064	0.0060	0.0062	0.0058	0.0062	0.0057	0.0061
60	aveSt	0.0008	0.0202	0.0113	0.0155	0.0080	0.0113	0.0046	0.0078	0.0026
	stDev	0.0110	0.0096	0.0096	0.0089	0.0096	0.0089	0.0095	0.0088	0.0094
	SR	0.0239	0.0178	0.0199	0.0156	0.0215	0.0195	0.0241	0.0233	0.0273
	VaR (1%)	0.0313	0.0264	0.0269	0.0252	0.0276	0.0248	0.0271	0.0247	0.0265
	VaR (5%)	0.0166	0.0141	0.0146	0.0132	0.0145	0.0133	0.0145	0.0130	0.0142
	CVaR (1%)	0.0333	0.0267	0.0291	0.0261	0.0298	0.0270	0.0288	0.0252	0.0282
	CVaR (5%)	0.0186	0.0165	0.0156	0.0153	0.0160	0.0134	0.0154	0.0141	0.0148
	MAD	0.0072	0.0063	0.0064	0.0060	0.0063	0.0059	0.0063	0.0058	0.0061

Table 13 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the CVaR (1%) on the S&P500 dataset

CVaR (1%)			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	1×10^{-16}	0.6818	0.1302	0.5923	0.0471	0.5575	0.0284	0.5478	0.0155
	stDev	0.0110	0.0094	0.0095	0.0091	0.0095	0.0090	0.0094	0.0091	0.0094
	SR	0.0242	-0.3351	-0.0386	-0.2965	0.0009	-0.2752	0.0115	-0.2629	0.0218
	VaR (1%)	0.0313	0.0304	0.0275	0.0287	0.0275	0.0285	0.0272	0.0287	0.0268
	VaR (5%)	0.0166	0.0179	0.0148	0.0169	0.0145	0.0167	0.0144	0.0166	0.0142
	CVaR (1%)	0.0333	0.0315	0.0288	0.0296	0.0290	0.0287	0.0278	0.0294	0.0289
	CVaR (5%)	0.0186	0.0188	0.0151	0.0189	0.0152	0.0186	0.0145	0.0170	0.0152
	MAD	0.0072	0.0064	0.0063	0.0063	0.0063	0.0062	0.0062	0.0062	0.0061
5	aveSt	0.0023	0.1752	0.0634	0.1321	0.0250	0.1215	0.0148	0.1188	0.0089
	stDev	0.0110	0.0094	0.0095	0.0092	0.0095	0.0091	0.0095	0.0091	0.0094
	SR	0.0232	-0.0681	-0.0059	-0.0475	0.0109	-0.0385	0.0183	-0.0323	0.0250
	VaR (1%)	0.0313	0.0285	0.0274	0.0269	0.0274	0.0258	0.0270	0.0272	0.0268
	VaR (5%)	0.0166	0.0154	0.0145	0.0148	0.0144	0.0145	0.0144	0.0143	0.0141
	CVaR (1%)	0.0334	0.0299	0.0281	0.0279	0.0280	0.0269	0.0292	0.0281	0.0269
	CVaR (5%)	0.0187	0.0156	0.0147	0.0155	0.0145	0.0151	0.0156	0.0149	0.0156
	MAD	0.0072	0.0065	0.0063	0.0063	0.0063	0.0061	0.0062	0.0061	0.0061
20	aveSt	0.0013	0.0572	0.0314	0.0414	0.0151	0.0358	0.0086	0.0318	0.0052
	stDev	0.0110	0.0093	0.0096	0.0092	0.0096	0.0091	0.0095	0.0091	0.0094
	SR	0.0237	-0.0024	0.0114	0.0079	0.0159	0.0067	0.0213	0.0097	0.0271
	VaR (1%)	0.0313	0.0272	0.0272	0.0265	0.0273	0.0257	0.0268	0.0265	0.0268
	VaR (5%)	0.0166	0.0146	0.0146	0.0141	0.0145	0.0139	0.0144	0.0137	0.0141
	CVaR (1%)	0.0333	0.0287	0.0294	0.0275	0.0274	0.0269	0.0272	0.0283	0.0285
	CVaR (5%)	0.0186	0.0148	0.0158	0.0144	0.0160	0.0160	0.0159	0.0141	0.0152
	MAD	0.0072	0.0064	0.0064	0.0063	0.0063	0.0061	0.0062	0.0060	0.0061
60	aveSt	0.0008	0.0205	0.0114	0.0177	0.0096	0.0140	0.0054	0.0120	0.0034
	stDev	0.0110	0.0097	0.0096	0.0094	0.0096	0.0094	0.0096	0.0094	0.0094
	SR	0.0239	0.0173	0.0205	0.0185	0.0220	0.0189	0.0231	0.0224	0.0288
	VaR (1%)	0.0313	0.0273	0.0270	0.0264	0.0275	0.0259	0.0269	0.0270	0.0270
	VaR (5%)	0.0166	0.0146	0.0145	0.0140	0.0146	0.0140	0.0145	0.0137	0.0141
	CVaR (1%)	0.0333	0.0280	0.0276	0.0272	0.0295	0.0261	0.0289	0.0282	0.0276
	CVaR (5%)	0.0186	0.0153	0.0164	0.0151	0.0159	0.0140	0.0155	0.0139	0.0143
	MAD	0.0072	0.0065	0.0064	0.0063	0.0064	0.0062	0.0063	0.0061	0.0061

Table 14 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the CVaR (5%) on the S&P500 dataset

