"My mind is getting used to always find a better solution process": Formative assessment and self-regulation in secondary school algebra

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We present and discuss the design of a sequence of activities, where formative assessment strategies are carried out, also with the support of technology. The analysis of data coming from a teaching experiment in grade 10, in the context of algebra teaching, shows the design is promising and suggests that the proposed activities were efficient in promoting students'self-regulation.

Keywords: Formative assessment, secondary school mathematics, self-regulation.

Introduction and background

We present and discuss the first steps of a long-term study concerning formative assessment classroom practices. We aim at contributing to the TWG21 discussion on the following theme, as evidenced in the call for papers: *how can student-centred assessment be designed and implemented*?

Black and Wiliam (2009) characterize formative assessment (FA) as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited". (p. 7). In order to implement formative assessment in classroom, five key strategies may be carried out (Wiliam & Thompson, 2007): FA1) clarifying and sharing learning intentions and criteria for success; FA2) engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding; FA3) providing feedback that moves learners forward; FA4) activating students as instructional resources for one another; FA5) activating students as the owners of their own learning. Within the FaSMEd project (Aldon et al., 2017; Cusi et al., 2017) the model of Wiliam and Thompson was extended so as to recognize the role of technology in promoting formative assessment strategy. The authors identified three functionalities through which technology can support formative assessment: T1) *sending and displaying* (for instance, the teacher may send a quiz or a poll and collect students' answers; a teacher can select, show to the whole class and discuss the written answer of a student); T2) *processing and analyzing* (for instance, the teacher may use technology to analyze data from a quiz or poll); T3) *providing an interactive environment* (for instance, the teacher may share with the students an interactive board). A key point is that formative assessment strategies, as described by Wiliam and Thompson (2007), may be activated by the teacher, but also by the peers and by the student himself. In this contribution we focus on formative assessment activities where students have a major role, becoming responsible for their own learning. This may also be linked to self-regulation, as already noted by Semana and Santos (2018), and in a broader sense to metacognition (Shilo & Kramarski, 2019). Self-regulation occurs in three phases (Zimmermann, 2000): SR1) *preparation*, when learners plan their behavior on the basis of the given task; SR2) *execution*, when learners monitor and control their performance, SR3) *self-reflection*, when learners reflect on the methods they used, the knowledge they gained, the usefulness of the solving strategy they employed. Schloemer and Brenan (2006) identify three components of self-regulation in learning: *goal setting*, *monitoring* of the learning process and *modification* of the learning strategies.

As evidenced by Wiliam and Thompson (2007)'s formative assessment strategy FA1 (*clarifying and sharing learning intentions and criteria for success*), as well by Schloemer and Brenan (2006)'s first component of self-regulation (*goal setting*), it is important to clarify what are the *learning intentions* of the activity at issue. The curricular topic for our study is algebraic thinking. We refer to Arcavi (1994)'s characterization of symbol sense as a major learning goal: students should develop the understanding of how and when using symbols to represent relationships, generalizations and proofs, the ability to manipulate and interpret symbolic expressions, the ability to select one possible symbolic representation for a problem, the awareness of the roles symbols can play in different contexts. Moreover, referring to Skemp (1976)'s distinction between relational and instrumental understanding, we assume relational understanding as a key learning goal: students should learn to carry out procedures, while at the same time understanding how and why the rules and procedures work; students should be led to establish connections and apply known concepts to newer problems.

In this contribution, we present the *task design* of the formative assessment activities and we analyse data coming from the first *implementation*. Such analysis will help us to identify key elements in our design choices, and possibly get insight for the redesign and improvement of the sequence.

