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Private and Social Welfare Gains in the Diamond-Dybvig Model: A Rationale for the Existence of Banks

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ABSTRACT

In this paper, I evaluate the private and the social welfare gains that in the Diamond-Dybvig model of bank runs characterise the switch from a decentralised to a centralised equilibrium that may hold even in an atomistic environment with banking intermediation. Relying on logarithmic preferences, I show that such a social welfare gain is an increasing function of the discount rate of more patient agents. Moreover, I demonstrate that for each level of the discount rate of agents who are willing to postpone consumption, there is an optimal value of the proportion of these agents in the economy that maximises the social welfare gain.

JEL Classification: D02, E02, E44, G21

1 | Introduction

In a renowned contribution, Diamond and Dybvig (1983) present a theoretical framework in which they show how demand deposit contracts offered by a bank can provide the public with superior allocations compared to those available in a decentralised environment where banking intermediation is absent. Within this setting, they also show that when atomistic depositors face a privately observed risk that leads to a sudden demand for liquidity, the bank can still attract deposits, but it is vulnerable to runs.[1]

After its publication, the seminal work by Diamond and Dybvig (1983) has been developed in many directions and it continues to be widely cited as providing a definitive theoretical setting for the design of financial-stability policies against runs. For instance, Gertler and Kiyotaki (2015) extend the Diamond and Dybvig (1983) model to a general-equilibrium setting with many banks by showing that anticipations of a run have harmful effects on the economy even if the run does not occur. Shell and Zhang (2018) introduce extrinsic (sunspot) uncertainty in the original setting by Diamond and Dybvig (1983) by distinguishing between pre-deposit and post-deposit incentives. More recently, Gu et al. (2023) embed the Diamond and Dybvig (1983) framework into an infinite-horizon environment to highlight the role of banks' reputation in preventing damaging runs.

In this paper, I dig into an aspect of the Diamond and Dybvig (1983) model of bank runs that—to the best of my knowledge—remained unexplored, that is, the strength of the incentives that may lead the society to the adoption of banking intermediation among individual agents instead of the «autarchic» management of private endowments. Specifically, relying on logarithmic preferences, I assess the private and the social welfare gains that characterise the switch from a decentralised to a centralised equilibrium that can be alternatively implemented by an omniscient social planner or—alternatively—by a bank on which depositors may decide to run.

From an analytical perspective, I show that the welfare gain experienced by the society as whole by switching to a centralised solution whose allocations can be also achieved through banking intermediation is an increasing function of the discount rate of more patient agents. Moreover, I demonstrate that for each level of the discount rate of agents who can afford to postpone consumption, there is an optimal value of the proportion of this kind of agents in the economy that maximises such a social welfare gain.

The paper is arranged as follows. Section 2 propaedeutically outlines the backbone of the Diamond and Dybvig (1983) model. Section 3 analyses the patterns of private and social welfare gains. Finally, Section 4 concludes.

2 | The Model

Diamond and Dybvig (1983) consider a three-period framework ($T = 0, 1, 2$) in which there is a continuum of economic agents whose mass is normalised to 1. In period 0, each agent does not consume but she/he is endowed with one unit of the unique consumption good that can be invested in a risk-free project. Specifically, whenever the unit of the consumption good is invested until period 2, the agent receives $R > 1$ units of the consumption good by earning a positive return. Otherwise, that is, whenever the investment is stopped in period 1, the investor receives back the same unit of the consumption good invested in period 0 by realising no return.

In period 0, all the agents feel to be alike because they cannot forecast their attitude towards consumption in the subsequent periods. However, in period 1—consistently with a Bernoulli distribution—a fraction $t \in (0,1)$ of agents realises that they are impatient and that they want to consume only in period 1 (type-1 agents), whereas the remaining $1 - t$ agents—despite some discounting—are more patient because they realise to be indifferent between consuming in period 1 or in period 2 (type-2 agents). Denoting by c_T^i the level of consumption in period T by type-i agent, the preferences over consumption of the two types of agents are assumed to be given by

$$\mathscr{U}^1 = u\left(c_1^1\right) \quad \text{and} \quad \mathscr{U}^2 = \rho u\left(c_1^2 + c_2^2\right) \tag{1}$$

where $\mathscr{U}^1$ ($\mathscr{U}^2$) is the utility function of type-1 (type-2) agents, $\rho \in (0,1)$, with $\rho R > 1$, is the discount factor of more patient agents, whereas $u(\bullet)$ is a continuous, concave twice-differentiable function with a relative-risk-aversion index higher or equal to 1.[2]

