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Analytical load flow solution for radial distribution networks

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ABSTRACT

This paper proposes an analytical method to solve the load flow problem for radial single and multi-feeder power distribution networks in three-phase balanced conditions. The Analytical Load Flow (ALF) formulation relies on a single assumption for the estimation of line losses and accounts for line susceptances. A set of comparative tests performed on a total active and passive 33 nodes benchmark network allowed showing that the accuracy of the proposed method is extremely high, if compared with the alternative numerical solution. Furthermore, a specific analysis is proposed to evaluate the impact of the approximation on the losses in the determination of the nodal voltage phasors. Finally, ALF is validated in a realistic scenario with high integration of Renewable Energy Sources (RESs), considering seasonal variations in production and consumption. In this context, it is shown that the proposed method outperforms existing approximate analytical approaches, such as the industrial voltage drop method. The ALF approach, being fully analytical, does not require any numerical solver and can be applied as a valid alternative to existing numerical and analytical methods in balanced multi-feeder networks.

1. Introduction

The solution of the Load Flow (LF) problem determines the nodal voltage values (amplitude and angle) under various operational conditions. Due to the intrinsic non-linear nature of its mathematical formulation, the resolution of the LF problem needs the utilization of numerical methods [1]. Several iterative methods have been developed to solve the LF problem, which requires computational software to manage the complexity of power systems [2]. Each method offers distinct advantages, depending on the characteristics of the network, such as whether it is meshed or radial, balanced or unbalanced. The Newton-Raphson method is an iterative method which is based on Jacobian matrix to relate power variations with voltage magnitudes and angles. This strategy is the most commonly implemented for complex systems, but it may be inefficient in networks with high R/X ratios or under unbalanced conditions [3]. The Symmetrical Component Method considers voltages and currents into positive, negative, and zero sequences provides detailed power flow analysis of three-phase unbalanced networks [4]. The Gauss-Zbus iteration method leverages the bus impedance matrix in an iterative process and is reputed effective for meshed passive networks [5]. The Backward/Forward Sweep Method is designed for the load-flow analysis of the radial distribution systems [6].

Several studies introduce improvements or variations to the main iterative strategies for solving the LF problem in order to enhance their accuracy, convergence, and applicability across different network types. In [7], a numerical power flow method based on a multi-port compensation technique is presented as an alternative to the Newton-Raphson approach for solving distribution and transmission networks. The method [8] uses oriented ordering of network elements, and it is based on voltage correction for the improving the iterative process and making the voltage calculation faster and more reliable for distribution networks. A three-phase power-flow solution using sequence components that saves time and memory compared to phase-coordinate Newton–Raphson methods, while maintaining similar convergence, is described in [9]. Newer and more recent algorithms have been developed to introduce modifications and improvements to previous numeric and iterative techniques [10,11].

In a scenario of extensive penetration of Renewable Energy Sources (RESs) into electrical grids, there is a growing need for user-friendly and computationally efficient tools to characterize in terms of currents and voltages Microgrids (MGs) [12,13] as well as distribution [14] and transmission networks [15,16].

To simplify LF studies and ensure numerical convergence, linear formulations of the LF equations have been derived using mathematical tools often unrelated to electrotechnical considerations. Among them, it is worth citing the approach proposed in [17], where an approximation of the geometric series formula allows to transform the LF problem into a linear algebraic system that can be solved via numerical matrix

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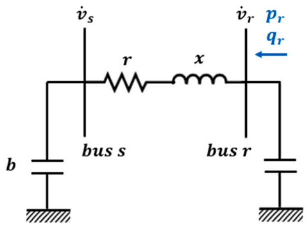

Fig. 1. Point-to-point case.

inversion. Following the same consideration, study [18] introduces a quadratic approximation formula based on a Taylor expansion for arbitrary dependent variables in power networks.

Considering radial networks, a simple algebraic expression of voltage magnitude is proposed in [19], however it is based on a complex algorithm for the identification of nodes and branches beyond a particular node. Cespedes G. proposed in [20] an analytical formulation based on an electric equivalent and in the elimination of the voltage phase angle in the equation to be solved. Alternatively, the only available and well-established method for analytically calculating voltage amplitudes in a radial network is the industrial voltage drop formula. Notably, this formulation incorporates multiple approximations, highlighted in [21], providing limited results accuracy especially in the case of networks with significant generation. However, these studies do not address the determination of voltage phase angles and these formulations are not applicable to lines where capacitance cannot be neglected.

This paper addresses these issues proposing an Analytical Load Flow (ALF) solution for determining the nodal voltage phasors in distribution networks based on Kirchhoff laws and on the estimation of the line losses at nominal voltage. Specifically, the ALF formulation provides the following key contributions to the existing literature gaps:

- It enables the analytical calculation of voltage phasors at the nodes of a balanced radial network,
- It is applicable to radial multi-feeder networks, accounting for line capacitances.
- It does not require the use of specific computational software for solving LF problems or for matrix inversion.

The ALF formulation is first derived in the point-to-point case and then extended to the single feeder and to a general the multi feeder case. The method is validated and compared with the numerical solution on the IEEE 33-bus benchmark network, under various load and generation conditions. Moreover, ALF is tested in a realistic scenario with high RESs penetration and compared, in terms of voltage magnitude errors, with the analytical method over four weeks to account for the seasonal variability of the profiles.

The paper is organized as follows: Section 2 details the ALF formulation in the point-to-point case and extends the procedure to multi-feeder radial network, Section 3 presents the case study used to test the proposed approach and discusses the related results considering a totally active/passive radial network and a realistic scenario. Finally, Section 4 provides some conclusive remarks and future developments.

2. Methodology

2.1. The point-to-point case

Let us consider the case where the sending bus (bus s) and the receiving bus (bus r) are connected via a π infrastructure model, as depicted in Fig. 1. Assuming p.u. quantities on suitable bases, application of KVL to the mesh of Fig. 1 allows writing:

$$\dot{v}_s = \dot{v}_r - (r + jx)\frac{p_r - jq_r}{\dot{v}_r^{\,*}} + jb(r + jx)\dot{v}_r \tag{1}$$

where the meaning of the symbols is clarified in Fig. 1.