CVaR (5%)			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	1×10^{-16}	0.6944	0.1293	0.6114	0.0544	0.5706	0.0322	0.5189	0.0191
	stDev	0.0110	0.0095	0.0094	0.0092	0.0094	0.0094	0.0094	0.0095	0.0093
	SR	0.0242	-0.3413	-0.0374	-0.3031	-0.0025	-0.2762	0.0093	-0.2436	0.0186
	VaR (1%)	0.0313	0.0310	0.0273	0.0292	0.0272	0.0302	0.0269	0.0306	0.0265
	VaR (5%)	0.0166	0.0183	0.0148	0.0175	0.0145	0.0173	0.0144	0.0171	0.0142
	CVaR (1%)	0.0333	0.0313	0.0274	0.0298	0.0292	0.0322	0.0287	0.0309	0.0265
	CVaR (5%)	0.0186	0.0187	0.0162	0.0191	0.0155	0.0184	0.0153	0.0175	0.0154
	MAD	0.0072	0.0065	0.0063	0.0064	0.0062	0.0064	0.0062	0.0064	0.0061
5	aveSt	0.0023	0.1708	0.0619	0.1351	0.0243	0.1261	0.0148	0.1166	0.0097
	stDev	0.0110	0.0094	0.0095	0.0093	0.0094	0.0093	0.0094	0.0095	0.0094
	SR	0.0232	-0.0648	-0.0049	-0.0487	0.0126	-0.0411	0.0174	-0.0337	0.0233
	VaR (1%)	0.0313	0.0277	0.0271	0.0269	0.0270	0.0280	0.0270	0.0283	0.0265
	VaR (5%)	0.0166	0.0154	0.0145	0.0150	0.0143	0.0151	0.0143	0.0151	0.0142
	CVaR (1%)	0.0334	0.0279	0.0287	0.0279	0.0287	0.0291	0.0290	0.0299	0.0283
	CVaR (5%)	0.0187	0.0155	0.0152	0.0152	0.0152	0.0164	0.0156	0.0169	0.0150
	MAD	0.0072	0.0065	0.0063	0.0063	0.0062	0.0063	0.0062	0.0063	0.0061
20	aveSt	0.0013	0.0556	0.0312	0.0395	0.0130	0.0352	0.0074	0.0313	0.0046
	stDev	0.0110	0.0094	0.0096	0.0093	0.0095	0.0093	0.0094	0.0094	0.0094
	SR	0.0237	-0.0034	0.0107	0.0043	0.0195	0.0078	0.0218	0.0110	0.0265
	VaR (1%)	0.0313	0.0270	0.0271	0.0269	0.0270	0.0268	0.0267	0.0276	0.0264
	VaR (5%)	0.0166	0.0147	0.0146	0.0143	0.0144	0.0143	0.0143	0.0144	0.0142
	CVaR (1%)	0.0333	0.0280	0.0289	0.0288	0.0287	0.0280	0.0267	0.0292	0.0279
	CVaR (5%)	0.0186	0.0166	0.0154	0.0164	0.0152	0.0148	0.0156	0.0153	0.0145
	MAD	0.0072	0.0064	0.0064	0.0063	0.0062	0.0062	0.0062	0.0062	0.0061
60	aveSt	0.0008	0.0195	0.0112	0.0161	0.0080	0.0138	0.0047	0.0116	0.0027
	stDev	0.0110	0.0095	0.0096	0.0096	0.0096	0.0095	0.0095	0.0097	0.0094
	SR	0.0239	0.0164	0.0199	0.0122	0.0215	0.0173	0.0239	0.0196	0.0275
	VaR (1%)	0.0313	0.0269	0.0269	0.0276	0.0276	0.0262	0.0271	0.0285	0.0265
	VaR (5%)	0.0166	0.0144	0.0146	0.0143	0.0145	0.0143	0.0145	0.0145	0.0142
	CVaR (1%)	0.0333	0.0271	0.0269	0.0290	0.0278	0.0272	0.0288	0.0299	0.0284
	CVaR (5%)	0.0186	0.0150	0.0158	0.0144	0.0164	0.0150	0.0155	0.0157	0.0150
	MAD	0.0072	0.0064	0.0064	0.0063	0.0063	0.0062	0.0063	0.0063	0.0061

C.3 Dataset MIB-Weekly

See Tables 15, 16, 17, 18, 19, 20.

Table 15 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the variance on the MIB-weekly dataset

variance			w_{in} = 20		w_{in} = 60		w_{in} = 120		w_{in} = 240	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.08×10^{-16}	–	0.2352	0.1330	0.0765	0.0702	0.0398	0.0386	0.0196
	stDev	0.0328	–	0.0235	0.0229	0.0233	0.0213	0.0222	0.0196	0.0206
	SR	0.0167	–	0.0153	0.0217	0.0208	0.0668	0.0488	0.0538	0.0454
	VaR (1%)	0.1036	–	0.0715	0.0756	0.0761	0.0678	0.0700	0.0625	0.0636
	VaR (5%)	0.0549	–	0.0410	0.0395	0.0393	0.0342	0.0350	0.0306	0.0321
	CVaR (1%)	0.1067	–	0.0740	0.0776	0.0779	0.0682	0.0702	0.0630	0.0649
	CVaR (5%)	0.0558	–	0.0418	0.0419	0.0420	0.0362	0.0357	0.0331	0.0325
	MAD	0.0234	–	0.0174	0.0162	0.0162	0.0146	0.0146	0.0118	0.0122
4	aveSt	0.0122	–	0.1282	0.0718	0.0455	0.0407	0.0250	0.0241	0.0138
	stDev	0.0328	–	0.0238	0.0235	0.0235	0.0216	0.0223	0.0199	0.0207
	SR	0.0149	–	0.0369	0.0333	0.0270	0.0742	0.0538	0.0573	0.0463
	VaR (1%)	0.1037	–	0.0745	0.0752	0.0768	0.0678	0.0704	0.0624	0.0636
	VaR (5%)	0.0549	–	0.0412	0.0388	0.0395	0.0340	0.0351	0.0307	0.0321
	CVaR (1%)	0.1068	–	0.0761	0.0760	0.0770	0.0686	0.0706	0.0626	0.0650
	CVaR (5%)	0.0559	–	0.0415	0.0401	0.0420	0.0365	0.0353	0.0320	0.0327
	MAD	0.0234	–	0.0174	0.0163	0.0163	0.0147	0.0147	0.0119	0.0123
12	aveSt	0.0078	–	0.0698	0.0411	0.0283	0.0249	0.0163	0.0149	0.0090
	stDev	0.0328	–	0.0246	0.0232	0.0237	0.0216	0.0225	0.0200	0.0207
	SR	0.0156	–	0.0359	0.0374	0.0242	0.0717	0.0519	0.0535	0.0468
	VaR (1%)	0.1036	–	0.0812	0.0712	0.0756	0.0675	0.0701	0.0620	0.0637
	VaR (5%)	0.0549	–	0.0402	0.0398	0.0401	0.0345	0.0350	0.0311	0.0320
	CVaR (1%)	0.1067	–	0.0819	0.0722	0.0762	0.0678	0.0703	0.0621	0.0648
	CVaR (5%)	0.0558	–	0.0423	0.0410	0.0410	0.0355	0.0380	0.0314	0.0325
	MAD	0.0234	–	0.0175	0.0163	0.0164	0.0146	0.0149	0.0119	0.0123

Table 16 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the MAD on the MIB—weekly dataset