In a decentralised equilibrium in which each agent manages its period-0 endowment in isolation, type-1 agents liquidate their investment in period 1 and consume the proceeds, whereas type-2 agents wait until period 2 to consume. Consequently, in that situation, the equilibrium allocations of individual consumption are given by

$$\begin{gathered} c_1^1 = 1 \\ c_2^1 = c_1^2 = 0 \\ c_2^2 = R \end{gathered} \tag{2}$$

The levels of consumption in Equation (2) imply that in a decentralised equilibrium the social welfare amounts to the following expression:

$$tu(1) + (1 - t)\rho u(R) \tag{3}$$

A centralised equilibrium can be framed instead as the solution of a social planner problem in which the planner is able to observe the type of each agent, and it is also aware of the investment opportunities available in the economy. As in the decentralised equilibrium described in the array in Equation (2), the social planner will fix to zero the level of consumption of type-1 (type-2) agents in period 2 (1), so that $c_2^{1*} = c_1^{2*} = 0$. Thereafter, the social planner will choose c_1^1 and c_2^2 by maximising the social welfare of the economy subject to the resource constraint according to which the yield on the investment projects which are not liquidated in period 1 by type-1 agents that want to consume, that is, $(1 - tc_1^1)R$, has to be equally divided among the $1 - t$ type-2 agents in period 2. Consequently, the social planner problem can be written as

$$\begin{gathered} \max_{c_1^1, c_2^2} tu\left(c_1^1\right) + (1 - t)\rho u\left(c_2^2\right) \\ \text{s.to} \\ c_2^2 = \frac{\left(1 - tc_1^1\right)R}{1 - t} \end{gathered} \tag{4}$$

Whenever the relative-risk-aversion index of $u(\bullet)$ is constant and equal to γ, that is, whenever $u = (1/(1 - \gamma))(c_T^j)^{1-\gamma}$ with $T, j = 1,2$, the solution of Equation (4) is given by the following pair:

$$\begin{gathered} c_1^{1*} = \frac{1}{t + (1 - t)\rho^{\frac{1}{\gamma}} R^{\frac{1-\gamma}{\gamma}}} \\ c_2^{2*} = \frac{(\rho R)^{\frac{1}{\gamma}}}{t + (1 - t)\rho^{\frac{1}{\gamma}} R^{\frac{1-\gamma}{\gamma}}} \end{gathered} \tag{5}$$

Recalling that $\rho R > 1$, the expressions in Equations (2) and (5) imply that in a centralised equilibrium the individual level of consumption realised by type-1 (type-2) agents in period 1 (2) is higher (lower) than the one obtained by this kind of agents in a decentralised equilibrium. Consequently, the implementation of a centralised equilibrium that increases the welfare of the society as a whole with respect to the level conveyed by the expression in Equation (3) requires to transfer some resources from type-2 agents to type-1 agents. Diamond and Dybvig (1983) shows that in a decentralised setting such a transfer can be effectively implemented by a bank that—by exploiting the risk-free project available in the economy—offers a demand deposit contract that promises to pay c_1^{1*} to all the depositors that withdraw their funds in period 1 by leaving all the remaining available resources to those who wait to withdraw until period 2.

Unfortunately, in the decentralised environment in which the bank is called in to manage the endowments of the two types of agents by creating liquidity for type-1 agents, the welfare-enhancing equilibrium described by the consumption allocations in Equation (5) is only one of two distinct Nash equilibria. Specifically, whenever each type-2 agent irrationally anticipates that all of her/his pairs will behave as a type-1 agent, there is another Nash equilibrium in which all the depositors withdraw their funds in period 1 and the bank fails.

3 | Private and Social Welfare Gains

Suppose that $u(\bullet)$ is a logarithmic function, so that $\gamma = 1$ for all c_T^j, with $T, j = 1{,}2$. In this case, recalling the expressions in Equations (2), (3) and (5), switching from a decentralised to a centralised solution, each type-1 agent faces an increase in her/his consumption by experiencing a welfare gain equal to

$$-\ln(t + (1 - t)\rho) \tag{6}$$

On the other side, each type-2 agent faces a reduction in her/his consumption, and she/he suffers a loss that amounts to

$$\rho \ln \frac{\rho}{t + (1 - t)\rho} \tag{7}$$

Equations (6) and (7) imply that the social gain experienced by the society depends on two parameters with finite upper and lower bounds; indeed, the algebraic sum of Equations (6) and (7) is given by the following expression:

$$(1 - t)\rho \ln \rho - (t + (1 - t)\rho)\ln(t + (1 - t)\rho) \tag{8}$$

As illustrated in the three-dimensional diagram of Figure 1, for all the eligible values of t and ρ, the social welfare gain from switching from a decentralised to a centralised solution is always positive by corroborating the superiority of the allocations that become possible even in an atomistic environment with banking intermediation. Obviously, this means that the welfare gain experienced by all the type-1's agents always exceeds the loss suffered by all the type-2's agents.