Indicating with δs the phase shift between the two voltages (bus s leading bus r), and separating real and imaginary parts of (1), one gets:

$$\begin{cases} v_s v_r \cos\delta_s = (1 - bx)v_r^2 - (rp_r + xq_r) \\ v_s v_r \sin\delta_s = brv_r^2 + rq_r - xp_r \end{cases} \tag{2}$$

Summing the square of both equations in (2) and summing up, one obtains:

$$v_r^4 - \left[2(rp_r + xq_r) + v_s^2 - 2bq_r\left(r^2 + x^2\right)\right]v_r^2 + \left(r^2 + x^2\right)\left(p_r^2 + q_r^2\right) = 0 \tag{3}$$

whose physically acceptable solution (if exists) reads:

$$v_r = f_{r,x}(v_s, p_r, q_r) = \sqrt{\frac{\left[2(rp_r + xq_r) + v_s^2\right] + \sqrt{\left[2(rp_r + xq_r) + v_s^2\right]^2 - 4\left[\left(1 - b^2r^2\right)^2 + b^2r^2\right]\left(r^2 + x^2\right)\left(p_r^2 + q_r^2\right)}}{2\left[\left(1 - b^2r^2\right)^2 + b^2r^2\right]}} \tag{4}$$

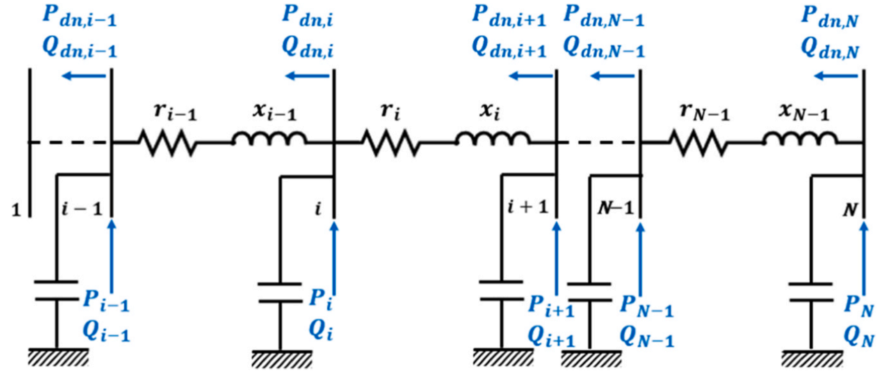

Fig. 2. Single-feeder radial network.

Eq. (4) states that one can evaluate the amplitude of the voltage at the receiving bus as function of the voltage amplitude at the sending bus and of the active and reactive power injected at the receiving bus.

Given the amplitudes of the voltages and assuming the receiving node as the reference node, one can determine the phase δs from relation (1).

2.2. Single feeder radial network

Let us now consider the situation of a N bus single feeder radial network, as sketched in Fig. 2. As a common practice, one can assume that bus 1 is the phase reference (i.e. $\dot{v}_1$=1) and all other busses are PQ ones (P_i and Q_i being the known active and reactive power net injections at the generic *i-th* bus, $i = 2...N$). Moreover, the *N-1* network branches are numbered in such a way that branch i connects bus *i-1* to bus i. Finally, for the future derivation, one indicates with $P_{\mathrm{dn}\ ,\mathrm{i}}$ ($Q_{\mathrm{dn}\ ,\mathrm{i}}$) the active (reactive) power injected by the downstream network of bus 1 (labelled as "dn" powers in the following for the sake of brevity). This means that $P_{\mathrm{dn}\ ,\mathrm{i}}$ ($Q_{\mathrm{dn}\ ,\mathrm{i}}$) is the sum of the injections of all nodes from the i-th to the last deprived of active (reactive) power losses $P_{loss,i}$ ($Q_{loss,i}$) in the branches from the *i-th* to the last. As losses are in principle unknown, their estimation can be done assuming rated voltage in the calculation of the current in the *i-th* branch, as follows:

$$\begin{cases} P_{loss,i} = r_i \dfrac{P^2_{dn,i+1} + \left(bv^2_{i+1} + Q_{dn,i+1}\right)^2}{v^2_{i+1}} \cong r_i \left[P^2_{dn,i+1} + \left(bv^2_{i+1} + Q_{dn,i+1}\right)^2\right] \\ Q_{loss,i} = x_i \dfrac{P^2_{dn,i+1} + \left(bv^2_{i+1} + Q_{dn,i+1}\right)^2}{v^2_{i+1}} - bv^2_{i+1} \cong x_i \left[P^2_{dn,i+1} + \left(bv^2_{i+1} + Q_{dn,i+1}\right)^2\right] - b \end{cases} \tag{5}$$

This allows to define the following recursive formula for the calculation of $P_{\mathrm{dn}\ ,\mathrm{i}}$ ($Q_{\mathrm{dn}\ ,\mathrm{i}}$):

$$\begin{aligned} & P_{dn,i} = P_i + P_{dn,i+1} - r_i \left[P^2_{dn,i+1} + \left(bv^2_{i+1} + Q_{dn,i+1}\right)^2\right] \\ & i = 2...N \\ & Q_{dn,i} = Q_i + Q_{dn,i+1} - x_i \left[P^2_{dn,i+1} + \left(bv^2_{i+1} + Q_{dn,i+1}\right)^2\right] + b \\ & i = 2...N \\ & P_{dn,N} = P_N \\ & Q_{dn,N} = Q_N \end{aligned} \tag{6}$$

At this point, one can calculate the voltage at all the network busses using (6) recursively, i.e.:

$$\begin{aligned} & v_i = f_{r_{i-1},x_{i-1}}\left(v_{i-1}, P_{dn,i}, Q_{dn,i}\right) \quad i = 2...N \\ & v_1 = 1 \end{aligned} \tag{7}$$

Then, assuming node N as the reference node, one can determine the phases of the voltages at all the network buses by applying KVL recursively, i.e.:

$$\begin{aligned} & \dot{v}_{i-1} = \dot{v}_i - (r_i + jx_i) \frac{P_{dn,i} - jQ_{dn,i}}{\dot{v}_i^{\,*}} + jb_i (r_i + jx_i) \dot{v}_i \quad i = 2...N \\ & \delta_N = 0 \end{aligned} \tag{8}$$

Note that, in principle, (8) allows to determine the phasor of voltage $\dot{v}_{i-1}$ whose amplitude is already known thanks to (7). As the approximation on the losses is the basis of the calculation of the powers entering both (7) and (8), the amplitude obtained with (8) is by definition the same as the one calculated with (7).