MAD			w_{in} = 20		w_{in} = 60		w_{in} = 120		w_{in} = 240	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.08×10^{-16}	0.5086	0.2061	0.2348	0.0669	0.1452	0.0330	0.0916	0.0179
	stDev	0.0328	0.0248	0.0234	0.0231	0.0229	0.0215	0.0220	0.0193	0.0201
	SR	0.0167	-0.0385	0.0229	-0.0071	0.0276	0.0376	0.0521	0.0447	0.0482
	VaR (1%)	0.1036	0.0833	0.0736	0.0795	0.0739	0.0673	0.0672	0.0599	0.0607
	VaR (5%)	0.0549	0.0444	0.0398	0.0401	0.0380	0.0349	0.0340	0.0304	0.0314
	CVaR (1%)	0.1067	0.0852	0.0746	0.0810	0.0762	0.0678	0.0676	0.0604	0.0635
	CVaR (5%)	0.0558	0.0447	0.0414	0.0428	0.0403	0.0355	0.0360	0.0309	0.0315
	MAD	0.0234	0.0184	0.0170	0.0163	0.0158	0.0143	0.0144	0.0115	0.0118
4	aveSt	0.0122	0.2164	0.1061	0.0998	0.0358	0.0622	0.0193	0.0349	0.0122
	stDev	0.0328	0.0259	0.0237	0.0236	0.0232	0.0217	0.0221	0.0195	0.0201
	SR	0.0149	0.0233	0.0437	0.0205	0.0352	0.0557	0.0564	0.0567	0.0495
	VaR (1%)	0.1037	0.0832	0.0758	0.0756	0.0746	0.0658	0.0676	0.0606	0.0607
	VaR (5%)	0.0549	0.0431	0.0395	0.0390	0.0379	0.0347	0.0337	0.0297	0.0314
	CVaR (1%)	0.1068	0.0859	0.0783	0.0766	0.0750	0.0671	0.0697	0.0615	0.0611
	CVaR (1%)	0.0559	0.0451	0.0421	0.0414	0.0396	0.0357	0.0341	0.0319	0.0315
	MAD	0.0234	0.0187	0.0170	0.0164	0.0159	0.0144	0.0144	0.0116	0.0118
12	aveSt	0.0078	0.1038	0.0588	0.0517	0.0213	0.0306	0.0123	0.0190	0.0074
	stDev	0.0328	0.0268	0.0244	0.0239	0.0234	0.0219	0.0222	0.0196	0.0202
	SR	0.0156	0.0151	0.0410	0.0304	0.0356	0.0601	0.0568	0.0550	0.0504
	VaR (1%)	0.1036	0.0880	0.0778	0.0769	0.0718	0.0649	0.0664	0.0605	0.0606
	VaR (5%)	0.0549	0.0446	0.0397	0.0393	0.0387	0.0346	0.0335	0.0305	0.0315
	CVaR (1%)	0.1067	0.0905	0.0781	0.0786	0.0746	0.0664	0.0695	0.0612	0.0610
	CVaR (5%)	0.0558	0.0465	0.0425	0.0397	0.0389	0.0350	0.0339	0.0316	0.0343
	MAD	0.0234	0.0190	0.0173	0.0167	0.0160	0.0145	0.0145	0.0116	0.0119

Table 17 Performance of the equally weighed portfolio (1/*n*). the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the VaR (1%) on the MIB—weekly dataset

VaR (1%)			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.08×10^{-16}	0.8077	0.2375	0.4987	0.0739	0.3942	0.0349	0.3796	0.0195
	stDev	0.0328	0.0266	0.0241	0.0246	0.0227	0.0230	0.0216	0.0207	0.0202
	SR	0.0167	-0.1068	0.0186	-0.0507	0.0143	-0.0187	0.0549	-0.0058	0.0409
	VaR (1%)	0.1036	0.0890	0.0793	0.0871	0.0764	0.0759	0.0665	0.0687	0.0604
	VaR (5%)	0.0549	0.0494	0.0417	0.0436	0.0401	0.0379	0.0345	0.0345	0.0329
	CVaR (1%)	0.1067	0.0915	0.0806	0.0883	0.0785	0.0777	0.0672	0.0702	0.0620
	CVaR (5%)	0.0558	0.0513	0.0428	0.0463	0.0409	0.0402	0.0361	0.0350	0.0329
	MAD	0.0234	0.0193	0.0178	0.0175	0.0161	0.0153	0.0143	0.0120	0.0116
4	aveSt	0.0122	0.2616	0.1366	0.1445	0.0526	0.1099	0.0257	0.1069	0.0180
	stDev	0.0328	0.0276	0.0247	0.0251	0.0233	0.0231	0.0219	0.0215	0.0203
	SR	0.0149	0.0018	0.0374	0.0121	0.0142	0.0371	0.0571	0.0356	0.0406
	VaR (1%)	0.1037	0.0917	0.0792	0.0856	0.0783	0.0717	0.0667	0.0693	0.0606
	VaR (5%)	0.0549	0.0442	0.0420	0.0419	0.0405	0.0377	0.0347	0.0331	0.0327
	CVaR (1%)	0.1068	0.0917	0.0798	0.0863	0.0789	0.0723	0.0684	0.0708	0.0628
	CVaR (5%)	0.0559	0.0463	0.0439	0.0421	0.0418	0.0392	0.0366	0.0337	0.0356
	MAD	0.0234	0.0193	0.0178	0.0174	0.0163	0.0155	0.0144	0.0124	0.0117
12	aveSt	0.0078	0.1130	0.0817	0.0639	0.0363	0.0477	0.0184	0.0402	0.0124
	stDev	0.0328	0.0285	0.0254	0.0249	0.0233	0.0229	0.0223	0.0211	0.0206
	SR	0.0156	0.0050	0.0342	0.0229	0.0131	0.0433	0.0494	0.0530	0.0390
	VaR (1%)	0.1036	0.0974	0.0792	0.0759	0.0796	0.0733	0.0675	0.0662	0.0604
	VaR (5%)	0.0549	0.0464	0.0416	0.0426	0.0406	0.0375	0.0353	0.0320	0.0332
	CVaR (1%)	0.1067	0.0999	0.0792	0.0786	0.0804	0.0739	0.0677	0.0666	0.0625
	CVaR (5%)	0.0558	0.0491	0.0420	0.0453	0.0429	0.0393	0.0357	0.0341	0.0351
	MAD	0.0234	0.0202	0.0182	0.0175	0.0162	0.0153	0.0146	0.0123	0.0118

Table 18 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the VaR (5%) on the MIB—weekly dataset