The expression in Equation (8) has two additional intriguing properties. First, the social welfare gain is an increasing (decreasing) function of the discount rate (factor) of type-2's agents and such a social welfare gain vanishes as ρ tends to unity as well as when t is equal to 1 or to 0. These results are far from surprising; indeed, when the discount factor tends to unity and—a fortiori—when there is no heterogeneity among the two types of agents, the transfer which is required to shift from a decentralised to a centralised solution becomes pointless, and the same will hold for banking intermediation. Moreover, for each value of ρ, there is a corresponding value of t that maximises the social welfare gain. Such a critical share of type-1's agents is equal to

$$\hat{t} = \frac{\exp\left(\frac{\rho(1 - \ln\rho) - 1}{1 - \rho}\right) - \rho}{1 - \rho} \tag{9}$$

Some interesting considerations about the economic interpretation of the expression in Equation (9) come from its evaluation at the extreme values of ρ. On the one hand, when ρ tends to $1/R$, $\hat{t}$ amounts to $(R\exp((1 - R - \ln R^{-1})/(R - 1)) - 1)/(R - 1)$ which is a monotonically decreasing function of R. Specifically, when R tends to 1 (infinity), $\hat{t}$ tends to $1/2$ ($\exp(-1) \cong 0.3678$). Interestingly, when the discount rate of more patient agents approaches its upper bound and—at the same time—the return on the risk-free project approaches its lower bound, the achievement of the maximum possible level of the social welfare gain requires that the proportions of the two types of agents tend to be equal. It is worth noticing that such a critical level of t is exactly the value that maximises the variance the Bernoulli distribution of the two types of agents. Consequently, when type-2 agents struggle to postpone consumption and the return on the risk-free project is low, the maximisation of the social welfare gain requires the highest degree of heterogeneity among the Bernoulli population in the economy. On the other hand, when ρ tends to unity the maximum value of the vanishing social welfare gain is once again achieved when the different types of agents are equally distributed among the two categories no matter the value of R.

In the model economy described in Section 2 the percentage of impatient agents (t) as well as the discount factor of more patient agents (ρ) are treated as fundamentals that the social planner involved in the solution of the problem in Equation (4)—or, eventually, a policy maker—cannot modify. However, in cost-benefit analyses as well as in experimental contributions, discount factors for postponed consumption streams are usually allowed to vary over space and time (cf., Arrow et al. 1996; Lloyd-Smith et al. 2021). Consequently, according to the theoretical analysis carried out above, whenever the allocations of a centralised equilibrium are actually implemented through banking intermediation, for a given value of t—which can be equal to $\hat{t}$ or not—there should be a positive link between the prevailing values of discount rates and the extensiveness of banking intermediation that may be empirically tested by finding a proxy for both references. An attempt to grasp such a relationship is sketched in Appendix A.

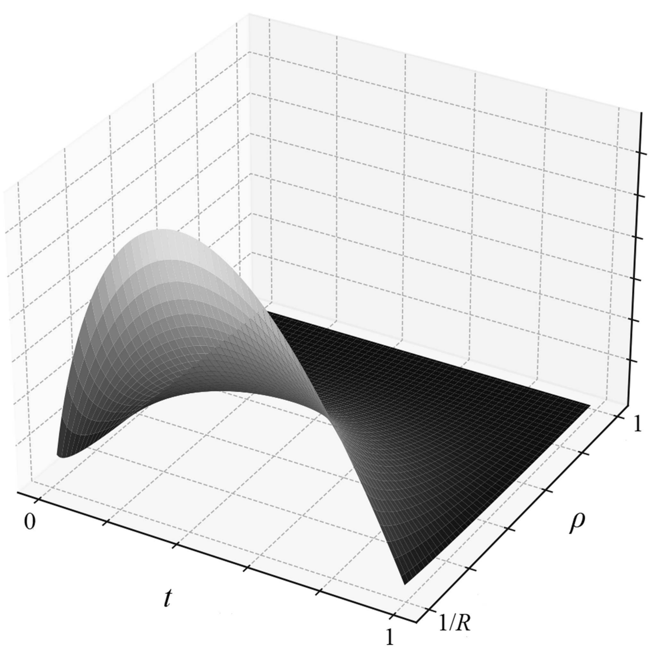

FIGURE 1 | Social welfare gain.