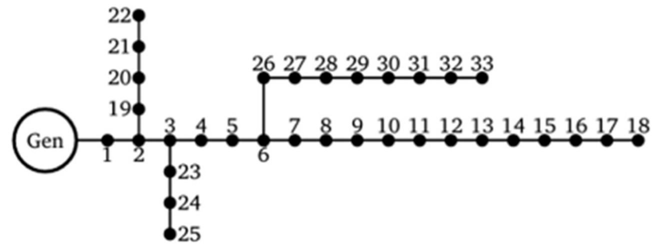

Fig. 3. Multi-feeder radial network.

2.3. Generalization to a multi-feeder radial network

The typical structure of a multi-feeder radial network consists of a main feeder with laterals departing from specified nodes of the main feeder itself. The extension of the developed procedure for this kind of networks basically requires an adjustment in the definition of $P_{\mathrm{dn}\ ,\mathrm{i}}$ and $Q_{\mathrm{dn}\ ,\mathrm{i}}$, which can be generally done as follows:

1. compute powers $P_{\mathrm{dn}\ ,\mathrm{i}}$ and $Q_{\mathrm{dn}\ ,\mathrm{i}}$ with (6) for all the buses in the laterals;
2. compute the "dn" powers in the junction buses as the sum of the contributions of the different laterals;
3. estimate voltages amplitude starting from the slack till the first junction bus with (7). Then, compute the voltages in the different feeders in parallel, as the substitution theorem allows to state that they are independent one another once the voltage at the junction point is known.

As a general comment, one can point out the following aspects:

- the ALF formulation only approximation is related to the estimation of losses in (5);
- the method is not iterative in the sense that many iterations are needed to get an accurate value for the bus voltages; the recurrence law (6) allows us to calculate the power injected by the downstream

Table 1
Line shunt susceptance values.

Branch	b [μS]	Branch	b [μS]
1–2	0.083	17–18	1.001
2–3	0.437	2–19	0.273
3–4	0.324	19–20	2.363
4–5	0.338	20–21	0.834
5–6	1.232	21–22	1.634
6–7	1.078	3–23	0.537
7–8	2.153	23–24	1.236
8–9	1.290	24–25	1.222
9–10	1.290	6–26	0.180
10–11	0.113	26–27	0.252
11–12	0.216	27–28	1.628
12–13	2.013	28–29	1.221
13–14	1.243	29–30	0.451
14–15	0.917	30–31	1.679
15–16	0.950	31–32	0.631
16–17	3.000	32–33	0.924

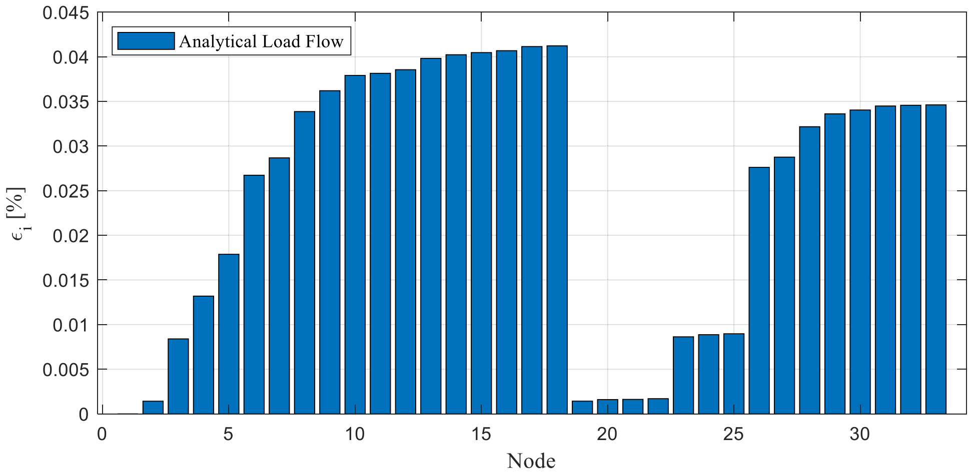

Fig. 4. Voltage error of the ALF for the 100 % load scenario.

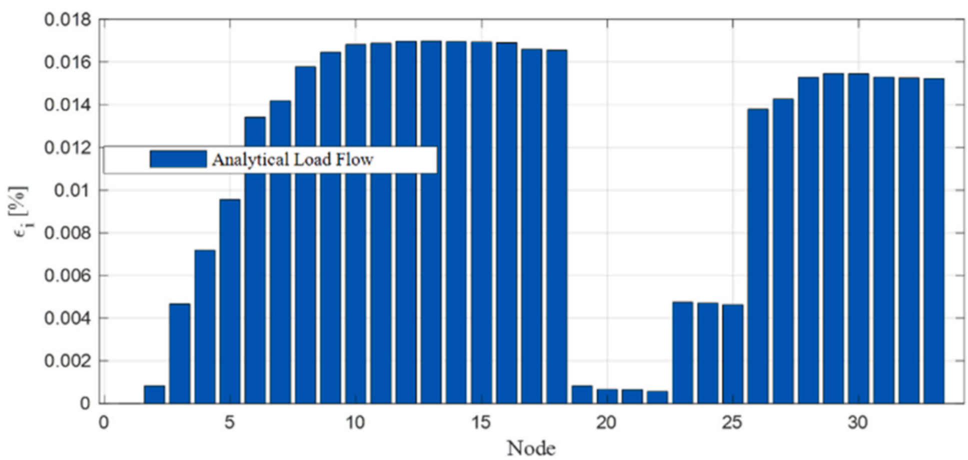

Fig. 5. Voltage error of the ALF for the 100 % generation scenario.

network of one bus starting from that of the subsequent one, while recurrence law (7) computes the voltage of a bus starting from that of the previous one;

- the method, being fully analytical, needs neither a (commercial or self-made) dedicated LF solver nor a general purpose mathematical software (such as Matlab), contrarily to what happens with approach presented in [2] in which the numerical inversion of a matrix is needed.