VaR (5%)			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.08×10^{-16}	0.6678	0.2406	0.4735	0.0793	0.3978	0.0364	0.3410	0.0198
	stDev	0.0328	0.0266	0.0238	0.0236	0.0224	0.0215	0.0215	0.0198	0.0200
	SR	0.0167	-0.0716	0.0124	-0.0621	0.0347	-0.0065	0.0579	0.0006	0.0546
	VaR (1%)	0.1036	0.0844	0.0765	0.0752	0.0721	0.0611	0.0633	0.0636	0.0607
	VaR (5%)	0.0549	0.0478	0.0409	0.0412	0.0379	0.0375	0.0339	0.0325	0.0318
	CVaR (1%)	0.1067	0.0852	0.0777	0.0769	0.0722	0.0640	0.0636	0.0639	0.0613
	CVaR (5%)	0.0558	0.0501	0.0416	0.0423	0.0398	0.0377	0.0353	0.0326	0.0320
	MAD	0.0234	0.0190	0.0176	0.0164	0.0158	0.0141	0.0142	0.0112	0.0118
4	aveSt	0.0122	0.2333	0.1354	0.1451	0.0478	0.1119	0.0231	0.0954	0.0141
	stDev	0.0328	0.0271	0.0244	0.0235	0.0229	0.0215	0.0216	0.0200	0.0200
	SR	0.0149	0.0050	0.0351	0.0117	0.0401	0.0483	0.0611	0.0432	0.0570
	VaR (1%)	0.1037	0.0839	0.0763	0.0715	0.0719	0.0612	0.0629	0.0614	0.0608
	VaR (5%)	0.0549	0.0460	0.0414	0.0392	0.0385	0.0375	0.0336	0.0325	0.0319
	CVaR (1%)	0.1068	0.0845	0.0772	0.0733	0.0744	0.0625	0.0640	0.0642	0.0632
	CVaR (5%)	0.0559	0.0492	0.0420	0.0396	0.0407	0.0385	0.0354	0.0350	0.0339
	MAD	0.0234	0.0189	0.0178	0.0164	0.0160	0.0143	0.0142	0.0116	0.0118
12	aveSt	0.0078	0.1114	0.0800	0.0602	0.0299	0.0467	0.0142	0.0327	0.0089
	stDev	0.0328	0.0284	0.0253	0.0239	0.0232	0.0217	0.0217	0.0199	0.0201
	SR	0.0156	0.0176	0.0348	0.0265	0.0333	0.0574	0.0626	0.0603	0.0581
	VaR (1%)	0.1036	0.0871	0.0782	0.0759	0.0714	0.0615	0.0626	0.0592	0.0610
	VaR (5%)	0.0549	0.0471	0.0418	0.0401	0.0393	0.0367	0.0336	0.0316	0.0320
	CVaR (1%)	0.1067	0.0894	0.0815	0.0781	0.0736	0.0616	0.0638	0.0602	0.0630
	CVaR (5%)	0.0558	0.0509	0.0436	0.0408	0.0399	0.0375	0.0351	0.0337	0.0337
	MAD	0.0234	0.0202	0.0182	0.0165	0.0161	0.0144	0.0143	0.0117	0.0119

Table 19 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the CVaR (1%) on the MIB—weekly dataset

CVaR (1%)			$w_{in}=20$		$w_{in}=60$		$w_{in}=120$		$w_{in}=240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.08×10^{-16}	0.8790	0.2398	0.7542	0.0888	0.6771	0.0464	0.6015	0.0275
	stDev	0.0328	0.0275	0.0241	0.0253	0.0227	0.0243	0.0216	0.0223	0.0202
	SR	0.0167	-0.1279	0.0176	-0.0948	0.0110	-0.0593	0.0536	-0.0729	0.0390
	VaR (1%)	0.1036	0.0953	0.0791	0.0867	0.0763	0.0794	0.0665	0.0767	0.0608
	VaR (5%)	0.0549	0.0501	0.0416	0.0435	0.0403	0.0441	0.0345	0.0382	0.0331
	CVaR (1%)	0.1067	0.0962	0.0795	0.0896	0.0785	0.0795	0.0693	0.0768	0.0622
	CVaR (5%)	0.0558	0.0512	0.0439	0.0457	0.0410	0.0442	0.0355	0.0399	0.0333
	MAD	0.0234	0.0199	0.0178	0.0180	0.0161	0.0166	0.0142	0.0135	0.0116
4	aveSt	0.0122	0.2756	0.1365	0.2053	0.0541	0.1910	0.0281	0.1877	0.0198
	stDev	0.0328	0.0283	0.0247	0.0259	0.0232	0.0249	0.0219	0.0233	0.0204
	SR	0.0149	-0.0095	0.0372	-0.0043	0.0143	0.0205	0.0567	-0.0046	0.0399
	VaR (1%)	0.1037	0.0918	0.0791	0.0835	0.0782	0.0788	0.0668	0.0764	0.0609
	VaR (5%)	0.0549	0.0474	0.0420	0.0435	0.0404	0.0413	0.0346	0.0379	0.0330
	CVaR (1%)	0.1068	0.0935	0.0799	0.0851	0.0785	0.0806	0.0687	0.0773	0.0633
	CVaR (5%)	0.0559	0.0504	0.0437	0.0442	0.0416	0.0419	0.0370	0.0389	0.0359
	MAD	0.0234	0.0199	0.0178	0.0178	0.0163	0.0165	0.0144	0.0133	0.0117
12	aveSt	0.0078	0.1108	0.0812	0.0814	0.0361	0.0685	0.0187	0.0669	0.0130
	stDev	0.0328	0.0286	0.0254	0.0257	0.0233	0.0247	0.0223	0.0224	0.0206
	SR	0.0156	0.0227	0.0342	0.0035	0.0139	0.0185	0.0491	0.0064	0.0386
	VaR (1%)	0.1036	0.0877	0.0793	0.0848	0.0796	0.0786	0.0673	0.0713	0.0607
	VaR (5%)	0.0549	0.0468	0.0416	0.0433	0.0407	0.0402	0.0354	0.0382	0.0332
	CVaR (1%)	0.1067	0.0882	0.0798	0.0850	0.0799	0.0806	0.0680	0.0725	0.0629
	CVaR (5%)	0.0558	0.0474	0.0427	0.0458	0.0426	0.0408	0.0361	0.0399	0.0354
	MAD	0.0234	0.0205	0.0182	0.0181	0.0162	0.0165	0.0146	0.0132	0.0118

Table 20 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the CVaR (5%) on the MIB—weekly dataset

CVaR (5%)			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.08×10^{-16}	0.8219	0.2466	0.6929	0.0936	0.5901	0.0497	0.4101	0.0300
	stDev	0.0328	0.0265	0.0238	0.0264	0.0225	0.0258	0.0215	0.0240	0.0200
	SR	0.0167	-0.1298	0.0123	-0.0897	0.0310	-0.0633	0.0556	-0.0439	0.0539
	VaR (1%)	0.1036	0.0901	0.0766	0.0929	0.0720	0.0870	0.0635	0.0801	0.0613
	VaR (5%)	0.0549	0.0492	0.0408	0.0473	0.0380	0.0468	0.0338	0.0417	0.0316
	CVaR (1%)	0.1067	0.0909	0.0769	0.0952	0.0725	0.0873	0.0643	0.0814	0.0622
	CVaR (5%)	0.0558	0.0509	0.0408	0.0491	0.0397	0.0471	0.0363	0.0418	0.0329
	MAD	0.0234	0.0194	0.0176	0.0188	0.0158	0.0176	0.0141	0.0141	0.0118
4	aveSt	0.0122	0.2516	0.1355	0.2011	0.0505	0.1671	0.0273	0.1338	0.0200
	stDev	0.0328	0.0272	0.0244	0.0272	0.0229	0.0258	0.0217	0.0247	0.0201
	SR	0.0149	-0.0085	0.0356	-0.0196	0.0379	0.0182	0.0609	-0.0090	0.0575
	VaR (1%)	0.1037	0.0903	0.0767	0.0916	0.0716	0.0833	0.0628	0.0866	0.0607
	VaR (5%)	0.0549	0.0453	0.0414	0.0448	0.0388	0.0425	0.0337	0.0402	0.0317
	CVaR (1%)	0.1068	0.0930	0.0773	0.0934	0.0743	0.0854	0.0635	0.0895	0.0631
	CVaR (5%)	0.0559	0.0476	0.0422	0.0453	0.0406	0.0438	0.0347	0.0433	0.0336
	MAD	0.0234	0.0193	0.0178	0.0187	0.0160	0.0171	0.0142	0.0139	0.0119
12	aveSt	0.0078	0.1028	0.0797	0.0749	0.0304	0.0655	0.0158	0.0516	0.0110
	stDev	0.0328	0.0280	0.0254	0.0279	0.0233	0.0257	0.0218	0.0246	0.0202
	SR	0.0156	0.0162	0.0349	-0.0043	0.0316	0.0291	0.0619	0.0050	0.0578
	VaR (1%)	0.1036	0.0887	0.0785	0.0949	0.0718	0.0858	0.0622	0.0755	0.0610
	VaR (5%)	0.0549	0.0446	0.0417	0.0452	0.0395	0.0417	0.0336	0.0397	0.0321
	CVaR (1%)	0.1067	0.0903	0.0798	0.0953	0.0736	0.0882	0.0632	0.0770	0.0629
	CVaR (5%)	0.0558	0.0448	0.0418	0.0467	0.0397	0.0438	0.0345	0.0399	0.0334
	MAD	0.0234	0.0201	0.0182	0.0189	0.0161	0.0170	0.0143	0.0139	0.0120