The formative assessment activity

Drawing from the aforementioned theoretical tools, we identified two design principles for Formative Assessment Activities (FAA): *1. FAA should be coherent with the learning intentions, that in turn should be informed by topic-specific mathematics education theoretical tools (in the present study, relational understanding and symbol sense); 2. FAA should exploit the opportunities provided by ICT*. Accordingly, we set up a teaching sequence on the topic "Quadratic functions and second-degree equations", that may be synthetised as it follows:

Step1: students filled online Questionnaire 1, containing two introductory self-assessment questions (1. *Do you think you are prepared on these topics? Multiple choice: Yes on both topics; Yes on second degree functions; Yes on second degree equations; Not at all; I don't know*. 2. *Explain accurately your previous answer*), 8 mathematical items (5 in form of multiple choice quizzes, 3 in form of open-ended questions), two "looking back" final questions (11. *Are you sure of the answers you just gave? If not, on which aspects are you uncertain?* 12. *Write here all your doubts, uncertainties, questions that this questionnaire aroused in you*).

Step 2: class discussion on the mathematical items. For each item, multiple solving strategies (in the algebraic and/or graphical register) were discussed. Also the "computational cost" of each strategy is discussed. The discussion was orchestrated by the teacher and all the students contributed with their ideas and comments.

Step 3: students were asked to write down a retrospective report on their solving strategies ("*In the light of the work done in class, analyze the aspects that worked or didn't work in your solution processes"*).

Step 4: students filled online Questionnaire 2, containing two introductory self-assessment questions (in bold the changes in comparison with step 1) (1. *Do you think you are prepared on these topics? Multiple choice: Yes on both topics,* ***after the work we did in class****; Yes on second degree functions,* ***after the work we did in class****; Yes on second degree equations,* ***after the work we did in class****; Not at all,* ***the work we did in class did not help me****; I don't know*. 2. *Explain accurately your previous answer*), 9 mathematical items (6 in form of multiple choice quizzes, 3 in form of open-ended questions), two "looking back" final questions (12. *Are you sure of the answers you just gave? If not, on which aspects are you uncertain?* 13. *Write here all your doubts, uncertainties, questions that this questionnaire aroused in you*).

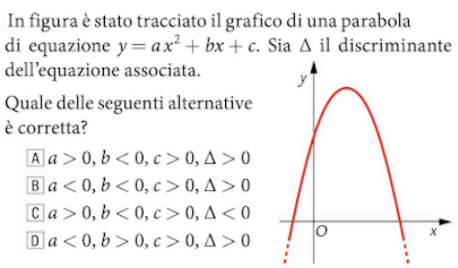
Step 5: identical to **Step 2**.

Step 6: students filled online Questionnaire 3, which had the same structure of Questionnaire 2 (introductory self-assessment questions, 11 mathematical items, two "looking back" questions).

Step 7: correction of the mathematical items of Questionnaire 3. For each item, the students were also asked to write down a reflection on their own solving process in comparison with the solving processes as presented by the teacher. Finally, the students were asked to write down an answer to the following question: *In the light of the work done in class, what changes did you notice in your solution processes? Explain them in detail.*

For space constraints, we report only an example of multiple-choice mathematical question (from step 1).

Table 1 – Item from Questionnaire 1

 In figura è stato tracciato il grafico di una parabola di equazione $y = ax^2 + bx + c$. Sia Δ il discriminante dell'equazione associata. Quale delle seguenti alternative è corretta? A $a > 0, b < 0, c > 0, \Delta > 0$ B $a < 0, b < 0, c > 0, \Delta > 0$ C $a > 0, b < 0, c > 0, \Delta < 0$ D $a < 0, b > 0, c > 0, \Delta > 0$	In the figure the graph of a parable of equation $y=ax^2+bx+c$ is plotted. Δ is the discriminant of the associated equation. Which is the correct option?