3. Test Results

This section presents the results of the proposed approach compared against exact LF Newton-Raphson numerical solution, chosen as benchmark. Moreover, a comparative analysis against the industrial voltage drop formula is proposed in a realistic scenario.

The comparison is performed by observing the voltage magnitudes deviation from the ones obtained with the Newton-Raphson numerical method. More in details, for the generic *i-th* node, the voltage error is defined as:

$$\varepsilon_i = \frac{|v_{LF,i} - v_{ALF,i}|}{v_{LF,i}} \cdot 100 \quad (9)$$

where $v_{LF,i}$ is the voltage at node i, calculated using the Newton-Raphson numerical method, while$v_{ALF,i}$ is the same voltage obtained with the proposed methodology. The same error is calculated with the voltages amplitude obtained by the industrial voltage drop formula when the comparison is provided.

Furthermore, to evaluate the performance of the ALF method compared to the numerical LF, phase, total active power, and reactive power errors are defined. Regarding the phases, the error $\varepsilon_{\delta i} = |\delta_{LF,i} - \delta_{ALF,i}|$is determined for each node, where $\delta_{LF,i}$ is the phase of the voltage at node i, calculated using the Newton-Raphson numerical method, while $\delta_{ALF,i}$is obtained with the ALF strategy. The error in total active losses $\varepsilon_{Ploss} = |P_{loss,LF} - P_{loss,ALF}|$is defined as the absolute difference between the total active losses calculated with the numerical LF $P_{loss,LF}$and with the ALF $P_{loss,ALF}$. Similarly, the error in total reactive power losses ε_{Qloss} is determined, being $Q_{loss,LF}$the LF reactive losses and $Q_{loss,ALF}$the ALF reactive losses.

The test is performed on a multi-feeder radial 33-node distribution network, whose topology is reported in Fig. 3 while all details about are reported respectively, R and X lines parameters and LF assignments can be found in [1]. Branch susceptances, not provided in [1], are reported in Table 1. The voltage base is 12.66 kV, and the power base is

Table 2
Results with different load values.

Load [%]	ε_{max} [%]	v_{max} [p.u]	$\varepsilon_{\delta\ max}$ [deg]
50 %	0.0048	0.9540	0.3196
75 %	0.0158	0.9296	0.4993
100 %	0.0412	0.9041	0.6954

Table 3
Active and reactive power loss with different load values.

Load [%]	$P_{loss,ALF}$ [MW]	$P_{loss,LF}$ [MW]	$Q_{loss,ALF}$ [MVar]	$Q_{loss,LF}$ [MVar]
50 %	0.0461	0.0480	0.0266	0.0276
75 %	0.1053	0.1122	0.0709	0.0714
100 %	0.1901	0.2078	0.1278	0.1364

Table 4
Results with different generation values.

Generation [%]	ε_{max} [%]	v_{max} [p.u]	$\varepsilon_{\delta\ max}$ [deg]
50 %	0.0022	1.0429	0.2773
75 %	0.0076	1.0631	0.4033
100 %	0.0169	1.0827	0.5221

Table 5
Active and reactive power loss with different generation values.

Generation [%]	$P_{loss,ALF}$ [MW]	$P_{loss,LF}$ [MW]	$Q_{loss,ALF}$ [MVar]	$Q_{loss,LF}$ [MVar]
50 %	0.0435	0.0420	0.0295	0.0229
75 %	0.0966	0.0920	0.0656	0.0565
100 %	0.1693	0.1591	0.1152	0.1016

Table 6
Active and Reactive power loss with different load values.

Node	Type	Size [MVA]	Node	Type	Size [MVA]
2	-	-	18	Load	0.50
3	PV	3.75	19	Load	0.25
4	Load	0.50	20	Load	0.50
5	WT	2.50	21	MG	4.00
6	Load	0.25	22	WT	2.50
7	Load	0.40	23	PV	3.25
8	PV	1.50	24	Load	0.40
9	WT	1.50	25	WT	2.50
10	Load	0.50	26	Load	0.50
11	PV	1.50	27	WT	3.00
12	Load	1.00	28	PV	1.50
13	WT	1.00	29	Load	0.25
14	Load	0.50	30	Load	0.50
15	MG	2.00	31	MG	2.00
16	PV	1.75	32	PV	1.75
17	Load	0.50	33	Load	0.25

1000 kVA. Specifically, it is noted that node 1 has been chosen as the reference node.

The general formulation proposed for multi-feeder network in Section 2.C is thus adapted for the proposed IEEE 33-node network. The powers $P_{dn,i}$ and $Q_{dn,i}$ are calculated with (6) for all the buses in the laterals, specifically from 18 to 7, from 33 to 26, from 25 to 23 and from 22 to 19. In the junction buses 2, 3, and 6, the "dn" powers are computed as the sum of the contributions from the different laterals. Therefore, $P_{dn,6}$ and $Q_{dn,6}$ is the sum of the contribution of feeders 6–33 and 6–18 calculated applying (6) to such feeders and the same logics is used for buses 3 and 2. Next, the voltage amplitude is estimated starting from the slack bus up to the first junction bus (bus 2) using formula (7). Afterward, the voltages in the parallel feeders are calculated independently.

3.1. Total passive/active 33-node network with π-model lines

In order to validate the effectiveness of the proposed ALF, extreme grid conditions are considered, corresponding to scenarios with total Generation/Load in the grid with progressively increasing power generation/demand. This choice is made to obtain very large voltage deviations, representing the most critical condition in the resolution of the grid.