Dataset FTSE

See Tables 21, 22, 23, 24, 25, and 26.

Table 21 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the variance on the FTSE dataset

variance			$w_{in}=20$		$w_{in}=60$		$w_{in}=120$		$w_{in}=240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.7×10^{-16}	–	0.3575	–	0.1378	0.1083	0.0759	0.0616	0.0419
	stDev	0.0121	–	0.0097	–	0.0093	0.0088	0.0092	0.0087	0.0092
	SR	0.0126	–	-0.1553	–	-0.0480	-0.0304	-0.0209	-0.0051	-0.0012
	VaR (1%)	0.0369	–	0.0294	–	0.0280	0.0267	0.0273	0.0266	0.0262
	VaR (5%)	0.0190	–	0.0163	–	0.0148	0.0138	0.0145	0.0137	0.0144
	CVaR (1%)	0.0388	–	0.0309	–	0.0282	0.0270	0.0275	0.0270	0.0269
	CVaR (5%)	0.0192	–	0.0178	–	0.0158	0.0144	0.0145	0.0146	0.0149
	MAD	0.0081	–	0.0067	–	0.0065	0.0060	0.0063	0.0058	0.0062
5	aveSt	0.0043	–	0.1798	–	0.0752	0.0548	0.0421	0.0318	0.0243
	stDev	0.0121	–	0.0099	–	0.0094	0.0088	0.0093	0.0087	0.0092
	SR	0.0108	–	-0.0561	–	-0.0156	-0.0011	-0.0016	0.0106	0.0089
	VaR (1%)	0.0369	–	0.0289	–	0.0272	0.0267	0.0265	0.0263	0.0258
	VaR (5%)	0.0190	–	0.0157	–	0.0145	0.0135	0.0143	0.0135	0.0142
	CVaR (1%)	0.0388	–	0.0297	–	0.0288	0.0279	0.0267	0.0265	0.0266
	CVaR (5%)	0.0192	–	0.0160	–	0.0161	0.0136	0.0153	0.0141	0.0145
	MAD	0.0081	–	0.0069	–	0.0065	0.0060	0.0063	0.0058	0.0062
20	aveSt	0.0023	–	0.0799	–	0.0409	0.0292	0.0239	0.0174	0.0142
	stDev	0.0121	–	0.0102	–	0.0096	0.0091	0.0094	0.0089	0.0093
	SR	0.0116	–	-0.0269	–	0.0013	0.0103	0.0074	0.0154	0.0128
	VaR (1%)	0.0369	–	0.0307	–	0.0278	0.0274	0.0271	0.0270	0.0260
	VaR (5%)	0.0190	–	0.0159	–	0.0148	0.0137	0.0143	0.0137	0.0143
	CVaR (1%)	0.0388	–	0.0325	–	0.0288	0.0288	0.0285	0.0285	0.0262
	CVaR (5%)	0.0192	–	0.0163	–	0.0160	0.0147	0.0161	0.0140	0.0157
	MAD	0.0081	–	0.0071	–	0.0066	0.0061	0.0064	0.0059	0.0062
60	aveSt	0.0013	–	0.0270	–	0.0236	0.0172	0.0148	0.0105	0.0090
	stDev	0.0121	–	0.0104	–	0.0095	0.0093	0.0093	0.0089	0.0094
	SR	0.0120	–	0.0065	–	0.0079	0.0207	0.0121	0.0187	0.0129
	VaR (1%)	0.0369	–	0.0310	–	0.0281	0.0282	0.0269	0.0268	0.0263
	VaR (5%)	0.0190	–	0.0157	–	0.0148	0.0141	0.0145	0.0136	0.0143
	CVaR (1%)	0.0388	–	0.0313	–	0.0286	0.0298	0.0269	0.0270	0.0264
	CVaR (5%)	0.0192	–	0.0157	–	0.0156	0.0147	0.0155	0.0147	0.0163
	MAD	0.0081	–	0.0071	–	0.0066	0.0063	0.0064	0.0060	0.0063

Table 22 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the MAD on the FTSE dataset