In terms of formative assessment strategies (Wiliam & Thompson, 2007), we may note that steps 1, 4 and 6 (online questionnaires) combine mathematical questions on the topics and self-assessment, thus realizing FA 5 (making students responsible for their own learning). Step 2 is the implementation of FA2 (Engineering effective classroom discussions), it allows the teacher to give feedback to students (FA3) and focus on the learning intentions and criteria for success. We recall that learning intentions refer to symbol sense and relational understanding. Steps 3 and 7 are a further implementation of FA5. Concerning self-regulated learning (Zimmermann, 2000), introductory questions in steps 1, 4 and 6 may also be linked to the preparation phase of self-regulation (SR1), while final questions in steps 1, 4 and 6 and the written reports in steps 3 and 7 may be related to the self-reflection phase (SR3). In terms of functionalities of technology (Aldon et al., 2017; Cusi et al., 2017) we may note that steps 1, 4 and 6 exploit the "sending and displaying" functionality of technology (T1), while the functionality "processing and analyzing" (T2) may be employed by the teacher in the preparation of the class discussions (steps 2 and 5).

The teaching sequence was carried out in two grade 10 classes (class A, 22 students; class C, 21 students) of a secondary school with scientific orientation, in the North of Italy. One of the authors (SQ) is the mathematics teacher for the two classes, as well as a member of the research team. The teaching sequence was implemented in January 2023 and it took 8 hours.

Method

Our study was conceived within the design-based approach (DBCR, 2003). Hence, we see the design of the activity as an important outcome of the research in itself. We are currently in the first cycle of design, enactment, analysis, and the results of the analysis will provide information for the redesign. Moreover, coherently with the DBCR approach, our research embodies "*specific theoretical claims about teaching and learning*" (DBCR, 2003, p.6), as evidenced by our assumptions concerning symbol sense and relational understanding. Finally, the research is characterized by close collaboration between researchers and teachers, as evidenced by the composition of the research team (see also the Italian paradigm of Research for Innovation, where teachers and researchers collaborate in all the phases of the research (planning, implementation and analysis) (Arzarello & Bartolini Bussi, 1998). All the students' written answers were collected and two levels of analysis were performed. The first level of analysis concerned the whole class: we studied data from questionnaires in steps 1, 4 and 6, treated as whole for each class. Such analysis is the kind of analysis the teacher may perform to adjust the teaching process and organize the subsequent class discussions. The second level of analysis concerned the evolution of each student. For each student we set up a file containing the sequence of all his/her written answers, so as to realize a collection of individual "stories". Such stories were analysed by means of the theoretical tools that we presented in the previous section, combining the mathematical dimension with the self-regulation dimension. From the mathematical standpoint, we asked ourselves whether the solving process of the mathematical tasks improved or the students were stuck in the same conceptual mistakes. Moreover, we searched for instances of symbol sense (Arcavi, 1994) and relational understanding (Skemp, 1976). For example, we ascribed to symbol sense the fact that a student was able to reflect on the meaning of symbols and get information from the interpretation of symbolic expressions; we ascribed to relational understanding the fact that a student was able to reflect on and explain the applied procedures, and not just carry out them. From the self-regulation standpoint, we scrutinized their written answers searching for evidences of self-regulation (Zimmermann, 2000), with a special focus on the components of *goal setting*, *monitoring of the learning process* and *modification of learning strategies*, according to Schloemer and Brenan (2006)'s construct. The analysis was performed by the first author (FM) and checked by the second author (SQ). There was not a coding guide, rather we interpreted data a posteriori according to the aforementioned theoretical tools.

Analysis

Concerning the first level of analysis, we may say that in both classes there was an improvement in terms of correctness of the answers to the mathematical items. As regards class C, in questionnaire 1 the average score was 59,30%, while in questionnaire 3 the average score was 71,73%. As regards class A, in questionnaire 1 the average score was 60,15%, while in questionnaire 3 the average score was 70,11%. Hereunder we focus on the first results from the second level of analysis. We present here the "story" of the student Gio, belonging to class A. We chose to focus on Gio because he had an improvement in terms of correctness of the answers to the mathematical items (from 46,15% score to 94,11% score), he showed a perception of such an improvement and was extremely accurate in reporting his reflections, as we will evidence in the subsequent analysis.

In the introductory question in **step 1**, Gio declares to feel comfortable with functions: "*concerning functions, I feel more secure, I think this security is also due to the fact that thanks to geogebra or other tools I was able to better understand their meaning and get their aspects*". In the 8 mathematical items of **Step 1**, Gio chooses the correct options, without providing any motivation.