With this aim in mind, active and reactive power of grid scenario reported in [1] are used considering three conditions, respectively 50 %, 75 % and 100 % of active and reactive power defined in [1]. Using an active sign convention, power are assumed all positive in the generation scenario and negative in the load one. As a result, six configurations have been analysed.

The percentage voltage errors at all busses of the 100 % load and 100 % generation scenarios are depicted respectively in Fig. 4 and Fig. 5. In general, one can notice that the error for proposed ALF increases with the distance from the slack node in both conditions.

The maximum magnitude and phase error and the minimum voltage value in all the load considered scenarios are reported in Table 2 while Table 3 reports the total active and reactive power losses obtained by the ALF formulation and compared to those obtained by numerical LF. Similarly, results for all the considered generation scenarios can be found in Table 4 and Table 5.

In conclusion, the voltage and phase errors calculated for the test network under various load and generation conditions highlight that the ALF yields highly accurate results compared to the numerical solution, despite the approximation introduced in the calculation of active and reactive power losses. For a more detailed and thorough analysis on the impact of losses approximation, one can refer to brief explanation included in Appendix A.

3.2. 33-node network with π-model lines in a realistic scenario

In this subsection, the ALF formulation is applied to determine the node voltages in the 33-node distribution network in a realistic scenario. Specifically, each node in the network represents one of the following technologies: a photovoltaic system (PV), a wind system (WT), a microgrid (MG), or a load. The connected technologies and their respective nominal power ratings are listed in Table 6. To validate the methodology, hourly sampled profiles for PV, WT, and loads were retrieved from a dataset referring to a realistic scenario in Northern Italy while for MGs real measurement of the University of Genova, Savona Campus MG Have been used after being properly scaled. Specifically, sets of four weeks representative of each season of the year 2018 were implemented. The detailed power profiles, for each node, technology and time instant are made available as supplementary material for this article. For sake of readability, the aggregated, total photovoltaic and wind production are shown in Fig. 6, while the aggregated total load consumption is represented in Fig. 7.

In this scenario, comparison with the industrial voltage drop formula is proposed. The distribution of the maximum voltage error ε_{max} for the industrial voltage drop and for the ALF is reported in Fig. 8. The average error is 0.051 % with a standard deviation of 0.0750 % for the proposed ALF method, whereas for the industrial voltage drop method, the average error is 0.8125 % with 0.7821 % standard deviation. As one can observe, the ALF exhibits significantly lower average error compared to the method based on industrial voltage drop and, in general, very accurate and reliable results with a majority of samples include in the first class of the distribution, that is the one with errors below 0.1 %. It is also worth stating that the proposed approach implies no computational effort at all, being fully analytical; so it can be seen as an improved version of the industrial voltage drop formula, because it presents higher accuracy with the same degree of computational complexity.

Regarding the maximum phase error and the power loss, the industrial voltage drop method totally neglects these quantities, since, unlike ALF, it does not allow for the calculation of voltage phases or system losses. The distribution of the maximum phase error $\varepsilon_{\delta\ max}$, along with the total active and reactive power losses (ε_{Ploss} and ε_{Qloss}), are presented respectively in Fig. 9, Fig. 10 and Fig. 11.

For sake of completeness, Table 7 summarizes the average values and

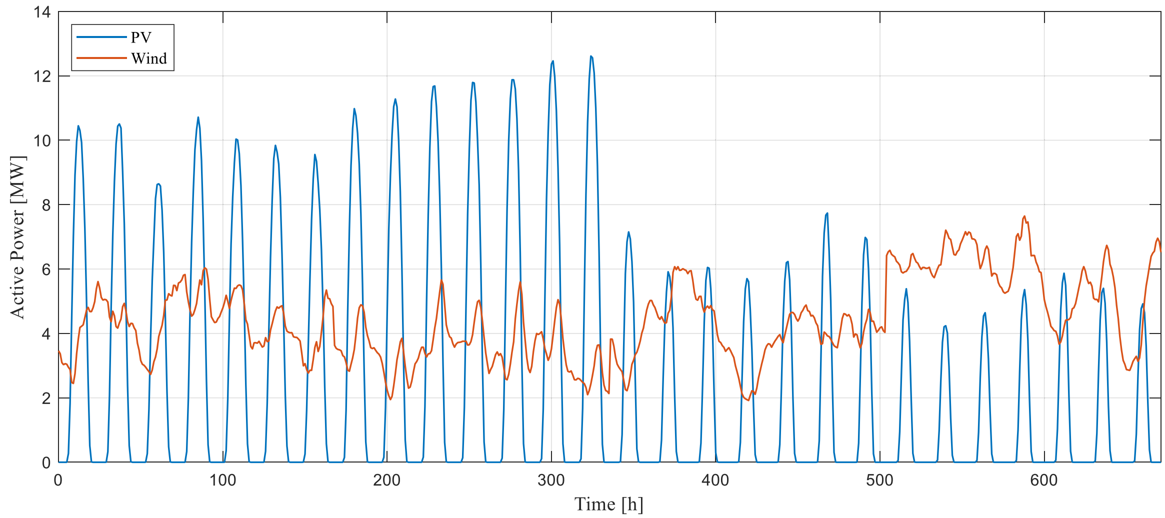

Fig. 6. Cumulative PV and Wind Production for the Spring, Summer, Autumn and Winter weeks.