MAD			$w_{in}=20$		$w_{in}=60$		$w_{in}=120$		$w_{in}=240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.7×10^{-16}	0.6070	0.3309	0.3100	0.1250	0.1924	0.0669	0.1187	0.0349
	stDev	0.0121	0.0100	0.0093	0.0091	0.0091	0.0090	0.0090	0.0089	0.0091
	SR	0.0126	-0.2807	-0.1512	-0.1486	-0.0450	-0.0799	-0.0162	-0.0397	0.0010
	VaR (1%)	0.0369	0.0316	0.0288	0.0282	0.0275	0.0287	0.0272	0.0268	0.0270
	VaR (5%)	0.0190	0.0182	0.0156	0.0157	0.0141	0.0145	0.0141	0.0141	0.0140
	CVaR (1%)	0.0388	0.0330	0.0302	0.0292	0.0280	0.0298	0.0283	0.0268	0.0287
	CVaR (5%)	0.0192	0.0186	0.0159	0.0165	0.0142	0.0157	0.0144	0.0154	0.0146
	MAD	0.0081	0.0069	0.0064	0.0063	0.0062	0.0061	0.0061	0.0059	0.0060
5	aveSt	0.0043	0.2253	0.1568	0.1191	0.0615	0.0730	0.0331	0.0443	0.0179
	stDev	0.0121	0.0101	0.0095	0.0092	0.0091	0.0090	0.0090	0.0089	0.0091
	SR	0.0108	-0.0955	-0.0495	-0.0401	-0.0104	-0.0138	0.0023	-0.0016	0.0097
	VaR (1%)	0.0369	0.0300	0.0278	0.0274	0.0269	0.0275	0.0270	0.0267	0.0269
	VaR (5%)	0.0190	0.0169	0.0149	0.0148	0.0140	0.0138	0.0139	0.0138	0.0140
	CVaR (1%)	0.0388	0.0308	0.0284	0.0289	0.0286	0.0283	0.0278	0.0281	0.0285
	CVaR (5%)	0.0192	0.0171	0.0158	0.0163	0.0148	0.0144	0.0159	0.0148	0.0144
	MAD	0.0081	0.0071	0.0066	0.0064	0.0062	0.0061	0.0061	0.0059	0.0060
20	aveSt	0.0023	0.0843	0.0739	0.0515	0.0327	0.0332	0.0177	0.0203	0.0101
	stDev	0.0121	0.0107	0.0099	0.0096	0.0093	0.0094	0.0091	0.0091	0.0092
	SR	0.0116	-0.0313	-0.0190	-0.0055	0.0065	0.0050	0.0108	0.0090	0.0124
	VaR (1%)	0.0369	0.0301	0.0300	0.0282	0.0276	0.0276	0.0271	0.0268	0.0272
	VaR (5%)	0.0190	0.0168	0.0152	0.0149	0.0141	0.0140	0.0140	0.0138	0.0140
	CVaR (1%)	0.0388	0.0309	0.0302	0.0290	0.0283	0.0292	0.0290	0.0274	0.0286
	CVaR (5%)	0.0192	0.0175	0.0153	0.0149	0.0157	0.0150	0.0150	0.0139	0.0145
	MAD	0.0081	0.0073	0.0068	0.0065	0.0063	0.0062	0.0061	0.0060	0.0061
60	aveSt	0.0013	0.0279	0.0250	0.0249	0.0192	0.0173	0.0110	0.0114	0.0066
	stDev	0.0121	0.0111	0.0100	0.0097	0.0093	0.0094	0.0092	0.0092	0.0093
	SR	0.0120	0.0003	0.0085	0.0078	0.0095	0.0155	0.0117	0.0143	0.0117
	VaR (1%)	0.0369	0.0325	0.0296	0.0294	0.0277	0.0282	0.0270	0.0278	0.0278
	VaR (5%)	0.0190	0.0170	0.0150	0.0147	0.0142	0.0142	0.0141	0.0139	0.0140
	CVaR (1%)	0.0388	0.0345	0.0313	0.0295	0.0288	0.0290	0.0271	0.0288	0.0284
	CVaR (5%)	0.0192	0.0172	0.0169	0.0159	0.0154	0.0155	0.0155	0.0152	0.0147
	MAD	0.0081	0.0074	0.0068	0.0066	0.0064	0.0063	0.0062	0.0061	0.0061

Table 23 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the VaR (1%) on the FTSE dataset

VaR (1%)			$w_{in} = 20$		$w_{in} = 60$		$w_{in} = 120$		$w_{in} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.7×10^{-16}	0.8531	0.3592	0.6603	0.1227	0.5666	0.0674	0.4920	0.0351
	stDev	0.0121	0.0101	0.0093	0.0095	0.0092	0.0090	0.0090	0.0090	0.0090
	SR	0.0126	-0.4071	-0.1779	-0.3311	-0.0498	-0.2790	-0.0193	-0.2429	0.0042
	VaR (1%)	0.0369	0.0329	0.0284	0.0316	0.0275	0.0303	0.0267	0.0289	0.0265
	VaR (5%)	0.0190	0.0204	0.0161	0.0182	0.0148	0.0166	0.0144	0.0165	0.0141
	CVaR (1%)	0.0388	0.0336	0.0294	0.0332	0.0284	0.0308	0.0275	0.0292	0.0277
	CVaR (5%)	0.0192	0.0206	0.0168	0.0186	0.0157	0.0173	0.0145	0.0175	0.0144
	MAD	0.0081	0.0072	0.0066	0.0066	0.0064	0.0062	0.0063	0.0062	0.0061
5	aveSt	0.0043	0.2416	0.1768	0.1617	0.0746	0.1304	0.0440	0.1078	0.0251
	stDev	0.0121	0.0104	0.0095	0.0096	0.0092	0.0090	0.0091	0.0091	0.0090
	SR	0.0108	-0.0955	-0.0719	-0.0652	-0.0210	-0.0460	-0.0042	-0.0419	0.0094
	VaR (1%)	0.0369	0.0301	0.0278	0.0286	0.0270	0.0282	0.0264	0.0276	0.0265
	VaR (5%)	0.0190	0.0178	0.0154	0.0159	0.0145	0.0145	0.0144	0.0145	0.0142
	CVaR (1%)	0.0388	0.0317	0.0285	0.0301	0.0285	0.0287	0.0268	0.0285	0.0279
	CVaR (5%)	0.0192	0.0185	0.0163	0.0173	0.0158	0.0157	0.0154	0.0151	0.0145
	MAD	0.0081	0.0074	0.0067	0.0066	0.0064	0.0063	0.0063	0.0061	0.0062
20	aveSt	0.0023	0.0852	0.0793	0.0563	0.0434	0.0424	0.0266	0.0331	0.0158
	stDev	0.0121	0.0103	0.0099	0.0097	0.0095	0.0094	0.0093	0.0093	0.0092
	SR	0.0116	-0.0267	-0.0267	-0.0066	0.0004	0.0003	0.0119	-0.0004	0.0167
	VaR (1%)	0.0369	0.0298	0.0296	0.0285	0.0278	0.0282	0.0269	0.0283	0.0266
	VaR (5%)	0.0190	0.0170	0.0159	0.0154	0.0147	0.0144	0.0143	0.0143	0.0141
	CVaR (1%)	0.0388	0.0314	0.0301	0.0288	0.0295	0.0298	0.0276	0.0297	0.0272
	CVaR (5%)	0.0192	0.0172	0.0165	0.0161	0.0160	0.0145	0.0160	0.0151	0.0156
	MAD	0.0081	0.0073	0.0069	0.0067	0.0065	0.0064	0.0064	0.0062	0.0062
60	aveSt	0.0013	0.0284	0.0269	0.0265	0.0252	0.0197	0.0163	0.0149	0.0104
	stDev	0.0121	0.0111	0.0100	0.0098	0.0096	0.0094	0.0093	0.0093	0.0093
	SR	0.0120	0.0045	0.0150	0.0076	0.0065	0.0112	0.0189	0.0073	0.0178
	VaR (1%)	0.0369	0.0311	0.0292	0.0287	0.0278	0.0276	0.0266	0.0274	0.0266
	VaR (5%)	0.0190	0.0173	0.0154	0.0155	0.0150	0.0146	0.0144	0.0144	0.0142
	CVaR (1%)	0.0388	0.0320	0.0304	0.0304	0.0285	0.0289	0.0277	0.0282	0.0269
	CVaR (5%)	0.0192	0.0187	0.0163	0.0162	0.0163	0.0164	0.0147	0.0158	0.0157
	MAD	0.0081	0.0075	0.0069	0.0068	0.0066	0.0064	0.0064	0.0063	0.0062