4 | Concluding Remarks

In this paper, drawing on the Diamond and Dybvig (1983) model of bank runs, I evaluated the private and the social welfare gains arising from switching from a decentralised to a centralised equilibrium that can be also achieved through banking intermediation. Assuming logarithmic preferences, I showed that the implied social welfare gain is an increasing function of the discount rate of more patient agents and that for each value of that discount rate there is an optimal level of the proportion of these agents for which the social welfare gain achieves its maximum level. These findings suggest that the implementation and the extensiveness of banking intermediation is more likely in economic environments characterised by substantial discount rates and a certain degree of heterogeneity in liquidity needs.

Recalling the expressions in Equation (5), when the function $u(\bullet)$ is a constant-relative-risk-aversion (CRRA) function with a value of γ strictly higher than 1, the social welfare gain arising from switching to a centralised equilibrium is still and increasing function of the discount rate of more patient agents but its level depends also on the unbounded values of R and γ. In this more general scenario, following the same procedure implemented above, it would be possible to show that for given values of t, ρ and R there is an optimal value of γ—say $\hat{\gamma}$—that maximises the social welfare gain arising from switching to a centralised equilibrium; indeed, for values of γ higher than $\hat{\gamma}$ the social welfare gain tends to vanish, whereas for values of γ lower than $\hat{\gamma}$ the social welfare gain shrinks by tending to the level achieved with logarithmic preferences. Moreover, for given values of t, ρ and γ, the value of the social welfare gain increases with R by tending to a positive upper bound equal to $t/(1-\gamma)(t^{\gamma-1}-1)$.[3]

The CRRA patterns described above suggest a positive observation and a normative issue. On the one hand, as a far as the behaviour of the social welfare gain with respect to γ is concerned, it is interesting to notice that whenever such a parameter tends to infinity the two types of agents end up to consuming the same amount of resources in the centralised equilibrium. Specifically, according to the expressions in Equation (5), when relative-risk-aversion index tends to infinity it holds that $c_1^{1*} = c_2^{2*} = c^* \equiv R/(1+t(R-1))$. In this case, since $c^* > 1$ and $c_1^{2*} = 0$, type-1 agents still do not envy type-2 agents (cf., Diamond and Dybvig 1983). However, given that $c_1^{2*} + c_2^{2*} = c^* = c_1^{1*} = c_1^{1*} + c_2^{1*}$ type-2 agents are indifferent with respect to the consumption bundle assigned to type-1 agents. Such an asymmetry may invalidate the process of self-selection between the two categories of agents by questioning the feasibility of the balanced pair $c_1^{1*} = c_2^{2*} = c^*$. Obviously, this may happen because some patient agents may decide to withdraw early in period 1 instead of waiting until the next period. Nonetheless, whenever γ tends to infinity, it would be possible to show that even when all the type-2 agents withdraw in period 2 there would be no gain to leave the decentralised equilibrium, because under unbounded risk-aversion the loss suffered by more patient agents is exactly equal to the gain experienced by type-1 agents.[4]

On the other hand, given the heterogenous preferences of depositors, the behaviour of the social welfare gain with respect to R suggests that over a certain range there may be a role to play for the central bank—when it discretionally sets interest rates on which the return on the project should align—to influence the incentives to switch to an institutional setting in which banking intermediation is allowed.

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Ethics Statement

The author has nothing to report.

Conflicts of Interest

The author declares no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available from the author upon reasonable request.

Endnotes

[1] In the original contribution the desire for liquidity arises because depositors want to consume. Similar arguments, however, also apply when depositors are presented with a favourable investment opportunity (cf., Holmström and Tirole 1998; Diamond and Rajan 2001).

[2] According to the Keynes-Ramsey rule, the inequality $\rho R > 1$ recalls that type-2 agents—in a dynamic context—should find profitable to increase their consumption (cf., Ramsey 1928; Cass 1965).

[3] Interestingly, while the welfare gain of type-1 agents is a monotonically increasing function of R, the loss of type-2's agents displays a u-shaped pattern with respect to that parameter. Consequently, for given values of t, ρ and γ, there is a value of R that maximises the loss of type-2's agents.

[4] Technical details are available from the author upon reasonable request.