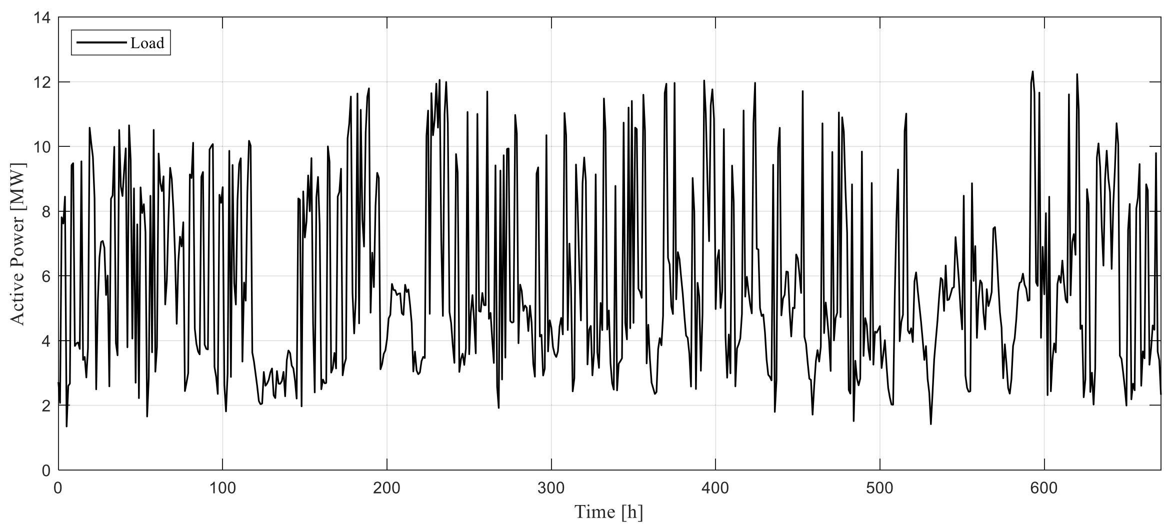

Fig. 7. Loads active power for the Spring, Summer, Autumn and Winter weeks.

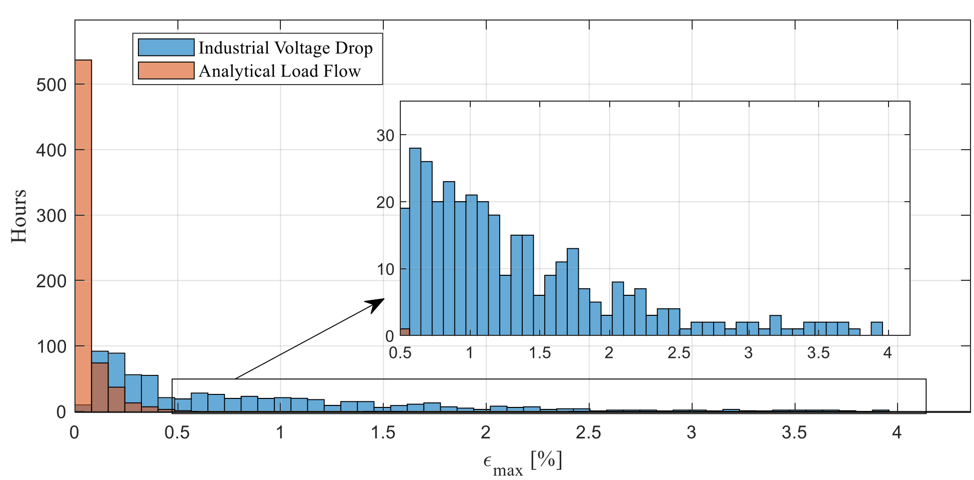

Fig. 8. Distribution of the maximum voltage magnitude error.

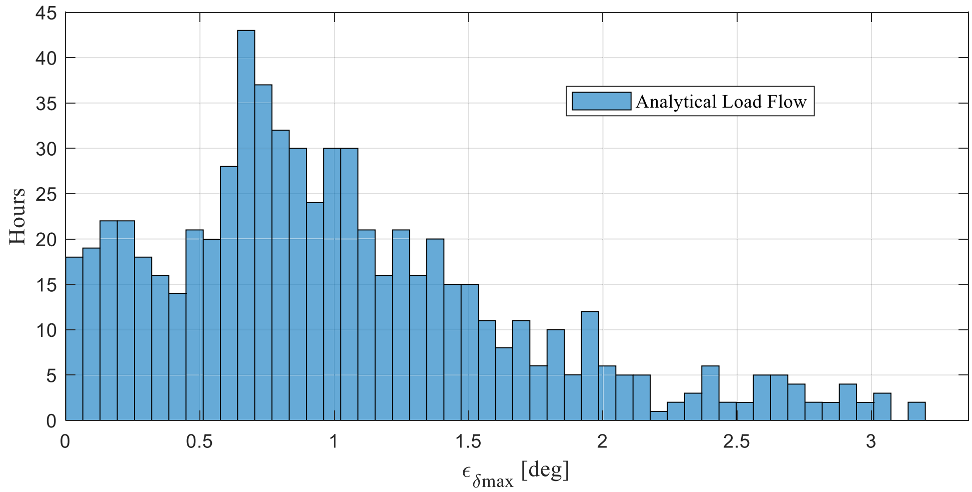

Fig. 9. Distribution of the maximum voltage phase error.

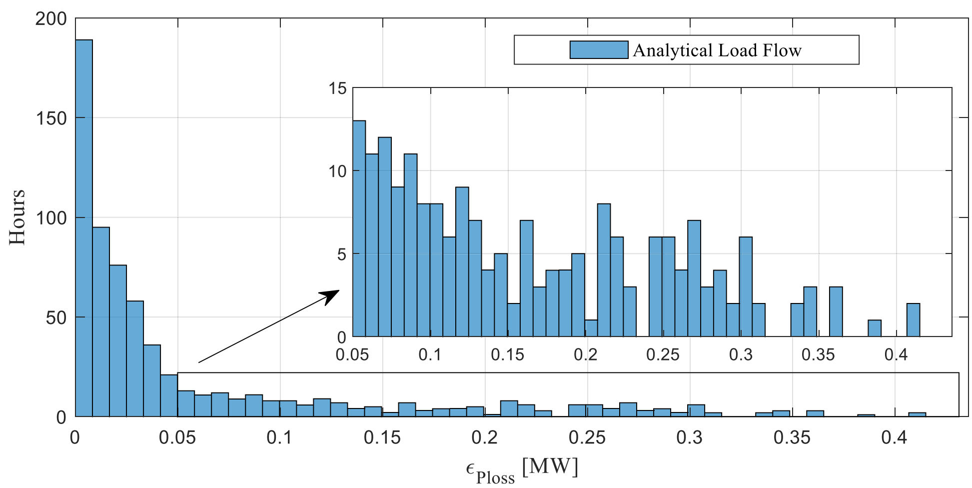

Fig. 10. Distribution of the active power error.