Table 24 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the VaR (5%) on the FTSE dataset

VaR (5%)			$w_{in}=20$		$w_{in}=60$		$w_{in}=120$		$w_{in}=240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.7×10^{-16}	0.7377	0.3556	0.6540	0.1365	0.5448	0.0738	0.4636	0.0393
	stDev	0.0121	0.0100	0.0093	0.0093	0.0090	0.0090	0.0090	0.0090	0.0090
	SR	0.0126	-0.3570	-0.1804	-0.3396	-0.0587	-0.2701	-0.0196	-0.2181	0.0012
	VaR (1%)	0.0369	0.0314	0.0290	0.0298	0.0275	0.0288	0.0266	0.0285	0.0268
	VaR (5%)	0.0190	0.0195	0.0161	0.0183	0.0145	0.0164	0.0143	0.0165	0.0140
	CVaR (1%)	0.0388	0.0320	0.0298	0.0300	0.0288	0.0299	0.0273	0.0288	0.0284
	CVaR (5%)	0.0192	0.0212	0.0164	0.0191	0.0151	0.0171	0.0154	0.0174	0.0143
	MAD	0.0081	0.0071	0.0066	0.0065	0.0063	0.0063	0.0062	0.0062	0.0060
5	aveSt	0.0043	0.2323	0.1768	0.1629	0.0743	0.1253	0.0413	0.1014	0.0230
	stDev	0.0121	0.0103	0.0095	0.0094	0.0090	0.0091	0.0090	0.0091	0.0091
	SR	0.0108	-0.0896	-0.0757	-0.0727	-0.0253	-0.0461	-0.0007	-0.0259	0.0104
	VaR (1%)	0.0369	0.0298	0.0280	0.0283	0.0280	0.0276	0.0266	0.0269	0.0267
	VaR (5%)	0.0190	0.0174	0.0153	0.0158	0.0142	0.0146	0.0140	0.0147	0.0139
	CVaR (1%)	0.0388	0.0311	0.0284	0.0287	0.0294	0.0276	0.0284	0.0274	0.0281
	CVaR (5%)	0.0192	0.0176	0.0159	0.0159	0.0157	0.0148	0.0145	0.0160	0.0139
	MAD	0.0081	0.0073	0.0067	0.0066	0.0063	0.0063	0.0062	0.0061	0.0060
20	aveSt	0.0023	0.0849	0.0794	0.0573	0.0405	0.0409	0.0227	0.0318	0.0131
	stDev	0.0121	0.0101	0.0099	0.0097	0.0094	0.0093	0.0092	0.0093	0.0092
	SR	0.0116	-0.0290	-0.0277	-0.0130	0.0011	0.0006	0.0133	0.0097	0.0158
	VaR (1%)	0.0369	0.0284	0.0296	0.0287	0.0279	0.0274	0.0270	0.0273	0.0272
	VaR (5%)	0.0190	0.0169	0.0158	0.0156	0.0144	0.0143	0.0140	0.0143	0.0139
	CVaR (1%)	0.0388	0.0285	0.0308	0.0289	0.0291	0.0280	0.0283	0.0291	0.0280
	CVaR (5%)	0.0192	0.0184	0.0158	0.0169	0.0145	0.0158	0.0160	0.0150	0.0158
	MAD	0.0081	0.0072	0.0069	0.0067	0.0065	0.0063	0.0063	0.0061	0.0061
60	aveSt	0.0013	0.0282	0.0270	0.0256	0.0226	0.0185	0.0134	0.0134	0.0082
	stDev	0.0121	0.0110	0.0101	0.0099	0.0094	0.0094	0.0093	0.0093	0.0093
	SR	0.0120	0.0019	0.0125	0.0085	0.0092	0.0133	0.0144	0.0161	0.0157
	VaR (1%)	0.0369	0.0316	0.0297	0.0290	0.0286	0.0280	0.0276	0.0278	0.0275
	VaR (5%)	0.0190	0.0171	0.0156	0.0153	0.0147	0.0144	0.0142	0.0141	0.0140
	CVaR (1%)	0.0388	0.0325	0.0297	0.0309	0.0295	0.0289	0.0284	0.0284	0.0280
	CVaR (5%)	0.0192	0.0184	0.0160	0.0155	0.0162	0.0149	0.0149	0.0152	0.0143
	MAD	0.0081	0.0074	0.0070	0.0068	0.0065	0.0063	0.0063	0.0062	0.0061

Table 25 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the CVaR (1%) on the FTSE dataset