[5] As far as the euro area is concerned, the series for the final consumption expenditure in constant 2015 US dollars (NE.CON.TOTL.KD) as well the one for the real interest rate (FR.INR.RINR) is provided by the World Bank (https://data.worldbank.org). Moreover, the series for the bank deposits to GDP (DDOI02EZA156NWDB) is provided by the Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org). The smoothing parameter of the HP filter has been set at the value of 6.25 which is consistent with annual data (cf., Ravn and Uhlig 2002). Negative figures of the proxy adopted for the discount rate are the upshot of the numerical procedure exploited to retrieve the trend when small values of the difference between the real interest rate and the growth rate of consumption are observed. Moreover, the sharp increase in the measure of the degree of impatience observed from 2007 to 2008 is realistically due to the strong cyclical component induced by the financial crisis not completely captured by the procedure used to extract the trend (cf., Hall 2017).

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Appendix A

Here I sketch an empirical test for the positive relationship between discounts rates and the incentives towards banking intermediation implied by the theoretical analysis carried out in the main text. On the one side, in the absence of systematic changes in the curvature of the utility function mirrored by the indexes of risk aversion, a proxy for the rate of intertemporal preference can be numerically retrieved by referring to a generic Euler equation for aggregate consumption according to which—in each period—the discount rate should be proportional to the difference between the observed real interest rate and the realised growth rate of consumption expenditure (cf., Cass 1965). Such a relation of proportionality follows from the fact that for any concave utility function, along the optimal path, the rate of reduction of the marginal utility of consumption has to be exactly equal to the difference between the real interest rate and the discount rate (cf., Ramsey 1928).

On the other side, a financial indicator that in macroeconomic studies has been often taken as a proxy for the pervasiveness of banking intermediation is certainly given by the share of bank deposits over GDP (cf., Rajan and Zingales 2003). Aiming at removing the cyclical components of the two involved series, the diagram in Figure A1 plots the relationship between the Hodrick-Prescott (HP) trend of the estimated proxy for the discount rates and the corresponding HP trend of the actual bank deposits-GDP ratio for the euro area over the period 1999–2015.[5]

The diagram in Figure A1 shows that in the countries that are adopting the euro there is a significant positive relationship between the trend components of the suggested proxy for discount rates and the share of bank deposits as a percentage of GDP mirrored by the value of the slope of the regression line reported together with its standard error in parenthesis. Consequently, on a time-series perspective, inferring the values of discount rates from an hypothetical Euler equation for aggregate consumption under the hypothesis of constant risk aversion, and taking the share of GDP represented by bank deposits as a measure for the extensiveness of banking intermediation, the available data for the euro area have a flavour of consistency with the theoretical pattern of the discount factor and social welfare gain achieved from switching from decentralised to the centralised allocations attainable with banking intermediation tracked in Figure 1.

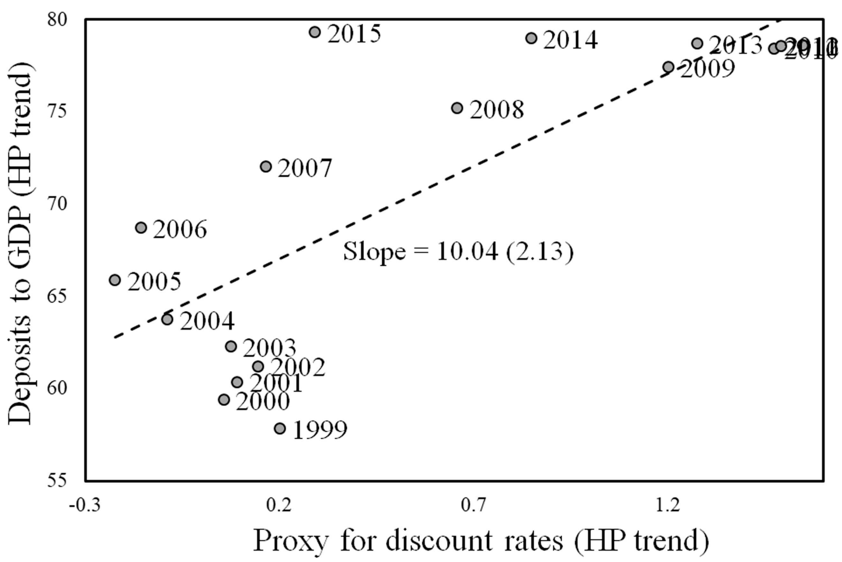

FIGURE A1 | The relationship between a proxy of discount rates and bank deposits to GDP.