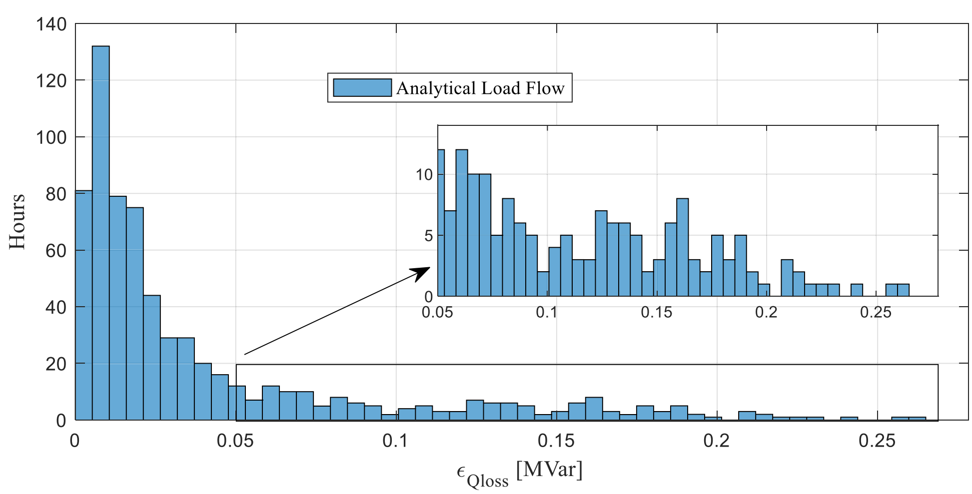

Fig. 11. Distribution of the reactive power error.

standard deviations of the errors of the ALF method.

In conclusion, the voltage errors calculated for the test network in a realistic scenario highlight that the ALF yields highly accurate results compared to the Industrial Voltage Drop relation. Furthermore, the distribution of the errors in the total active and reactive power losses highlights that the only approximation the ALF formulation relies on introduces acceptable errors in both the magnitudes and phases of the voltages at the network nodes.

Table 7
Average and Standard Deviation of the ALF errors.

	Average	Standard Deviation
$\varepsilon_{\max}$ [%]	0.0514	0.0754
$\varepsilon_{\delta\ \max}$ [deg]	1.0127	0.6701
ε_{Ploss} [MW]	0.1822	0.8496
ε_{Qloss} [MVar]	0.6131	0.7890

4. Conclusion

In this paper, an analytical solution is proposed for the LF problem in balanced multi-feeder radial three-phase power distribution networks. The only approximation on which the method is based consists in estimating line losses at nominal voltage. This allows to compute the voltage phasors at all the buses of a multi-feeder radial network in a fully analytical way.

Tests conducted on a benchmark network under various load and generation conditions demonstrated that the proposed formula provides very accurate results when compared with the numerical solution. Moreover, the performances of the proposed method are significantly better than the classical industrial voltage drop formula for what concerns the voltage amplitudes (maximum voltage error is approximately one order of magnitude lower).

The proposed ALF method is applicable only to radial networks and, however, with appropriate adjustments and modifications, could be extended to meshed networks. Future developments will focus on extending the ALF method to weakly meshed distribution networks to make the proposed approach useful also in future distribution power systems. Moreover, further investigation will delve into the ALF application for optimal power flow problems.

CRediT authorship contribution statement

Manuela Minetti: Writing – original draft, Software, Methodology, Data curation, Conceptualization. **Andrea Bonfiglio:** Writing – review & editing, Supervision. **Renato Procopio:** Writing – review & editing, Writing – original draft, Supervision, Methodology, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

This appendix aims to provide an analysis of the impact of the approximation on power losses in the ALF calculation of nodal voltage magnitudes in a radial network.

Specifically, a three-node, two-branch network is considered, with parameters corresponding to the first two branches of the 33-node network [1]. The cases of 100 % Load and 100 % Generation, presented in Section 3.1, are analyzed. For both cases, the active (reactive) power injected/absorbed at node 2 corresponds to the sum of the powers injected/absorbed at the nodes belonging to the lateral 2–22, while the active (reactive) power injected/absorbed at node 3 accounts for the sum of the injections/absorptions downstream, including that of node 3 itself. In this specific section, the power base is considered to be 4.4 MVA.

Assuming the minimum and maximum voltage values v_{max} and v_{min} equal to 0.9 p.u. and 1.10 p.u., it possible to determine the range of $P_{loss,2}$ and $Q_{loss,2}$, as follows:

$$
\begin{aligned}
P_{loss,2} &\in \left[r_2 \frac{P_{dn,3}^2 + Q_{dn,3}^2}{v_{\min}^2}; r_2 \frac{P_{dn,3}^2 + Q_{dn,3}^2}{v_{\max}^2} \right] \\
Q_{loss,2} &\in \left[x_2 \frac{P_{dn,3}^2 + Q_{dn,3}^2}{v_{\min}^2}; x_2 \frac{P_{dn,3}^2 + Q_{dn,3}^2}{v_{\max}^2} \right]
\end{aligned}
\tag{10}
$$

Consequently, by applying Eq. (6), the upper and lower limit for $P_{\mathrm{dn},2}$ and $Q_{\mathrm{dn},2}$ are obtained. For sake of completeness, these values are summarized in Table 8.

The dependence of voltage v_2 on $P_{\mathrm{dn},2}$ and $Q_{\mathrm{dn},2}$ is investigated using the relation (7). Fig. 12 and Fig. 13 report the results of the analysis for the cases of total active and passive network. As observed, the impact of the losses approximation can be considered acceptable, as it affects only the fifth significant digit of the voltage magnitude.