CVaR (1%)			$w_{\text{in}} = 20$		$w_{\text{in}} = 60$		$w_{\text{in}} = 120$		$w_{\text{in}} = 240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.7×10^{-16}	0.8872	0.3645	0.8569	0.1446	0.8310	0.0897	0.7682	0.0496
	stDev	0.0121	0.0102	0.0093	0.0098	0.0092	0.0101	0.0090	0.0103	0.0090
	SR	0.0126	-0.4151	-0.1805	-0.4171	-0.0609	-0.3894	-0.0309	-0.3392	-0.0031
	VaR (1%)	0.0369	0.0335	0.0284	0.0333	0.0277	0.0341	0.0268	0.0358	0.0265
	VaR (5%)	0.0190	0.0209	0.0160	0.0198	0.0149	0.0196	0.0146	0.0195	0.0142
	CVaR (1%)	0.0388	0.0352	0.0288	0.0345	0.0288	0.0360	0.0281	0.0372	0.0279
	CVaR (5%)	0.0192	0.0220	0.0161	0.0210	0.0161	0.0196	0.0151	0.0197	0.0144
	MAD	0.0081	0.0072	0.0066	0.0068	0.0064	0.0069	0.0063	0.0070	0.0061
5	aveSt	0.0043	0.2349	0.1771	0.1970	0.0768	0.1876	0.0481	0.1767	0.0291
	stDev	0.0121	0.0103	0.0095	0.0099	0.0092	0.0099	0.0091	0.0104	0.0090
	SR	0.0108	-0.0986	-0.0716	-0.0729	-0.0220	-0.0804	-0.0065	-0.0753	0.0071
	VaR (1%)	0.0369	0.0299	0.0279	0.0302	0.0270	0.0317	0.0264	0.0316	0.0266
	VaR (5%)	0.0190	0.0174	0.0155	0.0165	0.0145	0.0165	0.0144	0.0168	0.0142
	CVaR (1%)	0.0388	0.0315	0.0283	0.0309	0.0274	0.0336	0.0273	0.0320	0.0281
	CVaR (5%)	0.0192	0.0187	0.0161	0.0180	0.0147	0.0178	0.0160	0.0187	0.0146
	MAD	0.0081	0.0072	0.0067	0.0069	0.0064	0.0068	0.0063	0.0069	0.0062
20	aveSt	0.0023	0.0797	0.0792	0.0572	0.0436	0.0515	0.0272	0.0470	0.0166
	stDev	0.0121	0.0105	0.0099	0.0100	0.0095	0.0102	0.0093	0.0103	0.0092
	SR	0.0116	-0.0244	-0.0265	-0.0125	0.0006	0.0001	0.0114	-0.0105	0.0155
	VaR (1%)	0.0369	0.0294	0.0296	0.0289	0.0277	0.0307	0.0268	0.0313	0.0267
	VaR (5%)	0.0190	0.0173	0.0158	0.0162	0.0147	0.0155	0.0143	0.0156	0.0141
	CVaR (1%)	0.0388	0.0304	0.0297	0.0307	0.0295	0.0311	0.0280	0.0322	0.0275
	CVaR (5%)	0.0192	0.0188	0.0162	0.0174	0.0159	0.0165	0.0146	0.0171	0.0158
	MAD	0.0081	0.0073	0.0069	0.0069	0.0065	0.0068	0.0064	0.0067	0.0062
60	aveSt	0.0013	0.0266	0.0269	0.0240	0.0252	0.0194	0.0164	0.0179	0.0104
	stDev	0.0121	0.0111	0.0100	0.0103	0.0095	0.0104	0.0093	0.0103	0.0093
	SR	0.0120	0.0078	0.0151	0.0042	0.0065	0.0091	0.0184	0.0069	0.0175
	VaR (1%)	0.0369	0.0315	0.0293	0.0299	0.0276	0.0311	0.0266	0.0316	0.0266
	VaR (5%)	0.0190	0.0170	0.0154	0.0162	0.0150	0.0156	0.0145	0.0155	0.0141
	CVaR (1%)	0.0388	0.0327	0.0302	0.0305	0.0292	0.0321	0.0279	0.0322	0.0273
	CVaR (5%)	0.0192	0.0171	0.0160	0.0175	0.0152	0.0174	0.0149	0.0171	0.0160
	MAD	0.0081	0.0074	0.0069	0.0070	0.0066	0.0069	0.0064	0.0067	0.0062

Table 26 Performance of the equally weighed portfolio (1/*n*), the maximum diversification (MD) and the minimum risk (mr) strategies computed w.r.t. the CVaR (5%) on the FTSE dataset

CVaR (5%)			$w_{in}=20$		$w_{in}=60$		$w_{in}=120$		$w_{in}=240$	
w_{out}		1/*n*	mr	MD	mr	MD	mr	MD	mr	MD
1	aveSt	2.7×10^{-16}	0.8671	0.3666	0.7924	0.1676	0.6677	0.1042	0.5389	0.0648
	stDev	0.0121	0.0102	0.0093	0.0103	0.0090	0.0106	0.0090	0.0112	0.0090
	SR	0.0126	-0.4042	-0.1859	-0.3685	-0.0751	-0.3027	-0.0381	-0.2238	-0.0110
	VaR (1%)	0.0369	0.0340	0.0290	0.0351	0.0275	0.0360	0.0268	0.0365	0.0268
	VaR (5%)	0.0190	0.0205	0.0161	0.0200	0.0147	0.0197	0.0144	0.0200	0.0142
	CVaR (1%)	0.0388	0.0341	0.0300	0.0360	0.0291	0.0361	0.0284	0.0382	0.0269
	CVaR (5%)	0.0192	0.0210	0.0168	0.0210	0.0154	0.0198	0.0163	0.0218	0.0149
	MAD	0.0081	0.0072	0.0066	0.0070	0.0063	0.0072	0.0062	0.0075	0.0060
5	aveSt	0.0043	0.2268	0.1768	0.1908	0.0780	0.1603	0.0487	0.1364	0.0334
	stDev	0.0121	0.0105	0.0095	0.0102	0.0090	0.0106	0.0090	0.0110	0.0091
	SR	0.0108	-0.0911	-0.0745	-0.0758	-0.0260	-0.0638	-0.0058	-0.0516	0.0049
	VaR (1%)	0.0369	0.0314	0.0280	0.0307	0.0279	0.0325	0.0267	0.0346	0.0266
	VaR (5%)	0.0190	0.0177	0.0153	0.0171	0.0143	0.0171	0.0142	0.0175	0.0140
	CVaR (1%)	0.0388	0.0315	0.0288	0.0323	0.0280	0.0327	0.0274	0.0364	0.0283
	CVaR (5%)	0.0192	0.0185	0.0163	0.0189	0.0144	0.0182	0.0154	0.0176	0.0143
	MAD	0.0081	0.0074	0.0067	0.0070	0.0063	0.0071	0.0062	0.0072	0.0060
20	aveSt	0.0023	0.0770	0.0791	0.0547	0.0406	0.0447	0.0240	0.0367	0.0151
	stDev	0.0121	0.0106	0.0099	0.0104	0.0094	0.0107	0.0092	0.0111	0.0092
	SR	0.0116	-0.0136	-0.0264	-0.0054	0.0022	-0.0060	0.0107	-0.0073	0.0146
	VaR (1%)	0.0369	0.0301	0.0296	0.0304	0.0277	0.0324	0.0270	0.0350	0.0271
	VaR (5%)	0.0190	0.0168	0.0158	0.0164	0.0145	0.0165	0.0141	0.0167	0.0140
	CVaR (1%)	0.0388	0.0309	0.0308	0.0320	0.0281	0.0343	0.0281	0.0366	0.0284
	CVaR (5%)	0.0192	0.0178	0.0176	0.0178	0.0157	0.0184	0.0158	0.0182	0.0145
	MAD	0.0081	0.0074	0.0069	0.0071	0.0065	0.0071	0.0063	0.0072	0.0061
60	aveSt	0.0013	0.0261	0.0270	0.0227	0.0225	0.0177	0.0134	0.0147	0.0085
	stDev	0.0121	0.0110	0.0101	0.0107	0.0094	0.0109	0.0092	0.0111	0.0093
	SR	0.0120	0.0035	0.0129	0.0080	0.0097	0.0030	0.0147	0.0000	0.0163
	VaR (1%)	0.0369	0.0332	0.0297	0.0315	0.0284	0.0331	0.0278	0.0347	0.0275
	VaR (5%)	0.0190	0.0170	0.0156	0.0162	0.0147	0.0162	0.0141	0.0167	0.0140
	CVaR (1%)	0.0388	0.0335	0.0314	0.0326	0.0286	0.0342	0.0283	0.0349	0.0278
	CVaR (5%)	0.0192	0.0177	0.0159	0.0165	0.0153	0.0162	0.0148	0.0186	0.0141
	MAD	0.0081	0.0074	0.0070	0.0072	0.0065	0.0071	0.0063	0.0071	0.0061

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Declarations

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