In **step 3**, when reflecting after the class discussion, Gio writes down:

> After the correction I realized that my main lack is a reasoning one, indeed I think, and I am really sure about this, that I could have solved all the items if only I had been more careful. Unfortunately, I keep on doing this mistake and I don't know how to get rid of it. […] My habit of considering *b* negative if the translation is towards right and vice versa led me to the mistake. While I am happy that I found a technique for excluding more than one parable at once.

Gio efficiently looks back at Step 1 and recognizes that, although he provided correct answers, his solving process was not correct (indeed, choosing the sign of parameter *b* just on the basis of the position of the vertex is not a generalizable strategy, since the position of the vertex depends on both a and b). Gio is able to go beyond the correctness of results and critically examine the solving strategy. This instance of *self-reflection* (SR3) (Zimmermann, 2000) can be seen in terms of *monitoring of the learning process* (Schloemer & Brenan, 2006).

In the introductory question in **step 4**, Gio declares that the previous class work (step 3) was helpful:

> I think that the work done in class during the correction helped me a lot, especially because I was able to compare myself with the ideas of my classmates and I found resolutions I hadn't thought of. I also managed to optimize my reasoning, understanding the source of my mistakes. A mistake that I must not make again is to pay little attention while reading the problem, but I'm sure I can stay calmer while carrying out the task and pay more attention in solving it. I also think that using geogebra helped me a lot in understanding graphs and their various aspects.

In this piece of *self-reflection* (SR3), which is also *preparation* (SR1) for the next activity, Gio shows awareness of some recurrent mistakes and unfruitful habits and proposes to pay more attention on the problem. This may be seen as a *goal-setting* component of self-regulation.

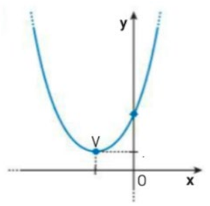
When facing a new mathematical item in Questionnaire 2 (the item is similar in content to the item in Questionnaire 1), Gio is able to select the correct option, but yet he does not motivate his choice. In the introductory question in **step 6**, Gio writes down:

> I'm sure I'm prepared on the covered topics, because thanks to the last two questionnaires I feel ready to be able to reflect on the appropriate and most effective methods to solve the given problems. I feel much more confident than in the previous two tests, perhaps also because my

mind is getting used to always find a better solution process. I don't think I have found, during these two weeks in which we carried out this work, that I have developed any particular skills, but simply that I have found security so that I can carry out each task calmly and consequently more effectively. Of course I am aware that I cannot solve all the problems in a short time but I am sure that I can overcome the difficulties without letting myself be influenced by other factors such as perhaps the pressure and the little time left.

In the *preparation* (SR1) for the mathematical activity, Gio values the fact that he worked on "*finding the better solution process*". Having at disposal "*appropriate and most effective methods*" makes Gio feel self-confident about the mathematical items he is expected to solve. As a matter of fact, in the mathematical items concerning functions, Gio provides correct answers accompanied by explanations:

Table 2 – Excerpts from Questionnaire 3

<table>
<tr>
<td>
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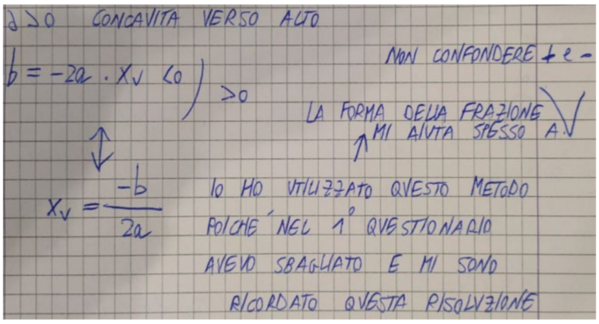
<td>
</td>
<td>a>0 concavity upward.
The shape of the fraction often helps me in not confusing + and -.
I used this method because in the first questionnaire I made a mistake and I remembered this solution.</td>
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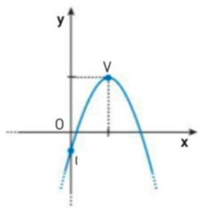
<td>
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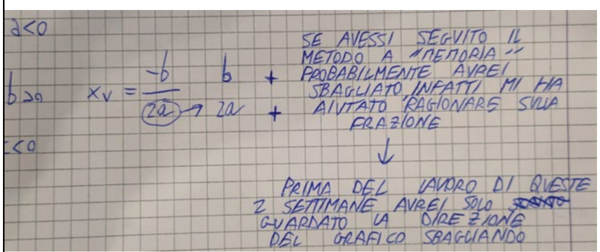
<td>
</td>
<td>If I had followed the method "by heart", I would have made a mistake. Indeed, reasoning on the fraction helped me. Before the work of these two weeks, I would have just looked at the direction of the graph, making a mistake.</td>
</tr>
</table>

In explaining his solving process, Gio is efficient in describing his new strategy, based on the study of the formula of the vertex, taking into account the parameters in a connected way. Since Gio is aware of the change of strategy and is able to describe and justify such a change, we ascribe this to the *modification* component of self-regulation. The change in the strategy reveals that Gio moved from a procedural to a relational understanding (Skemp, 1976) of functions, since he did not limit himself to apply a method "by heart". Moreover, when dealing with parameters and seeing "through symbols”, he showed a growing symbol sense (Arcavi, 1994). As a final comment in **Step 7**, Gio writes down:

> In the first questionnaire I had difficulty in reading the graphs to find whether *b* was greater, less than or equal to 0. After the correction, in a short time I can read the graph in such a way as to have all the information to find *b*. I took the three questionnaires as a personal test, and I am very satisfied with the progress I made. I never had mathematical growth, but above all mental growth in such a short time and I'm glad I dedicated my efforts to it.

In this final comment, Gio explicates the *modification* that occurred in his solving strategy and expresses personal satisfaction for the activity. Interestingly, he recognizes to have improved at mathematical and “mental” level, that is to say not only in terms of knowledge, but in terms of reasoning.

Discussion and preliminary conclusions

Drawing from existing frameworks concerning the formative assessment key strategies and the functionalities of technology in promoting formative assessment, we set up a series of formative activities and implemented them in two classes of level 10. The activities specifically aimed at making students responsible for their learning process, in a perspective that may be related to self-regulated learning. For this reason, we adopted the framework of self-regulated learning (Zimmermann, 2000) and the components of self-regulation (Schloemer & Brenan, 2006) to analyse students’answers. For space constraint, we presented the analysis of the story of one student, Gio. Data analysis shows the presence of self-regulation (both in terms of preparation, SR1, and self-reflection, SR3) and an evolution in terms of components of self-regulated learning: Gio moves from *monitoring* the solving process to *goal setting* and, finally, *modification* of the solving strategy. The growth in self-regulation may also be found in the improvement of the argumentation process. While answers to questionnaires 1 and 2 are not motivated, in questionnaire 3 Gio is able not only to provide the correct answers, but to explain his solving strategy. Data analysis provides information on Gio’s perception of the sources of difficulty and mistake: looking for quick solutions rather than a general method. We may say that Gio moves from a procedural knowledge to a relational knowledge about functions and that he is developing symbol sense. Finally, data analysis suggests that the student recognizes the value of the whole designed sequence. For instance, he values the comparison with peers and class discussion as a source of reflection on the process and a way of finding out other solving procedures.

The analysis of a collection of individual stories is in progress. Such analysis is the final step of the first cycle of our research, according to the DBCR (2003) approach. Data analysis are helping us to identify key elements in our design choices (for instance, the importance of peer comparison and class discussion), and possibly get insight for the redesign and improvement of the sequence (for instance, we are thinking about inserting more direct questions related to the components of self-regulation in steps 2, 5, 7).

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