After analysing the voltage magnitude as the active and reactive power losses vary, its behaviour around the operating point is studied using (11). The operating point is defined by the voltage v_1= 1 p.u. and the values $P_{\mathrm{dn},2}$ and $Q_{\mathrm{dn},2}$ calculated using the approximated relation (6) and obtaining the results reported in Table 9. Fig. 14 and Fig. 15 show the results of the analysis around the defined operating point for the 100 % load and generation scenario and further support the previous considerations related to the losses approximation.

$$
v_2 = 1 + \frac{\partial v}{\partial p}(p_2 - P_{nd2}) + \frac{\partial v}{\partial q}(q_2 - Q_{nd2}) \tag{11}
$$

Table 8
Lower and Upper Limits of power range.

	100 % Load		100 % Generation	
	Lower Limit	Upper Limit	Lower Limit	Upper Limit
$P_{\mathrm{dn},2}$ [p.u.]	−0.8632	−0.8589	0.8373	0.8415
$Q_{\mathrm{dn},2}$ [p.u.]	−0.5330	−0.5308	0.5198	0.5220

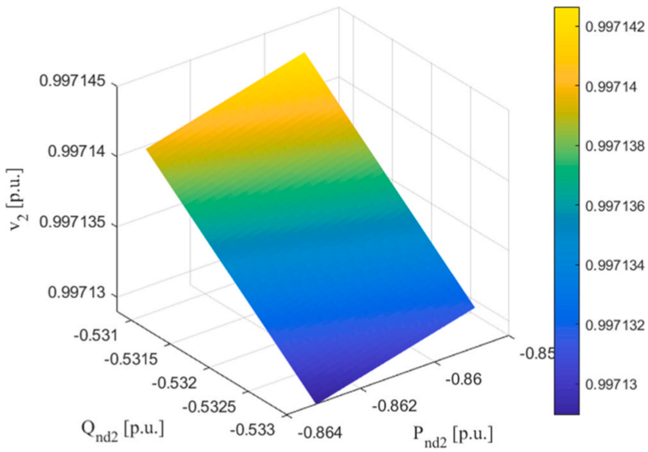

Fig. 12. Voltage dependence on the active and reactive power injected by the downstream network of bus 2, 100 % Load.

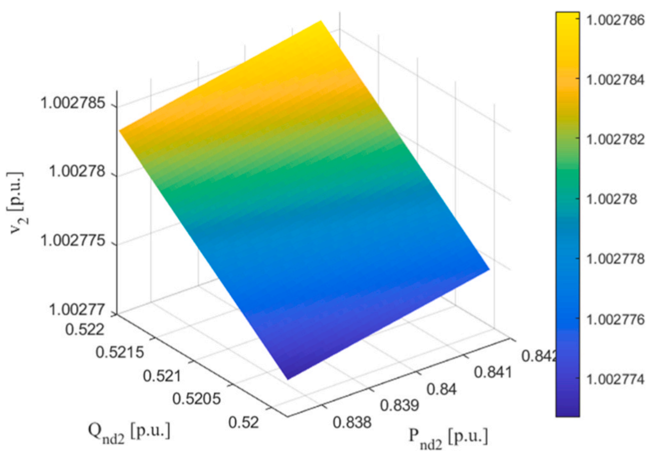

Fig. 13. Voltage dependence on the active and reactive power injected by the downstream network of bus 2, 100 % Generation.

Table 9
Operating point values.

	100 % Load	100 % Generation
$P_{dn,2}$ [p.u.]	−0.8607	0.8397
$Q_{dn,2}$ [p.u.]	−0.5317	0.5210

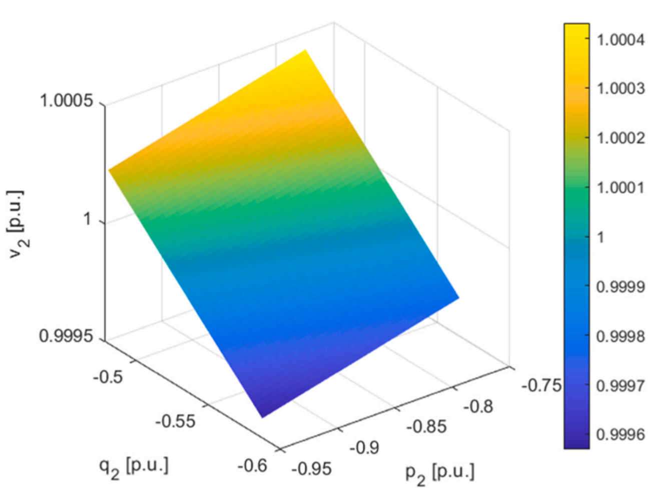

Fig. 14. Voltage local analysis around the defined operating point, 100 % Load.

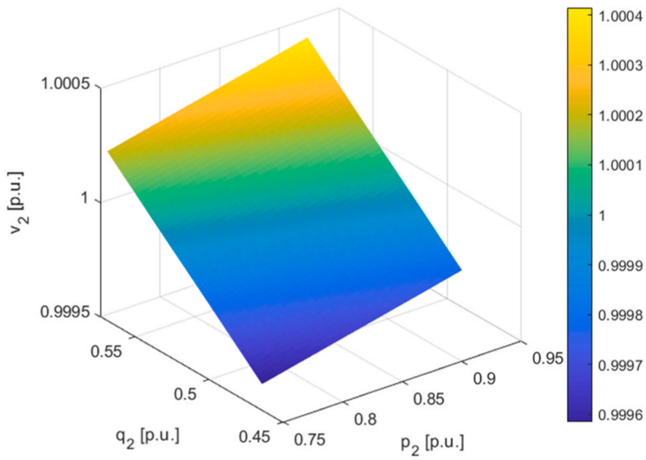

Fig. 15. Voltage local analysis around the defined operating point, 100 % Generation.

Appendix B. Supporting information

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.segan.2025.101871.

Data availability

Data will be made available on request.

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